1. Binomial distribution

- Four conditions for a binomial random variable:
  - Binary outcomes: there are two possible outcomes for each trial (success and failure)
  - Independent trials: the outcomes of the trials are independent of each other
  - $n$ is fixed: the number of trials, $n$, is fixed in advance
  - Same value of $p$: the probability of a success on a single trial is the same for all trials

- The binomial distribution formula
  For a binomial random variable, the probability that the $n$ trials result in $j$ successes (and $n - j$ failures) is given by the following formula:
  $$Pr\{j \text{ successes}\} = nC_j p^j (1 - p)^{n-j}$$
  where $nC_j = \frac{n!}{j!(n-j)!}$ and $x! = x(x-1)(x-2)...(2)(1)$, $0! = 1$.
  $nC_j$ has some properties:
  $$nC_0 = nC_n = 1$$
  $$nC_j = nC_{n-j}$$

- Properties of binomial distribution:
  - expectation (or mean) of $X$ is $np$.
  - variance is $np(1 - p)$; standard deviation is $\sqrt{np(1 - p)}$
  - the binomial distribution is symmetric if and only if $p = 0.5$
2. Normal distribution

- If $Y$ follows a normal distribution with mean $\mu$ and standard deviation $\sigma$, then it is common to write $Y \sim N(\mu, \sigma)$. Its density function is

$$f(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(\frac{y-\mu}{\sigma})^2}$$

- By standardization formula

$$Z = \frac{Y - \mu}{\sigma}$$

random variable $Z$ has density function

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

which is called standard normal distribution with mean 0 and standard deviation 1.

- $\Pr\{Z\text{ is between } a\text{ and } b\} = \text{area under the standard normal curve between } a\text{ and } b$. The table in the book gives the area under the normal curve below a specified value of $z$.

$\Pr\{Z \leq z\} = \text{area to the left of } z$ (given in table)

$\Pr\{Z \geq z\} = \text{area to the right of } z = 1 - \text{area to the left of } z$

$\Pr\{a \leq Z \leq b\} = \text{area to the left of } b - \text{area to the left of } a$

- Given a probability $\alpha$, from the normal table we can get $Z_\alpha$ such that $\Pr\{Z \leq Z_\alpha\} = 1 - \alpha$, then $Y_\alpha = Z_\alpha \sigma + \mu$ satisfying that $\Pr\{Y \leq Y_\alpha\} = 1 - \alpha$. 
