CS559: Computer Graphics

Lecture 35: Shape Modeling

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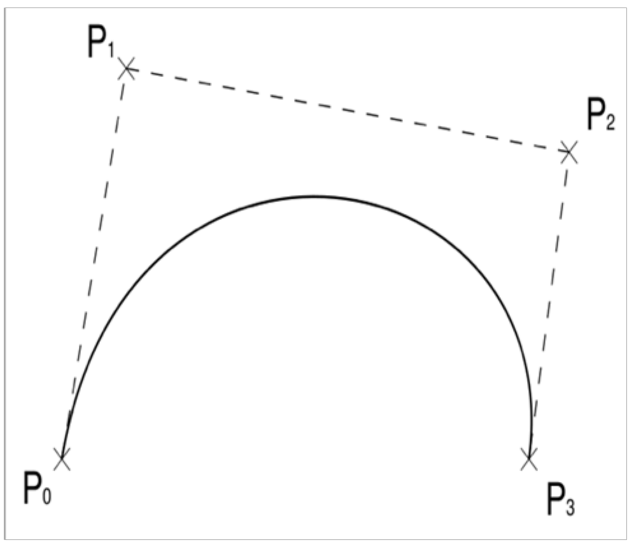
Today

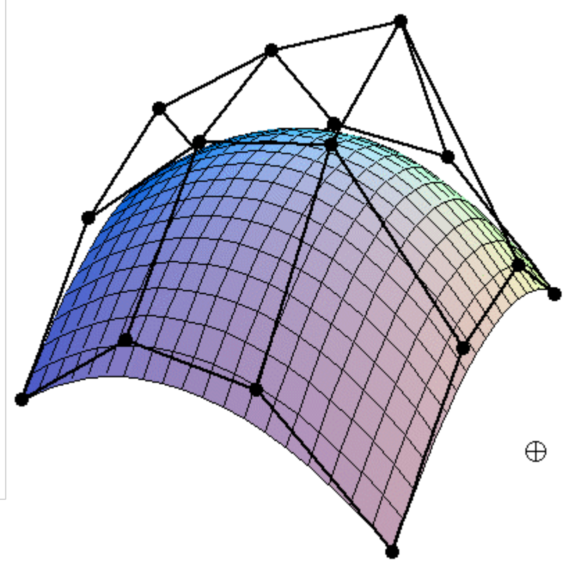
- Shape Modeling

- Reading
  - Real-Time Rendering, 3e, 13.2.1 (except Rational Bezier Patches)
    - Linux: /p/course/cs559-lizhang/public/readings/13_surfs_gleicher.pdf
    - Windows: P:\course\cs559-lizhang\public\readings\13_surfs_gleicher.pdf

Parametric surface

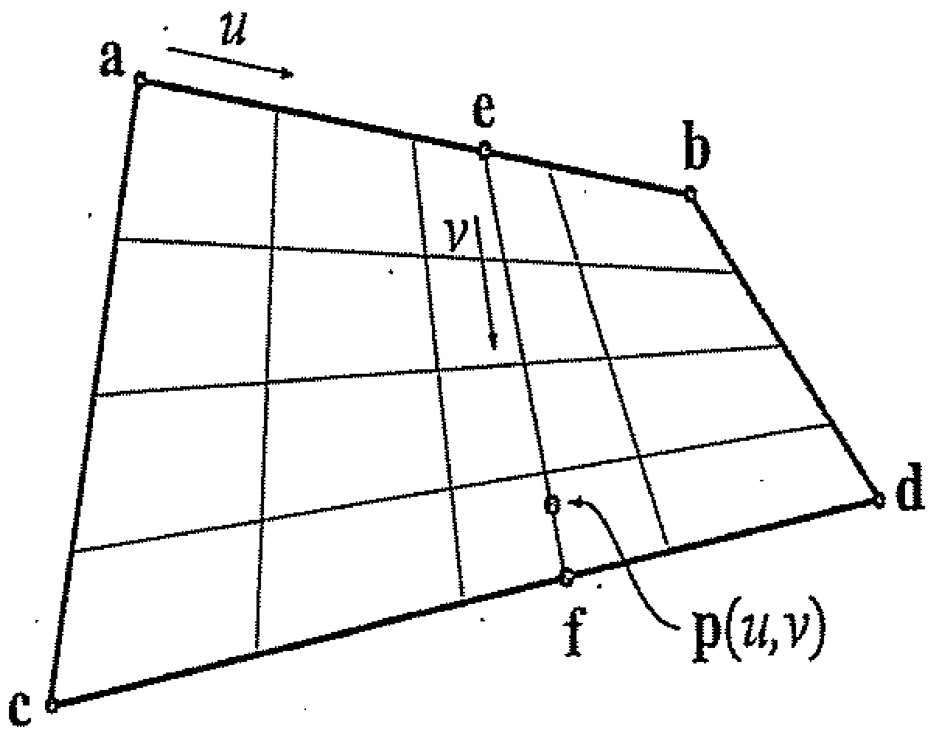
- Line Segments (1D) -> polygon meshes (2D)
- Cubic curves (1D) -> BiCubic Surfaces (2D)
  - Bezier curve -> Bezier surface





Bilinear Bezier Patch

- Define a surface that passes through a, b, c, d?



$$\mathbf{e} = (1-u)\mathbf{a} + u\mathbf{b},$$

$$\mathbf{f} = (1-u)\mathbf{c} + u\mathbf{d}.$$

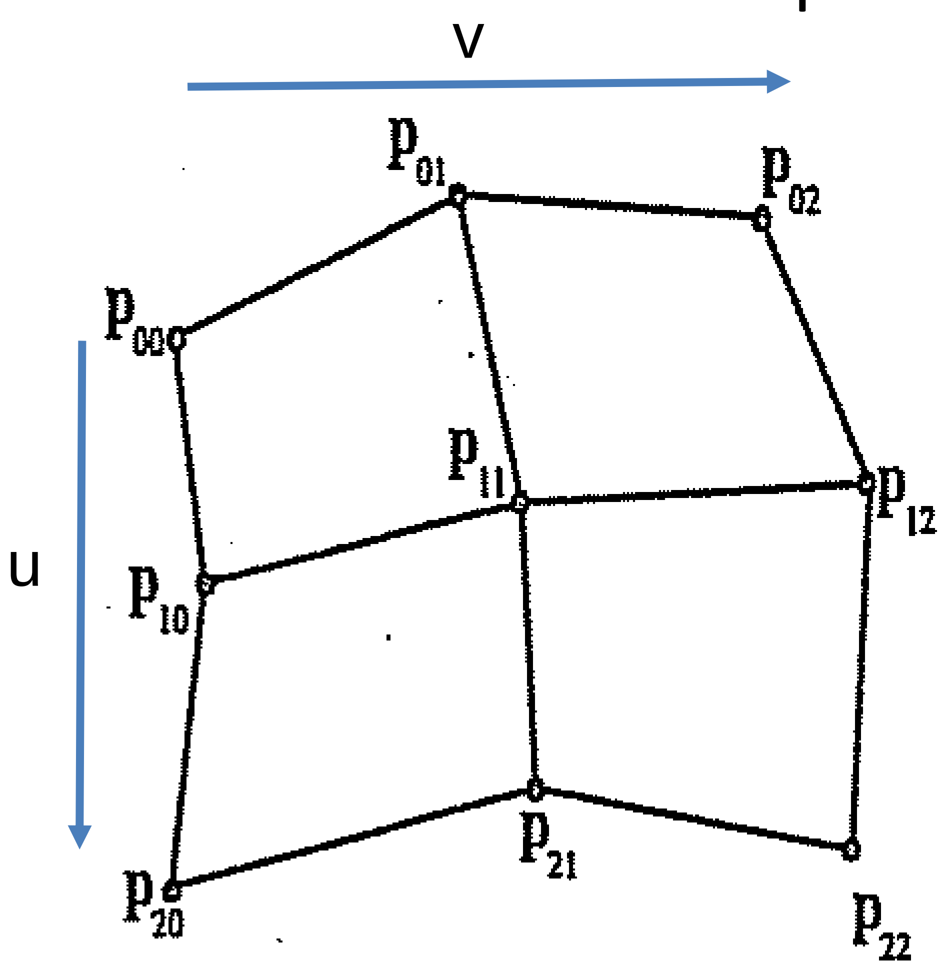
Looks familiar?

$$\mathbf{p}(u,v) = (1-v)\mathbf{e} + v\mathbf{f}$$

$$= (1-u)(1-v)\mathbf{a} + u(1-v)\mathbf{b} + (1-u)v\mathbf{c} + uv\mathbf{d}.$$

Biquadratic Bezier Patch

- Define a surface that passes a 3x3 control lattice.



$$p(u,0) = (1-u)^2\, p_{00} + 2(1-u)u\, p_{10} + u^2\, p_{20}$$

$$p(u,1) = (1-u)^2\, p_{01} + 2(1-u)u\, p_{11} + u^2\, p_{21}$$

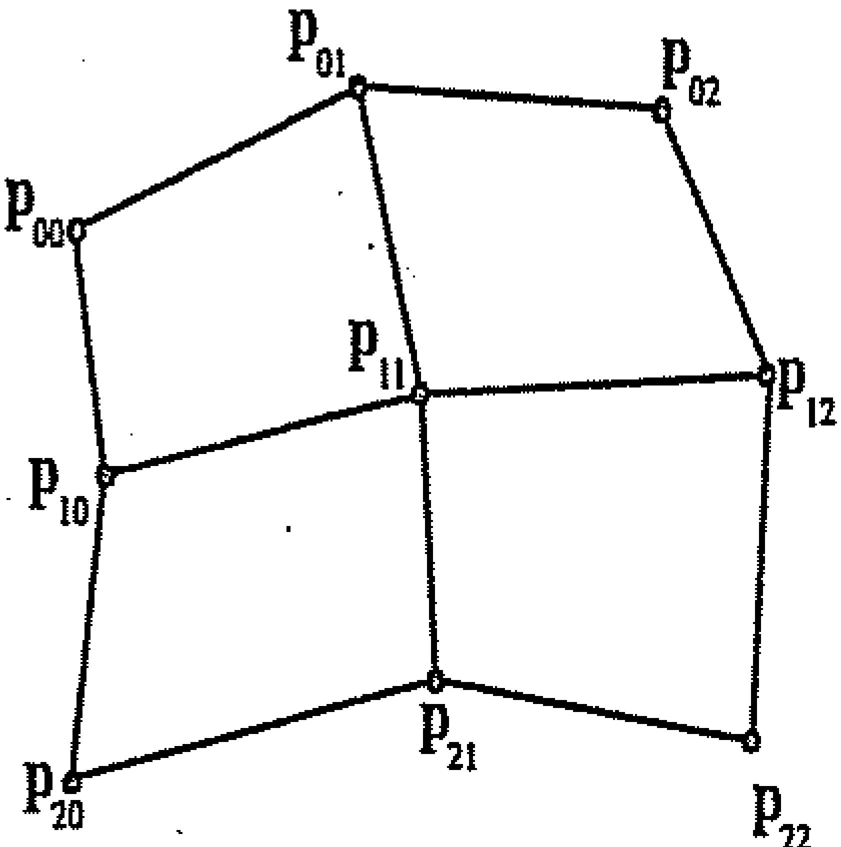
$$p(u,2) = (1-u)^2\, p_{02} + 2(1-u)u\, p_{12} + u^2\, p_{22}$$

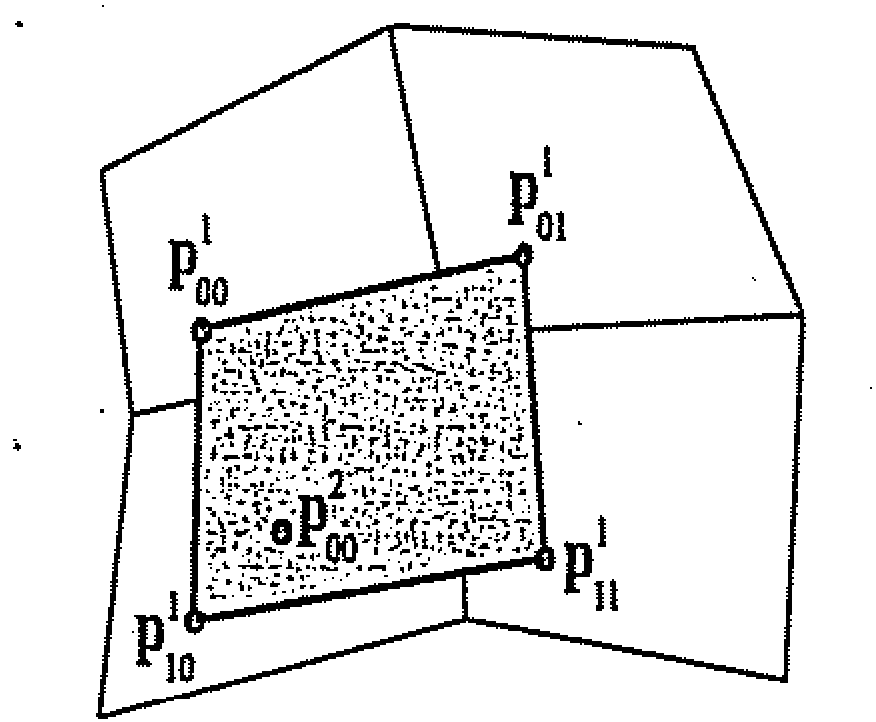
$$p(u,v) = (1-v)^2\, p(u,0) + 2(1-v)v\, p(u,1) + v^2\, p(u,2)$$

Bicubic Bezier Patch

- 4x4 control points?
- Demo: http://www.nbb.cornell.edu/neurobio/land/OldStudentProjects/cs490-96to97/anson/BezierPatchApplet/index.html
- Connecting Bezier Patches, demo on the same page.

De Casteljau algorithm in 2D





$p^1_{00}(u,v)$ = Bilinear(p_{00}, p_{10}, p_{01}, p_{11}; u, v)

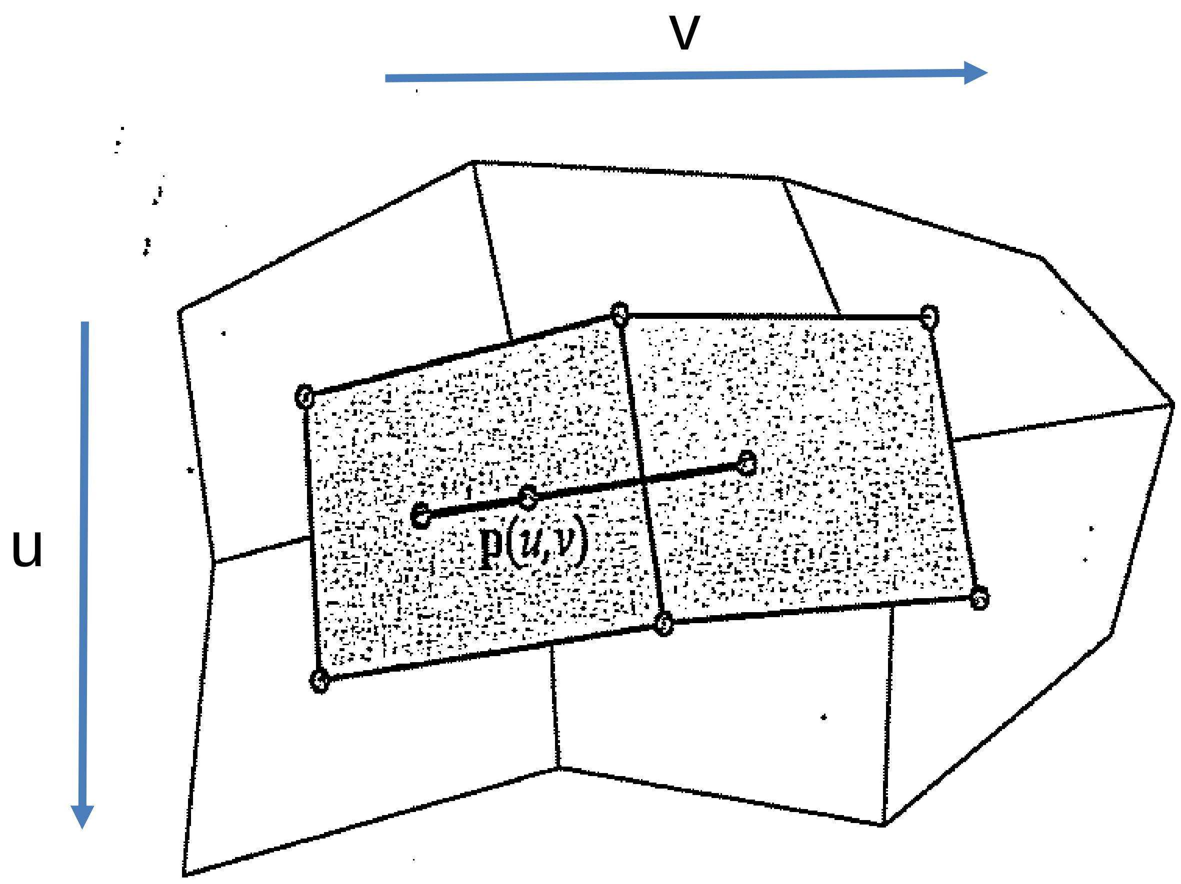
$p^1_{01}(u,v)$ = Bilinear(p_{01}, p_{11}, p_{02}, p_{12}; u, v)

$p^1_{10}(u,v)$ = Bilinear(p_{10}, p_{20}, p_{11}, p_{21}; u, v)

$p^1_{11}(u,v)$ = Bilinear(p_{11}, p_{21}, p_{12}, p_{22}; u, v)

$p^1_{00}(u,v)$ = Bilinear(p_{00}, p_{10}, p_{01}, p_{11}; u, v)

Different degree in different directions



General Formula for Bezier Patch

- If we have contronl points $p_{i,j}$ on a m by n lattice,

$$\mathbf{p}(u,v) = \sum_{i=0}^{m} B_i^m(u) \sum_{j=0}^{n} B_j^n(v)\mathbf{p}_{i,j} = \sum_{i=0}^{m}\sum_{j=0}^{n} B_i^m(u)B_j^n(v)\mathbf{p}_{i,j}$$

$$= \sum_{i=0}^{m}\sum_{j=0}^{n} \binom{m}{i}\binom{n}{j} u^i(1-u)^{m-i}v^j(1-v)^{n-j}\mathbf{p}_{i,j}$$

- Properties
  - Invariant to affine transform
  - Convex combination,
  - Used for intersection

$$\sum_{i=0}^{m}\sum_{j=0}^{n} B_i^m(u)B_j^n(v) = 1$$

General Formula for Bezier Patch

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$$= \sum_{i=0}^{m}\sum_{j=0}^{n} \binom{m}{i}\binom{n}{j} u^i(1-u)^{m-i} v^j (1-v)^{n-j}\mathbf{p}_{i,j}$$

- Surface Normal

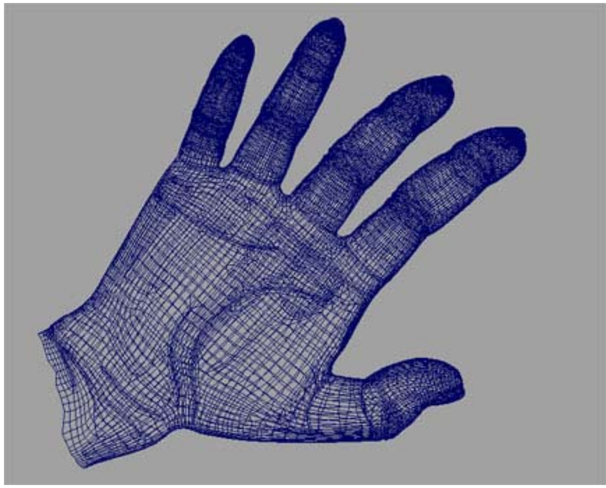
$$\frac{\partial \mathbf{p}(u,v)}{\partial u} = m\sum_{j=0}^{n}\sum_{i=0}^{m-1} B_i^{m-1}(u)B_j^n(v)[\mathbf{p}_{i+1,j}-\mathbf{p}_{i,j}]$$

$$\mathbf{n}(u,v) = \frac{\partial \mathbf{p}(u,v)}{\partial u} \times \frac{\partial \mathbf{p}(u,v)}{\partial v}.$$

$$\frac{\partial \mathbf{p}(u,v)}{\partial v} = n\sum_{i=0}^{m}\sum_{j=0}^{n-1} B_i^{m}(u)B_j^{n-1}(v)[\mathbf{p}_{i,j+1}-\mathbf{p}_{i,j}]$$

Issues with Bezier Patches

- With Bézier or B-spline patches, modeling complex surfaces amounts to trying to cover them with pieces of rectangular cloth.



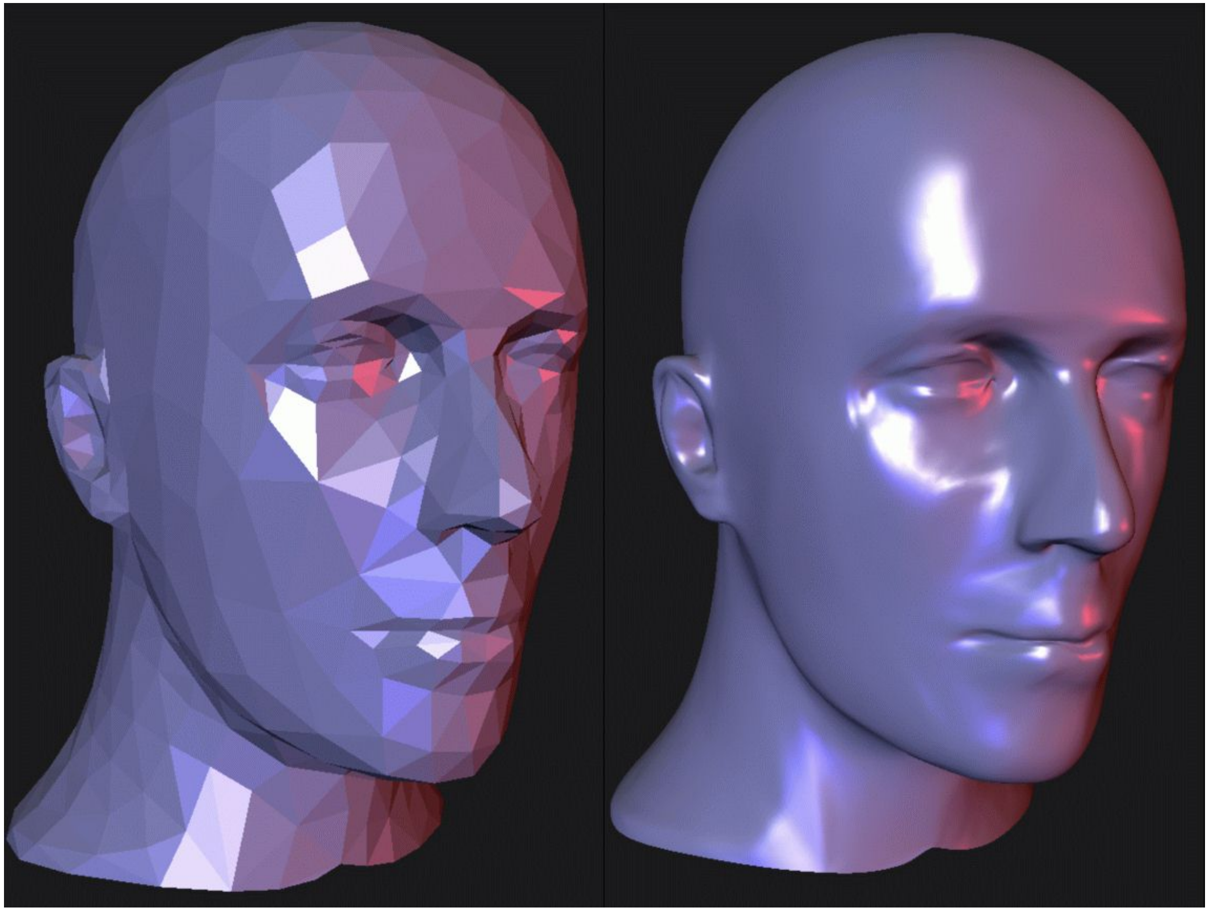
- It's not easy, and often not possible if you don't make some of the patch edges degenerate (yielding triangular patches).

- Trying to animate that object can make continuity very difficult, and if you're not very careful, your model will show creases and artifacts near patch seams.

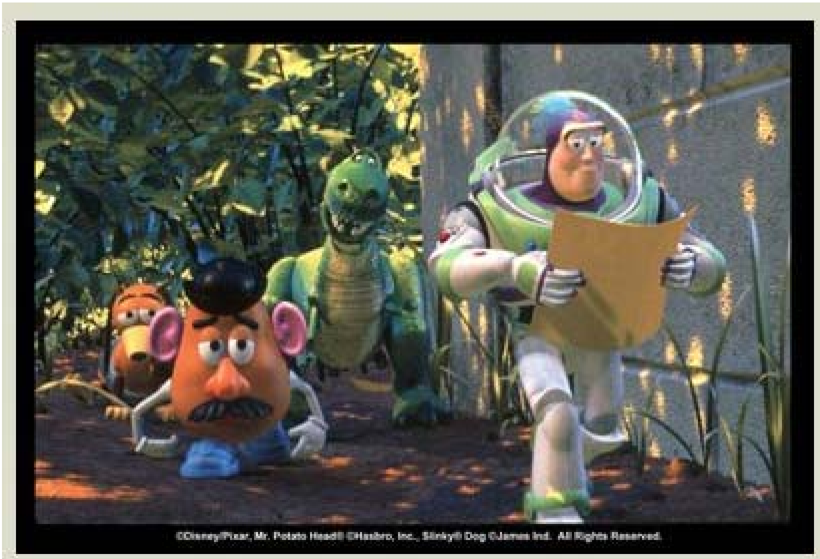
- Subdivision Surface is a promising solution.

Subdivision Surface

- From a coarse control mesh to smooth mesh with infinite resolution

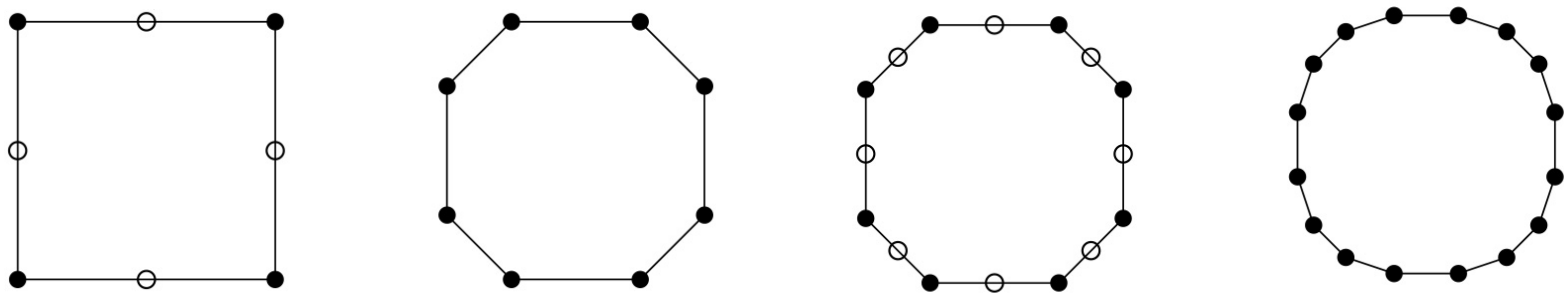


Example: Toy story 2

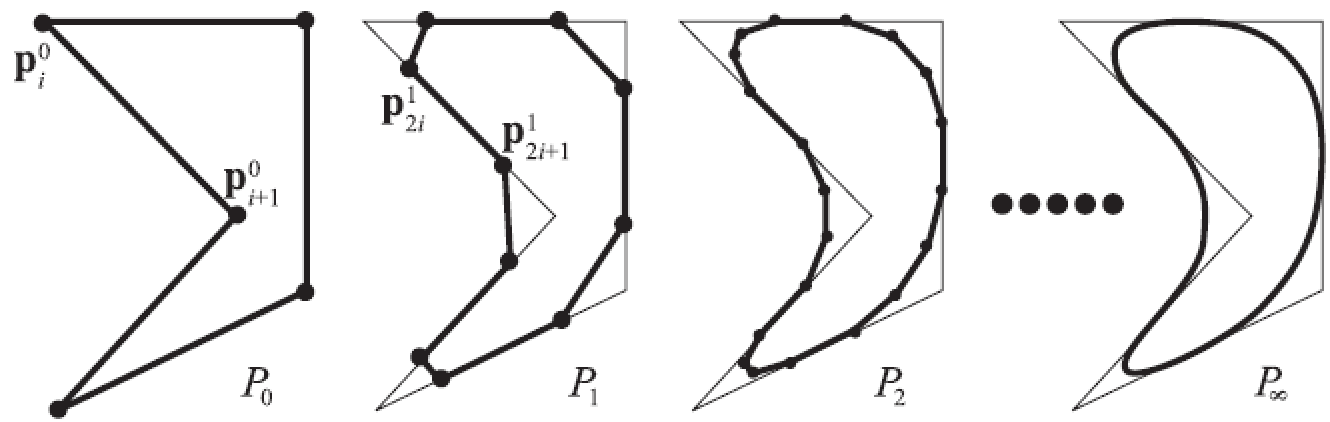


Subdivision Curve

We have seen this idea before



Shirley, Figure 15.15, The limiting curve is a *quadratic Bezier Curve*



RTR 3e, Figure 13.29, The limiting curve is a *quadratic B-spline*

Both are correct – why they say different things?