CS559: Computer Graphics

Lecture 38: Animation

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Spring 2008

Slides from Brian Curless at U of Washington

Today

- Computer Animation, Particle Systems

- Reading
  - (Optional) Shirley, ch 16, overview of animation

  - Witkin, *Particle System Dynamics*, SIGGRAPH ’01 course notes on Physically Based Modeling.

  - Witkin and Baraff, *Differential Equation Basics*, SIGGRAPH ’01 course notes on Physically Based Modeling.

Animation

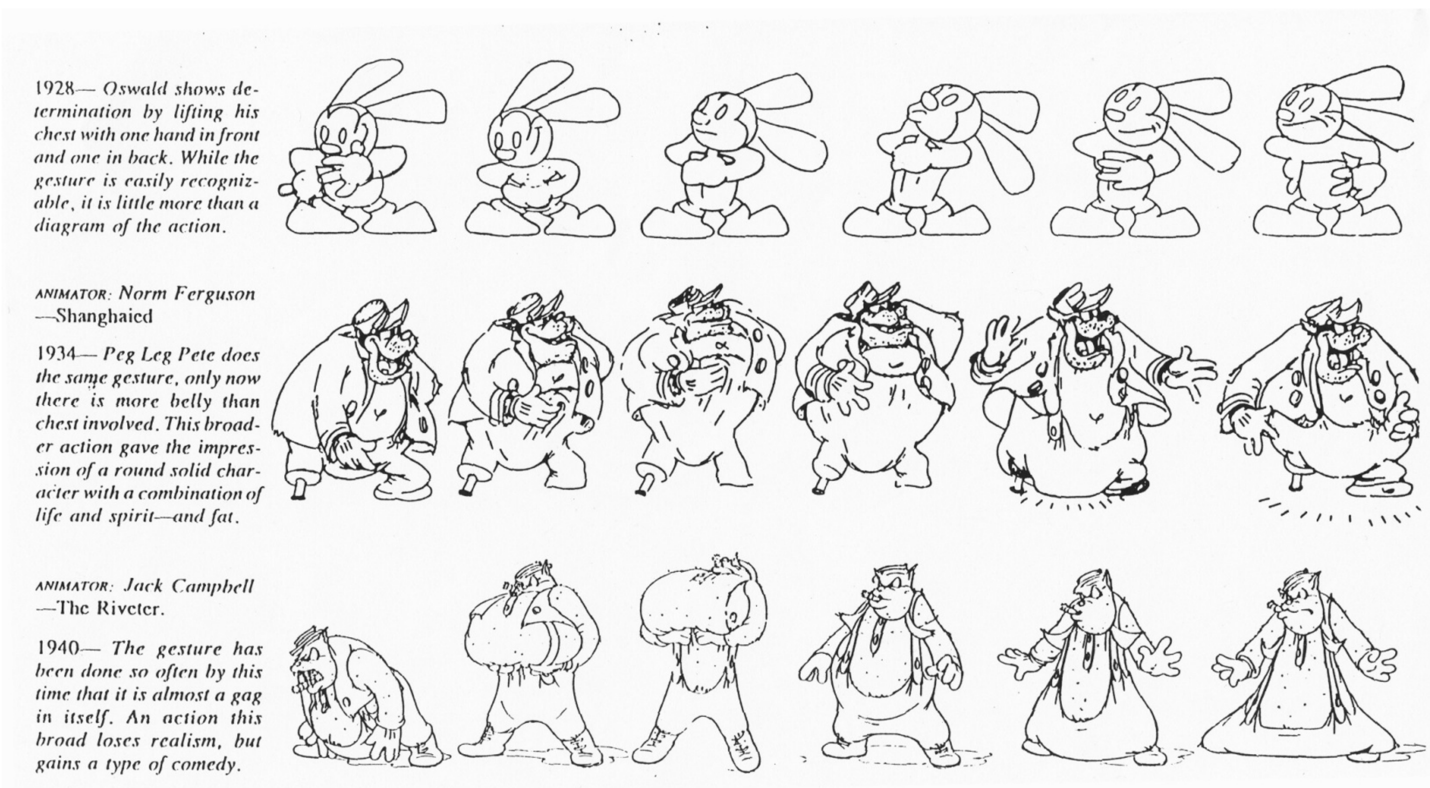
- Traditional Animation – without using a computer

Animation

- Computer Animation

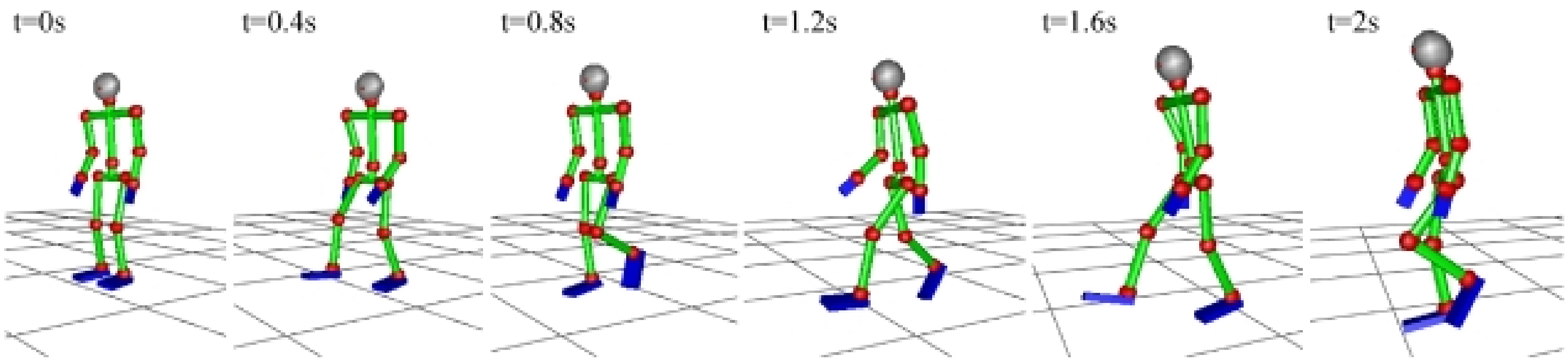
Types of Animation

- Cartoon Animation



Types of Animation

- Cartoon Animation
- Key Frame Animation



Types of Animation

- Cartoon Animation
- Key Frame Animation
- Physics based animation

Nguyen, D., Fedkiw, R. and Jensen, H., "Physically Based Modeling and Animation of Fire", SIGGRAPH 2002

Types of Animation

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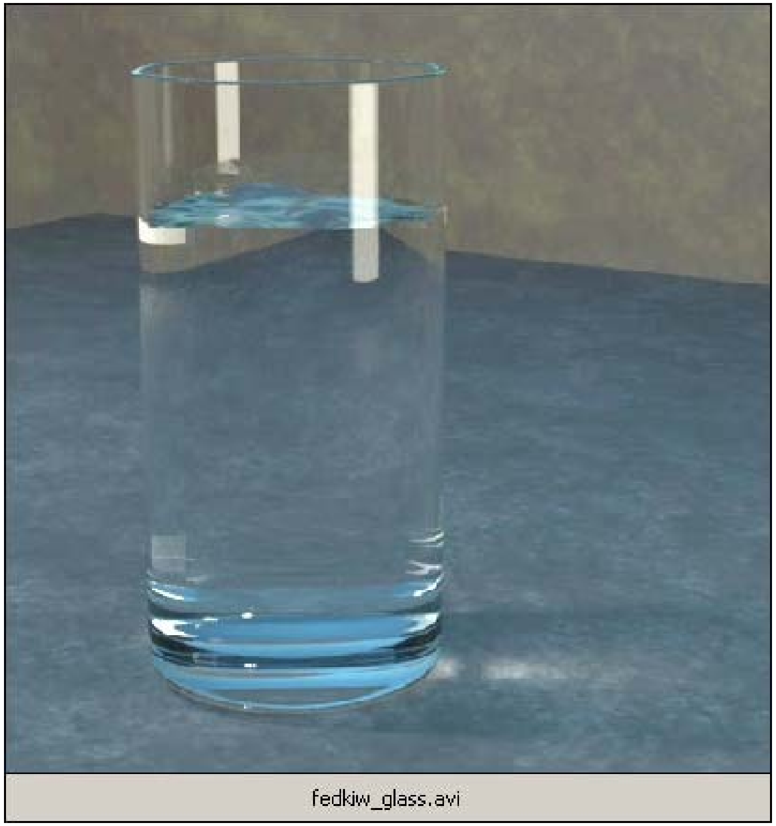
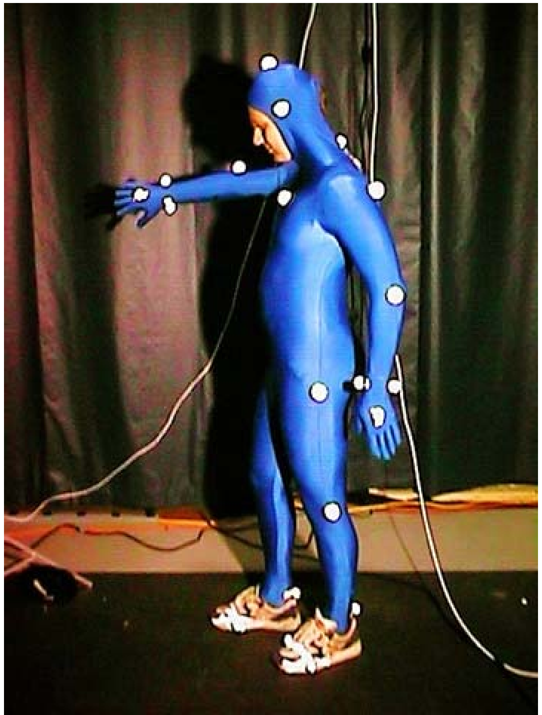


[Enright, D., Marschner, S. and Fedkiw, R., "Animation and Rendering of Complex Water Surfaces", SIGGRAPH 2002]

Types of Animation

- Cartoon Animation
- Key Frame Animation
- Physics based animation
- Data driven animation



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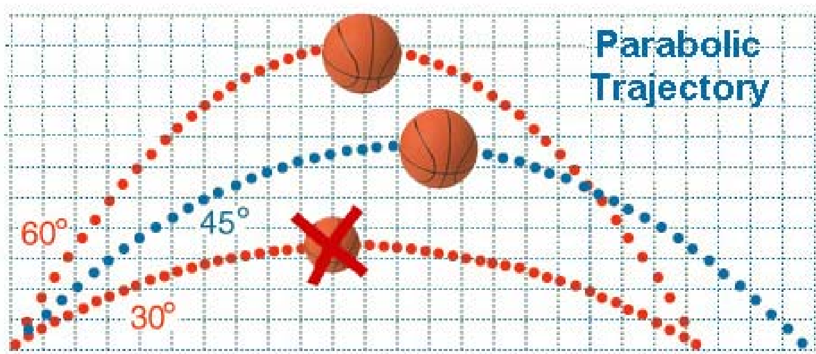
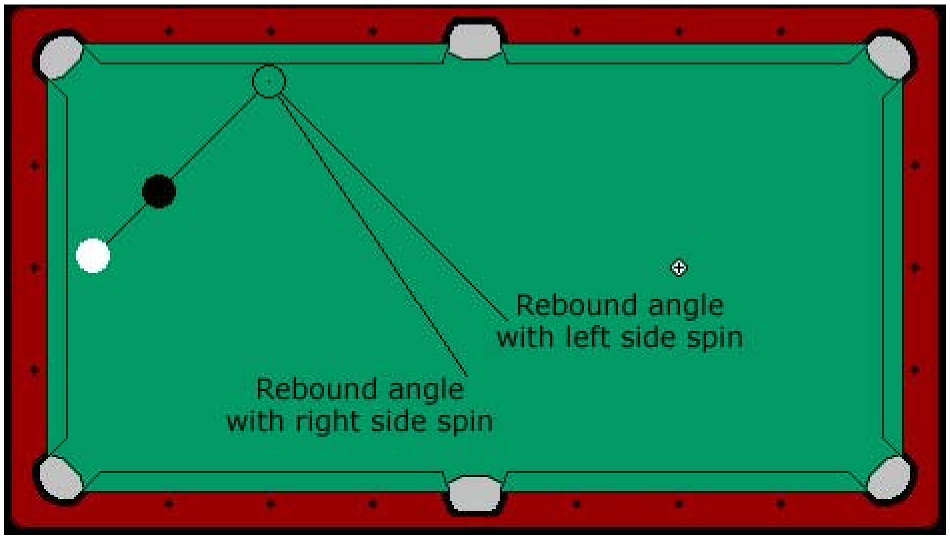
Particle Systems

- What are particle systems?
  - A **particle system** is a collection of point masses that obeys some physical laws (e.g, gravity, heat convection, spring behaviors, …).

- Particle systems can be used to simulate all sorts of physical phenomena:

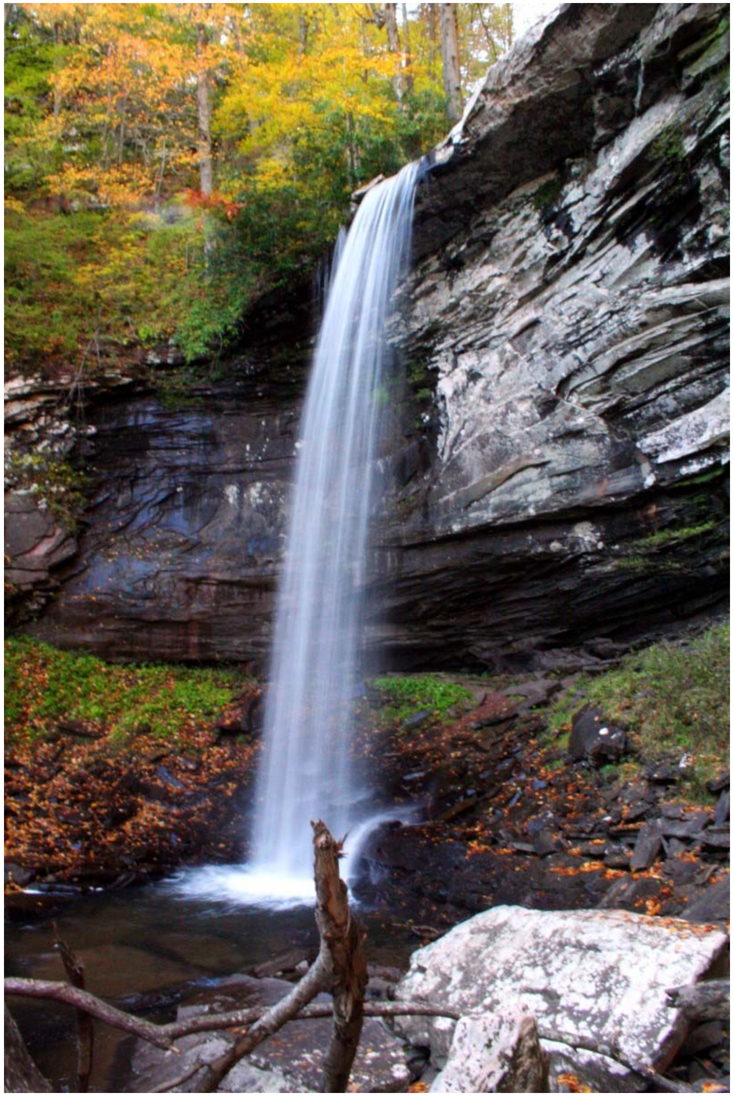
Balls in Sports



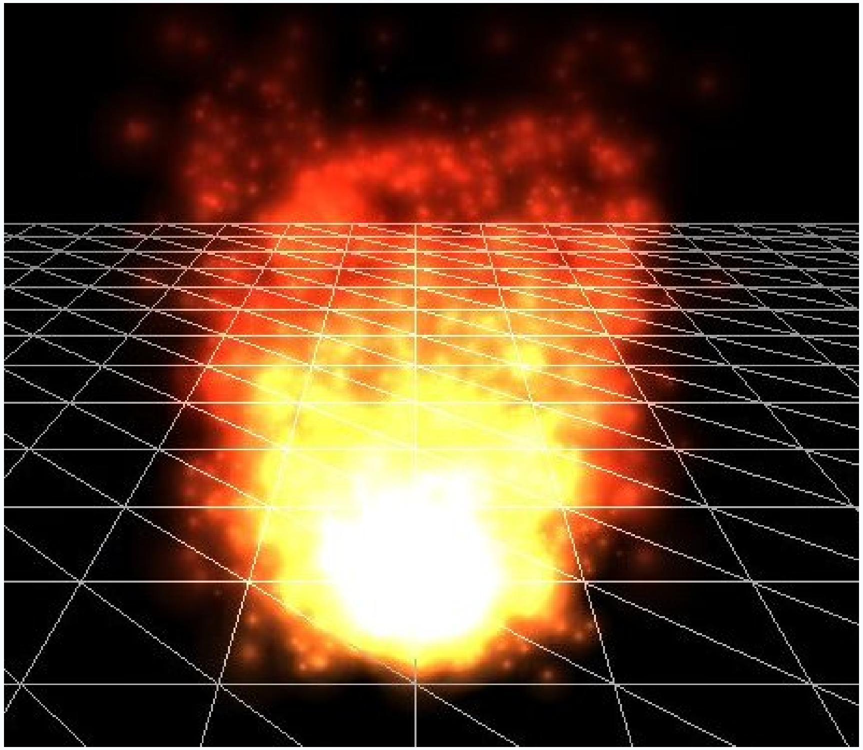


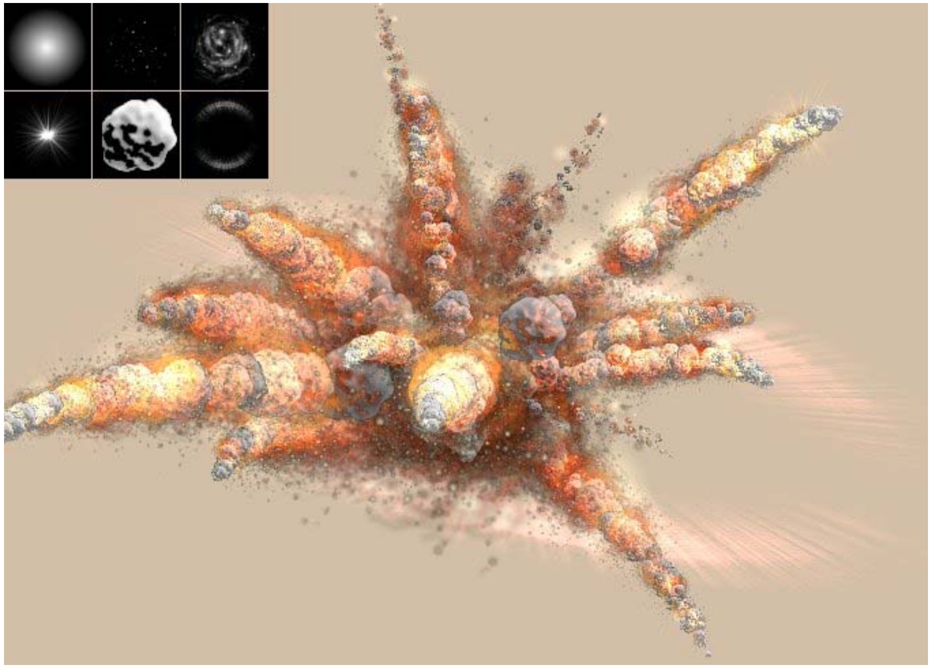
Fireworks

Water



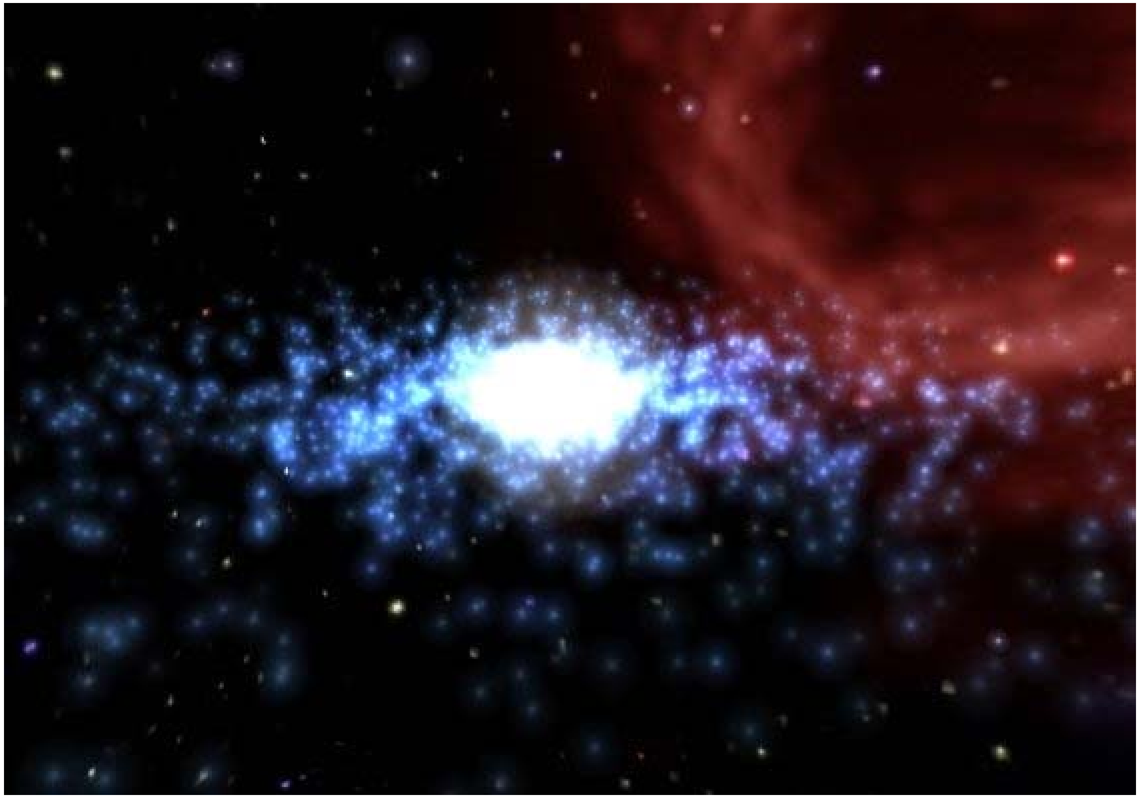
Fire and Explosion





http://en.wikipedia.org/wiki/Particle_system

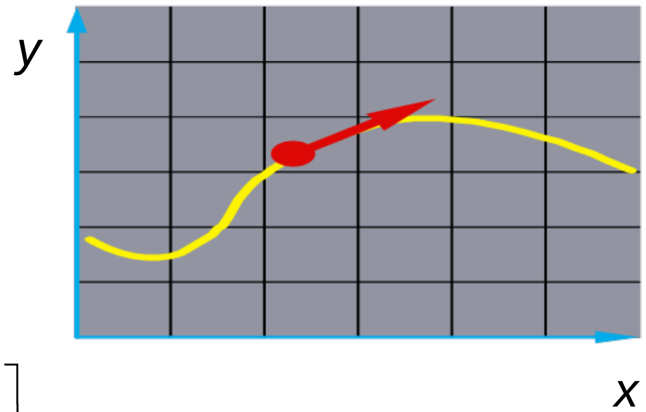
Galaxy



http://en.wikipedia.org/wiki/Particle_system

Particle in a flow field

- We begin with a single particle with:
  - Position, $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$



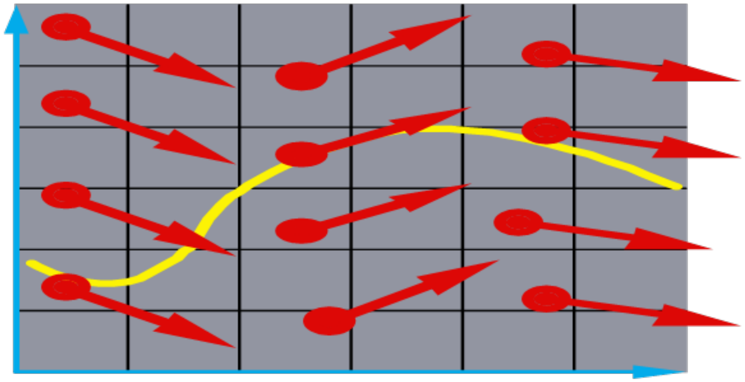
  - Velocity, $\mathbf{v} \equiv \dot{\mathbf{x}} = \frac{d\mathbf{x}}{dt} = \begin{bmatrix} dx/dt \\ dy/dt \end{bmatrix}$

- Suppose the velocity is actually dictated by some driving function **g**:

$$\dot{\mathbf{x}} = \mathbf{g}(\mathbf{x}, t)$$

Vector fields

- At any moment in time, the function **g** defines a vector field over **x**:
  - Wind
  - River



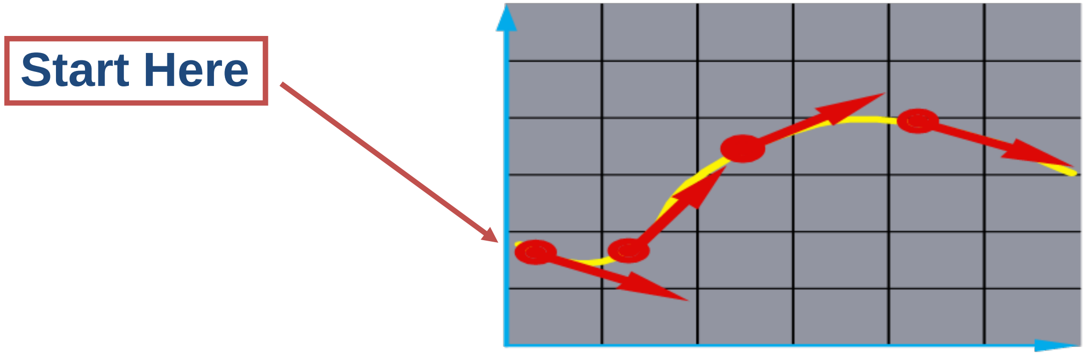
- How does our particle move through the vector field?

Diff eqs and integral curves

- The equation

$$\dot{\mathbf{x}} = \mathbf{g}(\mathbf{x}, t)$$

- is actually a **first order differential equation.**
- We can solve for **x** through time by starting at an initial point and stepping along the vector field:



- This is called an **initial value problem** and the solution is called an **integral curve**.
  - Why do we need initial value?

Euler's method

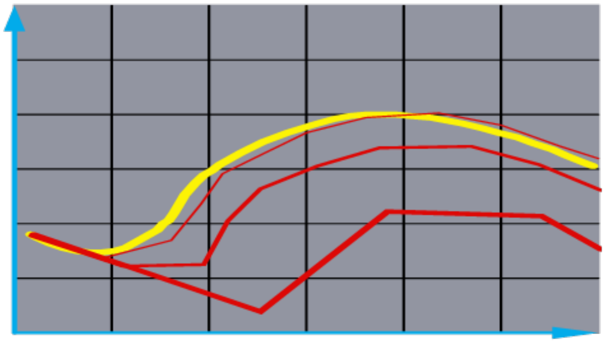
- One simple approach is to choose a time step, Δt, and take linear steps along the flow:

$$\begin{aligned} \mathbf{x}(t + \Delta t) &\approx \mathbf{x}(t) + \Delta t \cdot \dot{\mathbf{x}}(t) \\ &\approx \mathbf{x}(t) + \Delta t \cdot \mathbf{g}(\mathbf{x}, t) \end{aligned}$$

- Writing as a time iteration:

$$\mathbf{x}^{i+1} = \mathbf{x}^{i} + \Delta t \cdot \mathbf{v}^{i}$$

- This approach is called **Euler's method** and looks like:



- Properties:
  - Simplest numerical method
  - Bigger steps, bigger errors. Error ~ $O(\Delta t^2)$.
- Need to take pretty small steps, so not very efficient. Better (more complicated) methods exist, e.g., "Runge-Kutta" and "implicit integration."

Particle in a force field

- Now consider a particle in a force field **f**.
- In this case, the particle has:
  - Mass, *m*
  - Acceleration,

$$\mathbf{a} \equiv \ddot{\mathbf{x}} = \dot{\mathbf{v}} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{x}}{dt^2}$$

$$\mathbf{f} = m\mathbf{a} = m\ddot{\mathbf{x}}$$

- The particle obeys Newton's law:

$$\ddot{\mathbf{x}} = \frac{\mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}, t)}{m}$$

- The force field **f** can in general depend on the position and velocity of the particle as well as time.
- Thus, with some rearrangement, we end up with:

Second order equations

This equation:

$$\ddot{\mathbf{x}} = \frac{\mathbf{f}(\mathbf{x}, \mathbf{v}, t)}{m}$$

is a **second order differential equation**.

Our solution method, though, worked on first order differential equations.

We can rewrite this as:
$$\begin{bmatrix} \dot{\mathbf{x}} = \mathbf{v} \\ \dot{\mathbf{v}} = \dfrac{\mathbf{f}(\mathbf{x}, \mathbf{v}, t)}{m} \end{bmatrix}$$

where we have added a new variable **v** to get a pair
of coupled first order equations.

Phase space

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{v} \end{bmatrix}$$

- Concatenate **x** and **v** to make a 6-vector: position in **phase space**.

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{bmatrix}$$

- Taking the time derivative: another 6-vector.

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ \mathbf{f} / m \end{bmatrix}$$

- A vanilla 1st-order differential equation.

Differential equation solver

Starting with:

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ \mathbf{f}/m \end{bmatrix}$$

Applying Euler's method:

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \Delta t \cdot \dot{\mathbf{x}}(t)$$
$$\dot{\mathbf{x}}(t+\Delta t) = \dot{\mathbf{x}}(t) + \Delta t \cdot \ddot{\mathbf{x}}(t)$$

And making substitutions:

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \Delta t \cdot \mathbf{v}(t)$$
$$\mathbf{v}(t+\Delta t) = \dot{\mathbf{x}}(t) + \Delta t \cdot \frac{\mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}, t)}{m}$$

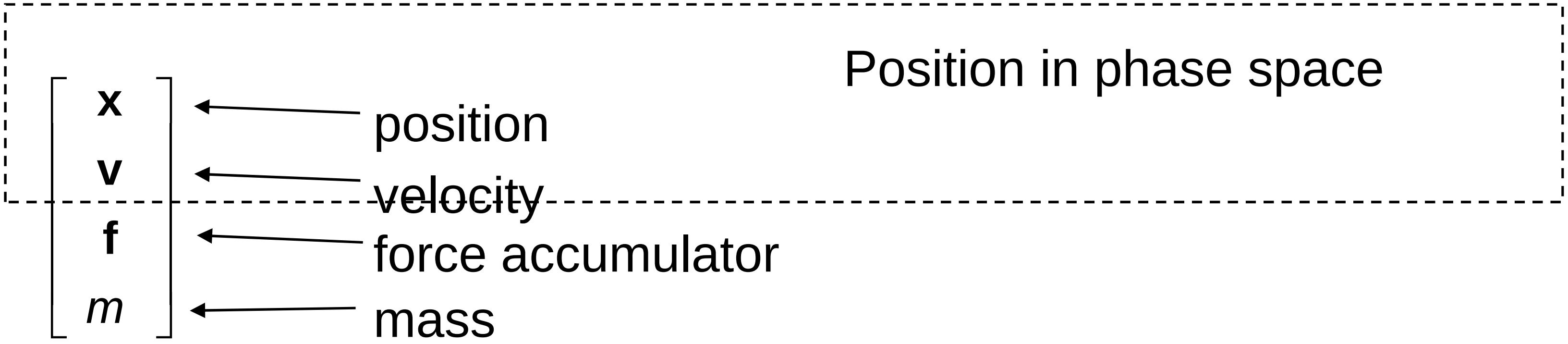
Writing this as an iteration, we have:

$$\mathbf{x}^{i+1} = \mathbf{x}^{i} + \Delta t \cdot \mathbf{v}^{i}$$
$$\mathbf{v}^{i+1} = \mathbf{v}^{i} + \Delta t \cdot \frac{\mathbf{f}^{i}}{m}$$

Again, performs poorly for large Δt.

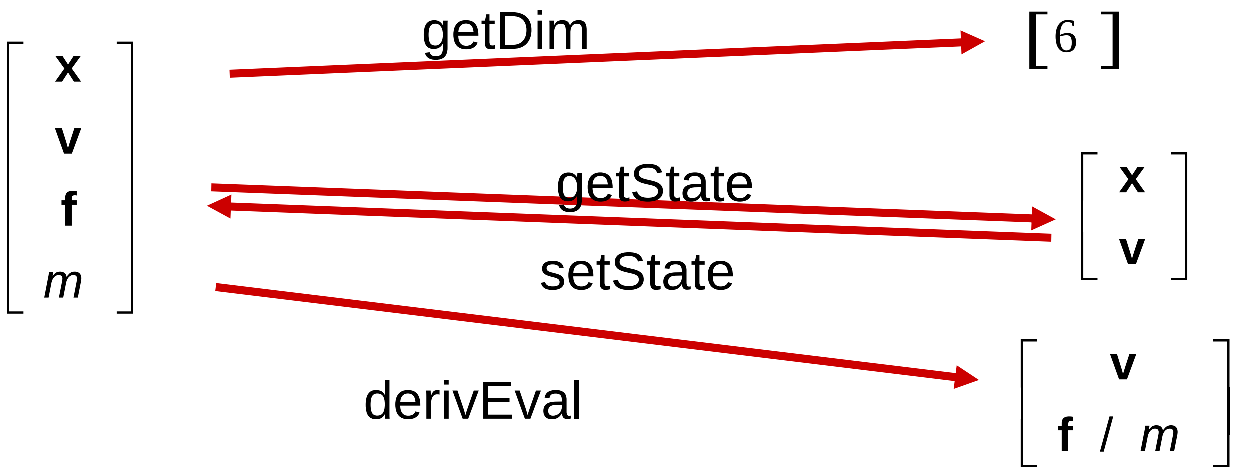
Particle structure

How do we represent a particle?




```
typedef struct{
float m; /* mass */
float *x; /* position vector */
float *v; /* velocity vector */
float *f; /* force accumulator */
} *Particle;
```

Single particle solver interface

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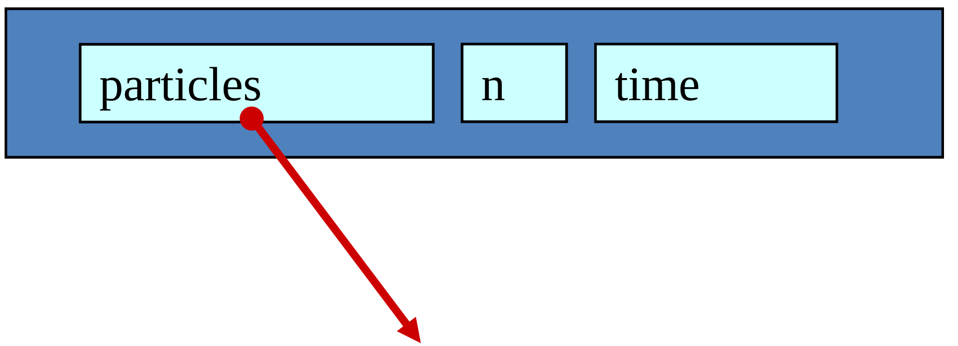
Particle systems

In general, we have a particle system consisting of *n* particles to be managed over time:



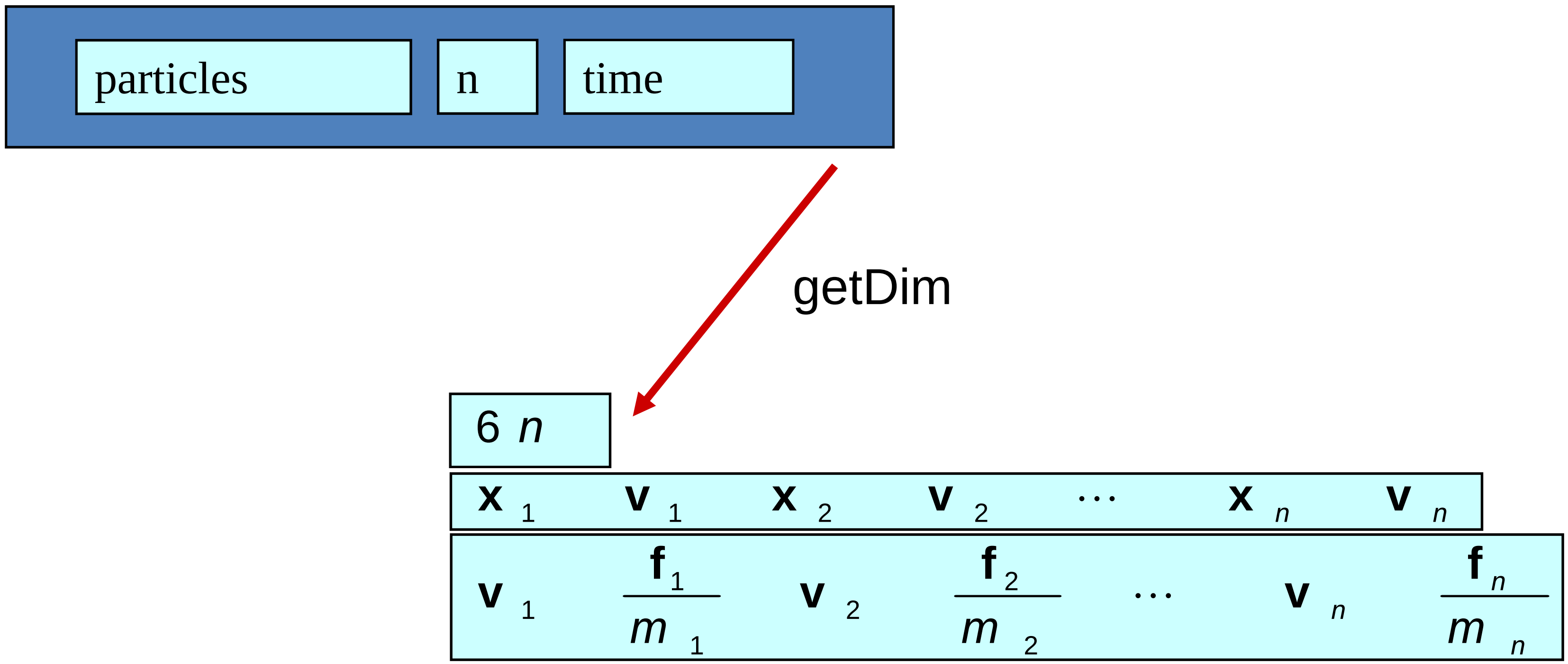
$$\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{v}_1 \\ \mathbf{f}_1 \\ m_1 \end{bmatrix} \begin{bmatrix} \mathbf{x}_2 \\ \mathbf{v}_2 \\ \mathbf{f}_2 \\ m_2 \end{bmatrix} \cdots \begin{bmatrix} \mathbf{x}_n \\ \mathbf{v}_n \\ \mathbf{f}_n \\ m_n \end{bmatrix}$$

```
typedef struct{
float m; /* mass */
float *x; /* position vector */
float *v; /* velocity vector */
float *f; /* force accumulator */
} *Particle;
```

```
typedef struct{
Particle *p; /* array of pointers to particles */
int n; /* number of particles */
float t; /* simulation clock */
} *ParticleSystem
```

Particle system solver interface

For *n* particles, the solver interface now looks like:




```
int ParticleDims(ParticleSystem p){
return(6 * p->n);
};
```

Particle system solver interface

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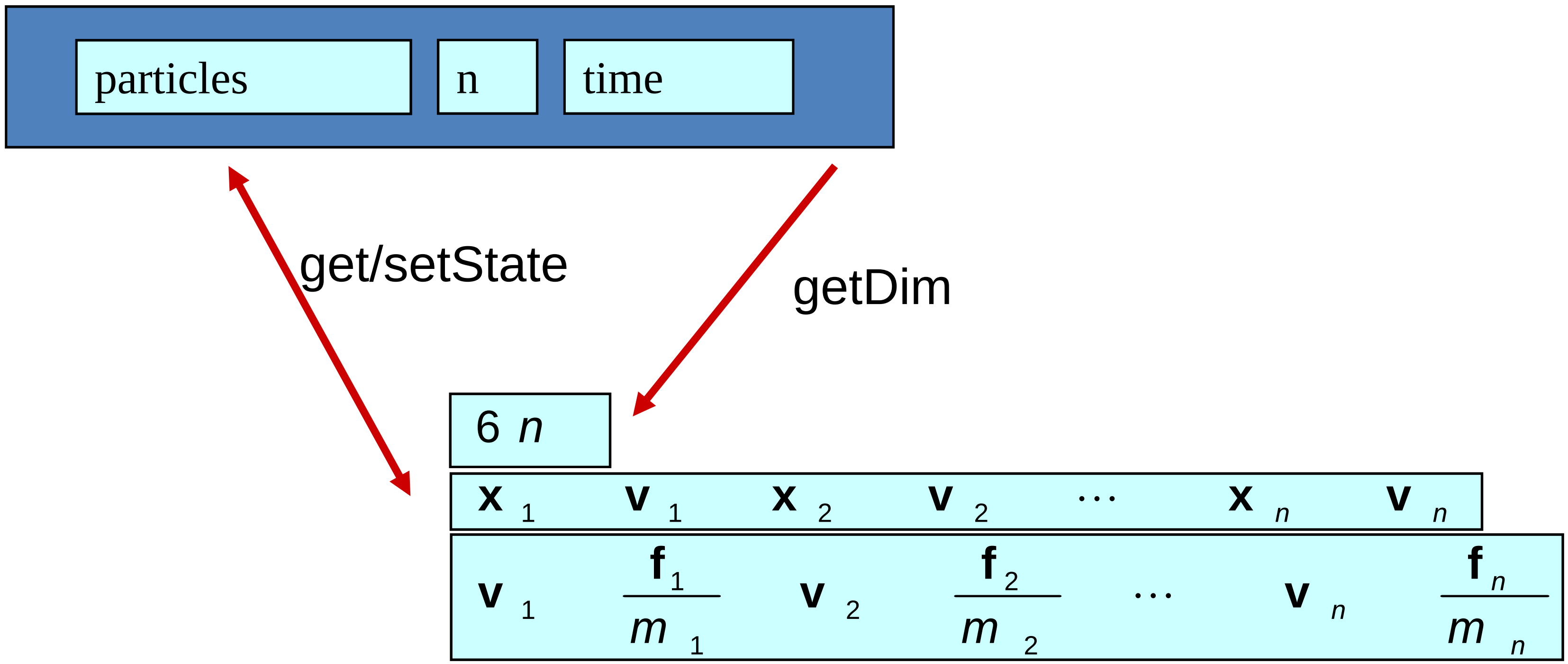


```
int ParticleGetState(ParticleSystem p, float *dst){
for(int i=0; i < p->n; i++){
*(dst++) = p->p[i]->x[0];    *(dst++) = p->p[i]->x[1];    *(dst++) = p->p[i]->x[2];
*(dst++) = p->p[i]->v[0];    *(dst++) = p->p[i]->v[1];    *(dst++) = p->p[i]->v[2];
}
}
```

Particle system solver interface

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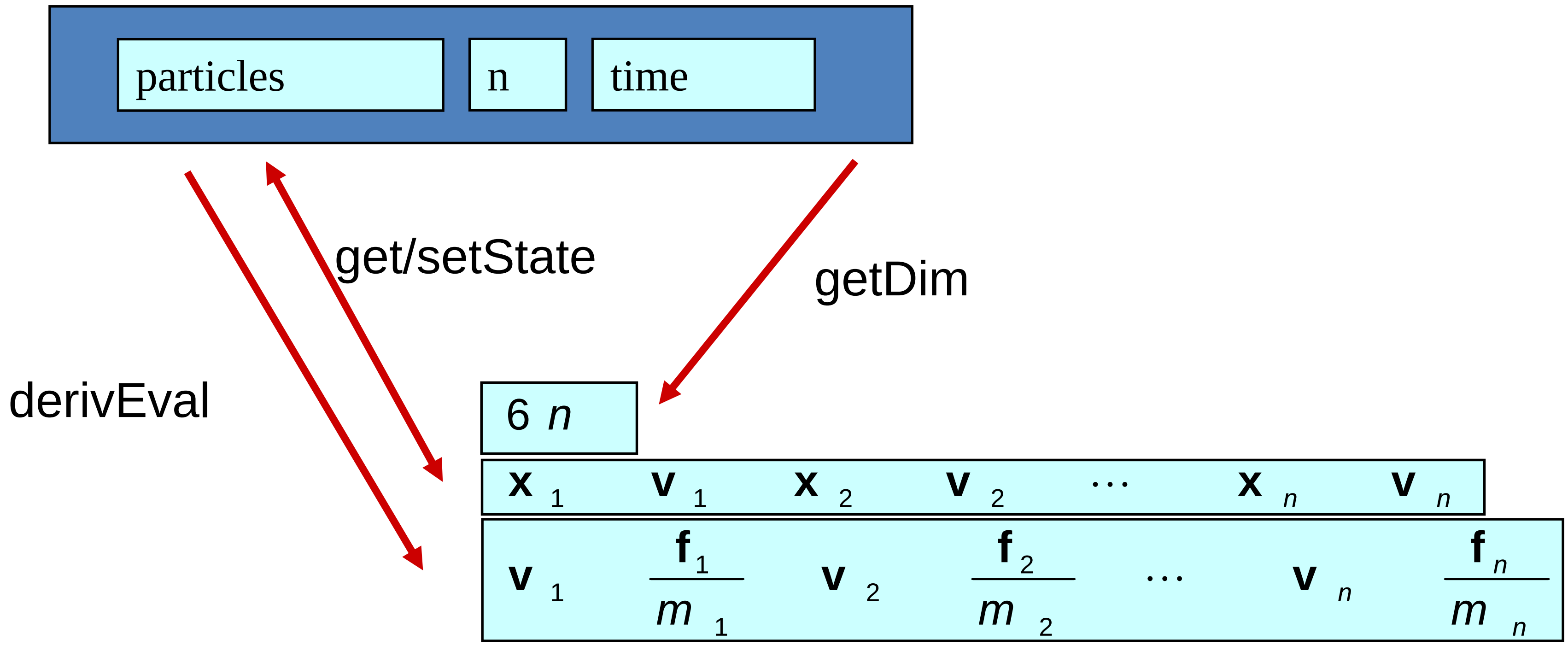


Particle system diff. eq. solver

We can solve the evolution of a particle system again using the Euler method:

$$\begin{bmatrix} \mathbf{x}_1^{i+1} \\ \mathbf{v}_1^{i+1} \\ \vdots \\ \mathbf{x}_n^{i+1} \\ \mathbf{v}_n^{i+1} \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1^{i} \\ \mathbf{v}_1^{i} \\ \vdots \\ \mathbf{x}_n^{i} \\ \mathbf{v}_n^{i} \end{bmatrix} + \Delta t \begin{bmatrix} \mathbf{v}_1^{i} \\ \mathbf{f}_1^{i} / m_1 \\ \vdots \\ \mathbf{v}_n^{i} \\ \mathbf{f}_n^{i} / m_n \end{bmatrix}$$

```
void EulerStep(ParticleSystem p, float DeltaT){
	ParticleDeriv(p,temp1); /* get deriv */
	ScaleVector(temp1,DeltaT) /* scale it */
	ParticleGetState(p,temp2); /* get state */
	AddVectors(temp1,temp2,temp2); /* add -> temp2 */
	ParticleSetState(p,temp2); /* update state */
	p->t += DeltaT; /* update time */
}
```

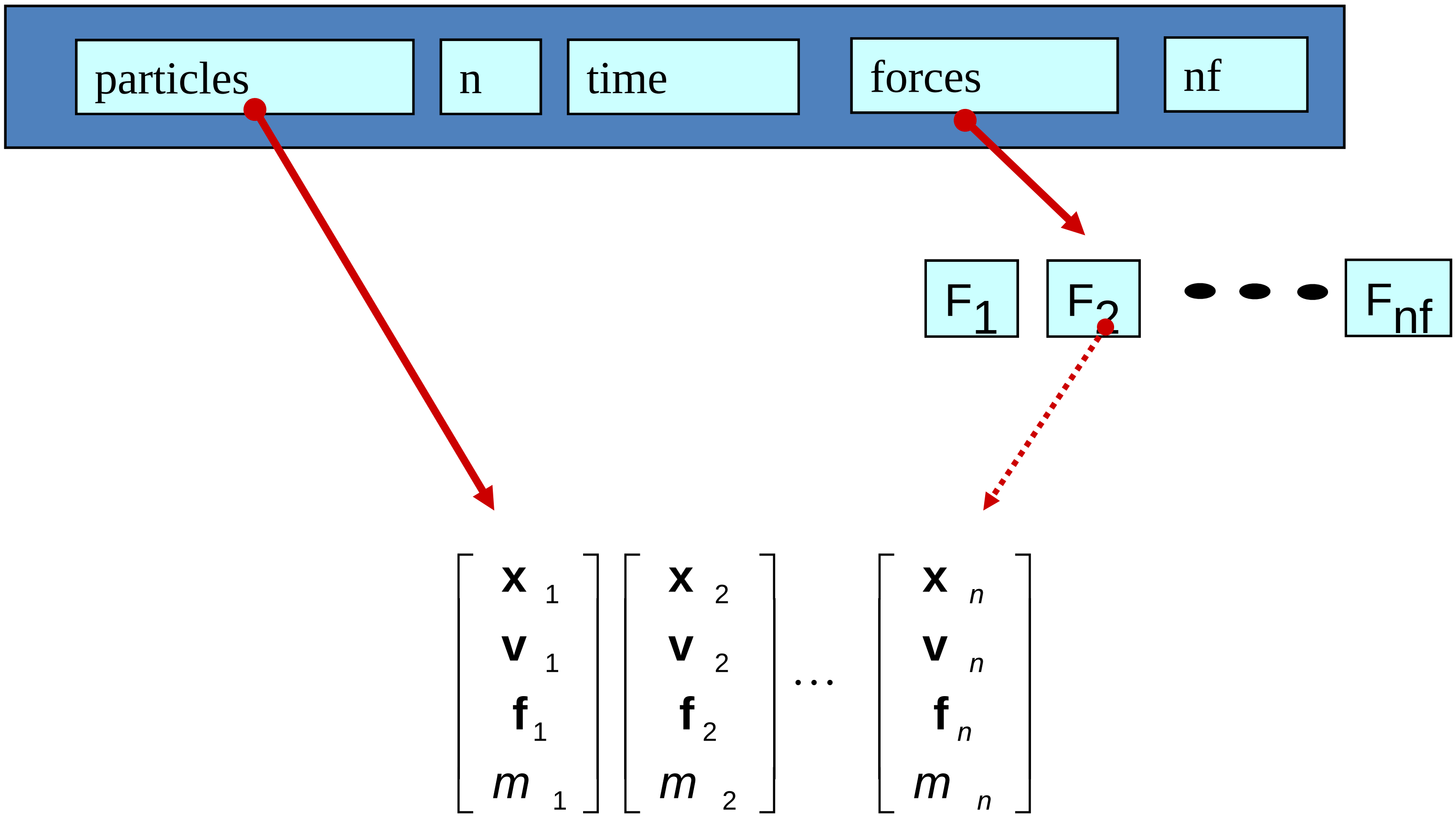
Forces

- Each particle can experience a force which sends it on its merry way.
- Where do these forces come from? Some examples:
  - Constant (gravity)
  - Position/time dependent (force fields)
  - Velocity-dependent (drag)
  - N-ary (springs)

- How do we compute the net force on a particle?

Particle systems with forces

- Force objects are black boxes that point to the particles they influence and add in their contributions.
- We can now visualize the particle system with force objects:



Gravity and viscous drag

The force due to **gravity** is simply:

$$\mathbf{f}_{grav} = m\,\mathbf{G}$$

Often, we want to slow things down with **viscous drag**:

$$\mathbf{f}_{drag} = -k_{drag}\,\mathbf{v}$$

Damped spring

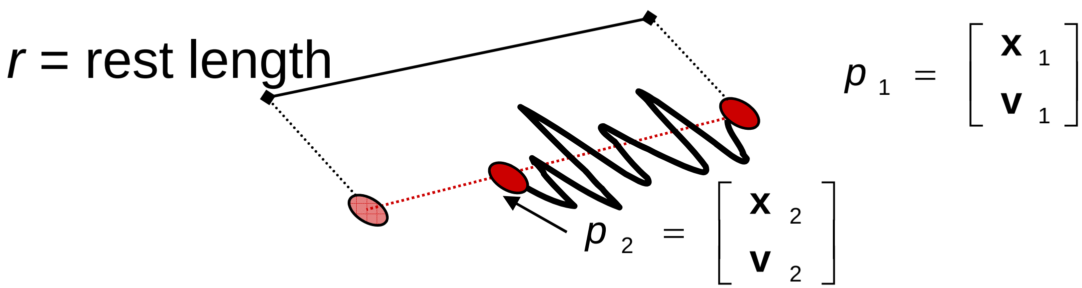
A spring is a simple examples of an “N-ary” force.

Recall the equation for the force due to a spring:

$$f = -k_{spring}(x - r)$$

We can augment this with damping:

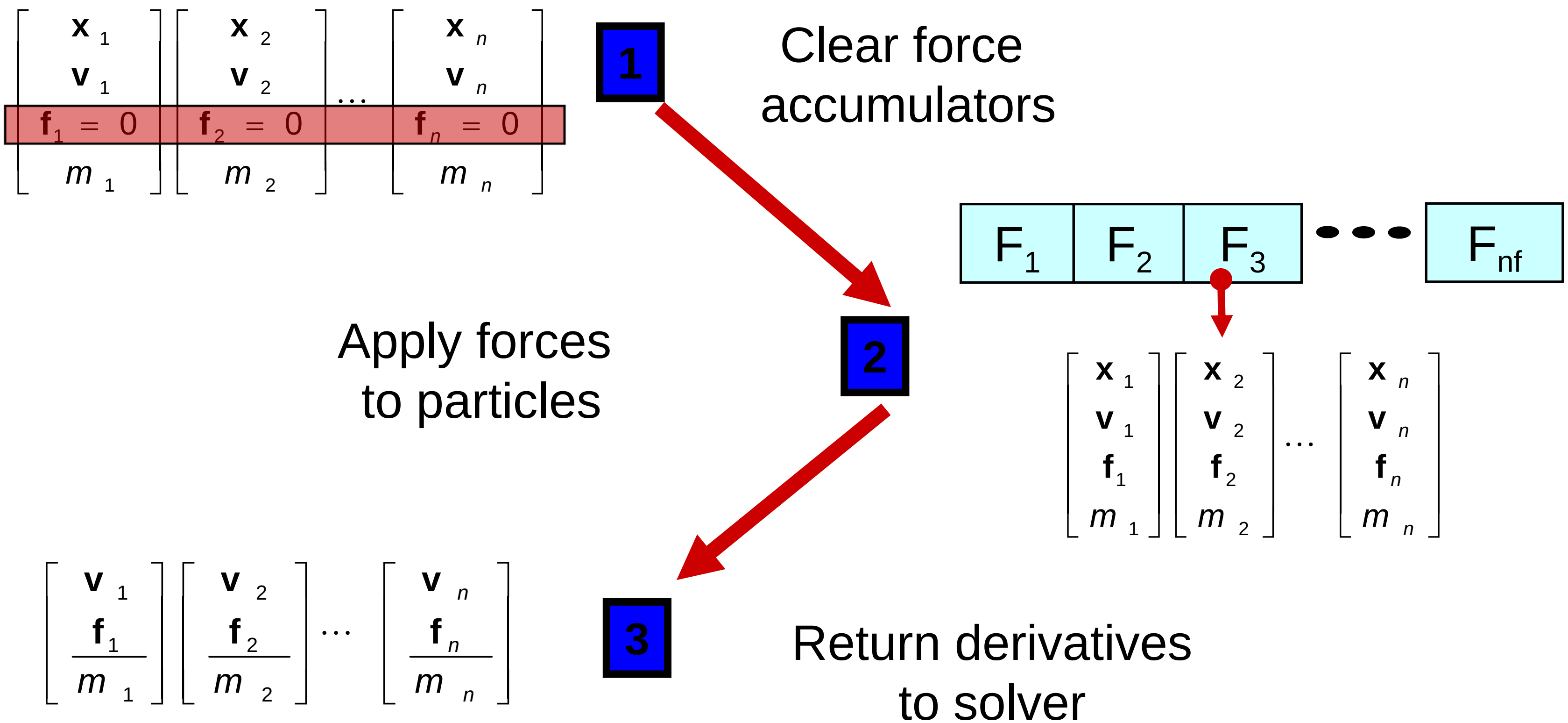
$$f = -[k_{spring}(x - r) + k_{damp}v]$$



Note: stiff spring systems can be very unstable under Euler integration. Simple solutions include heavy damping (may not look good), tiny time steps (slow), or better integration (Runge-Kutta is straightforward).

derivEval

1. Clear forces
   - Loop over particles, zero force accumulators
2. Calculate forces
   - Sum all forces into accumulators
3. Return derivatives
   - Loop over particles, return **v** and **f**/*m*



Particle system solver interface

```
int ParticleDerivative(ParticleSystem p, float *dst){
	Clear_Forces(p); /* zero the force accumulators */
	Compute_Forces(p); /* magic force function */
	for(int i=0; i < p->n; i++){
		*(dst++) = p->p[i]->v[0]; /* xdot = v */
		*(dst++) = p->p[i]->v[1];
		*(dst++) = p->p[i]->v[2];
		*(dst++) = p->p[i]->f[0]/m; /* vdot = f/m */
		*(dst++) = p->p[i]->f[1]/m;
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	}
}
```

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