

1. **Computing individual binomial probabilities.** For $n = 5, 10,$ or $20,$ and $p = 0.1, 0.5,$ and $0.7,$ find the probability of exactly three successes. There are nine different problems here. Just do enough to be confident that you could do any single similar problem.

Here is a detailed solution to the first. For $n = 5$ and $p = 0.1,$

$$\Pr\{Y = 3\} = {}_5C_3(0.1)^3(0.9)^2 = \frac{5!}{2!3!}(0.1)^3(0.9)^2 = \frac{5 \cdot 4 \cdot 3!}{2 \cdot 1 \cdot 3!}(0.1)^3(0.9)^2 = 20(0.1)^3(0.9)^2 \doteq 0.0081.$$

Here are all of the answers.

Binomial Probabilities

n	p		
	0.1	0.5	0.7
5	0.0081	0.3125	0.3087
10	0.0574	0.1172	0.009
20	0.1901	0.0011	0

2. **Computing sums of binomial probabilities.** For $n = 5, 10$ and $15,$ and $p = 0.25,$ find the probability of: (a) two or more successes; and (b) two or fewer successes.

Here is a solution to (a) when $n = 5.$

$$\begin{aligned} \Pr\{Y \geq 2\} &= 1 - \Pr\{Y < 2\} = 1 - \Pr\{Y \leq 1\} = 1 - \Pr\{Y = 0\} - \Pr\{Y = 1\} \\ &= 1 - (0.75)^5 - 5(0.25)(0.75)^4 \doteq 1 - 0.2373 - 0.3955 = 0.3672 \end{aligned}$$

If you round off individual probabilities to four digits, you should find these answers: $n = 5,$ (a) 0.3672, and (b) 0.8965; $n = 10,$ (a) 0.756, and (b) 0.5256; $n = 15,$ (a) 0.9198, and (b) 0.2361;

3. **Finding areas under normal curves.** Consider a normal distribution with mean $\mu = 125$ and standard deviation $\sigma = 14.$

- (a) Find the area to the left of 100. [0.0367]
- (b) Find the area to the right of 140. [0.1423]
- (c) Find the area between 110 and 150. [0.9633 - 0.1423 = 0.821]
- (d) Find the probability of being more than 20 from the mean. [$2 \times 0.0764 = 0.1528$]

4. **Finding quantiles from normal curves.** Consider a normal distribution with mean $\mu = 125$ and standard deviation $\sigma = 14.$

- (a) Find the 20th and 70th percentiles. [$z = -0.84, z = 0.52, 113.2, 132.3$]
- (b) Find the values that cut off the middle 99 percent. [$z = \pm 2.575, 88.9, 161.1$]

5. **Sampling distribution of $\bar{Y}.$** Consider a random sample of size $n = 15, 20, 25, 30$ from a normal population with $\mu = 125$ and standard deviation $\sigma = 14.$

- (a) For each $n,$ find the probability that the sample mean is within one of the population mean. [0.2206, 0.251, 0.2812, 0.3034.]
- (b) For each $n,$ find the 0.975 quantile. [132.1, 131.1, 130.5, 130.]