CS559: Computer Graphics

Lecture 7: Image Warping and Morphing

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Spring 2010

Most slides borrowed from [Yungyu Chuang]

Last time: edge dection

Lat time: edge detection

- Edge detection algorithms typically proceed in three or four steps:
  - **Filtering**: cut down on noise
  - **Enhancement**: amplify the difference between edges and non-edges
  - **Detection**: use a threshold operation
  - **Localization** (optional): estimate geometry of edges, which generally pass between pixels

The Canny Edge Detector

original image (Lena)

The Canny Edge Detector

magnitude of the gradient

The Canny Edge Detector

After non-maximum suppression

Lat time: edge detection

- Edge detection algorithms typically proceed in three or four steps:
  - **Filtering**: cut down on noise
  - **Enhancement**: amplify the difference between edges and non-edges
  - **Detection**: use a threshold operation
  - **Localization** (optional): estimate geometry of edges, which generally pass between pixels

Last time: Mattes

- A *matte* is an image that shows which parts of another image are foreground objects
- Term dates from film editing and cartoon production
- How would I use a matte to insert an object into a background?
- How are mattes usually generated for television?

Basic Compositing Operation

- At each pixel, combine the pixel data from *f* and the pixel data from *g* with the equation:

$$c_f = [\alpha_f R_f, \alpha_f G_f, \alpha_f B_f, \alpha_f]$$

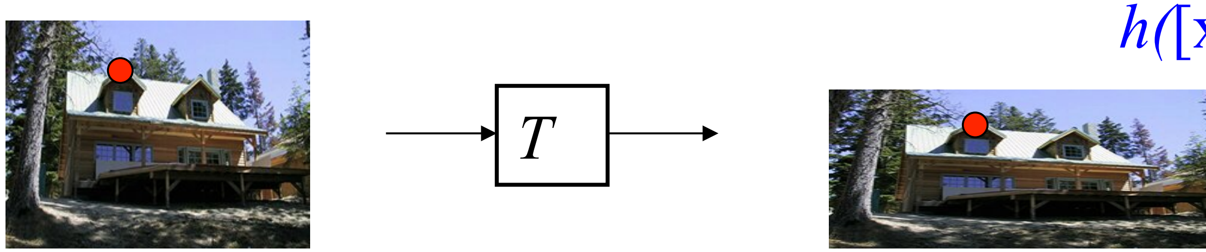
$$c_g = [\alpha_g R_g, \alpha_g G_g, \alpha_g B_g, \alpha_g]$$

$$c_o = 1 \cdot c_f + (1 - \alpha_f) \cdot c_g$$

“Over” Operator

Last time: 2D Transformations

$h([x,y])=[x,y/2]$



$\mathbf{p} = (x,y)$ $\qquad$ $\mathbf{p'} = (x',y')$

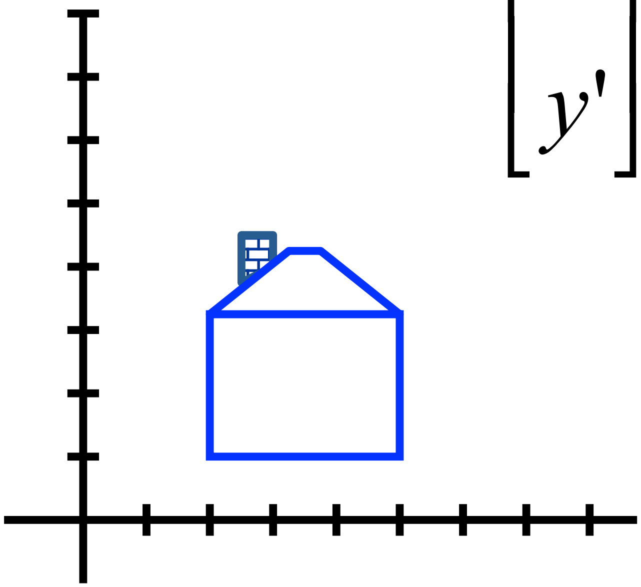
- Transformation T is a coordinate-changing machine: p' = *T*(p)
- What does it mean that *T* is global?
  - can be described by just a few numbers (parameters)
  - the parameters are the same for any point p
- Represent *T* as a matrix: p' = **M**p

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{M} \begin{bmatrix} x \\ y \end{bmatrix}$$

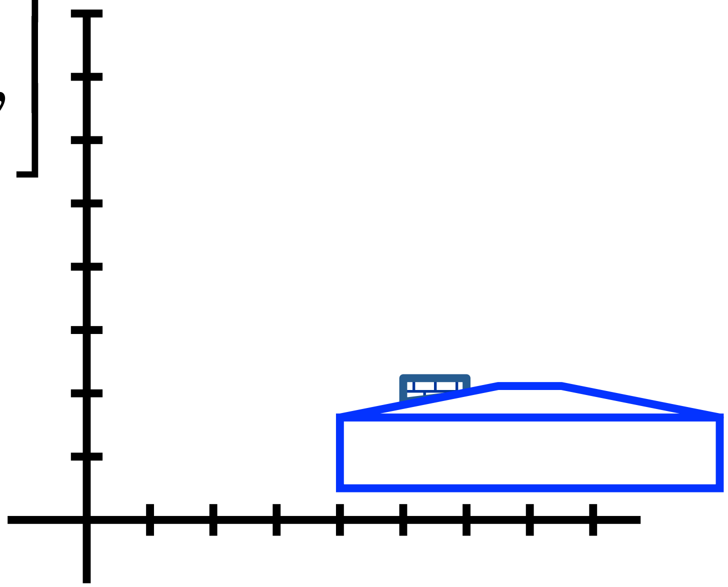
Scaling

- *Non-uniform scaling*: different scalars per component:

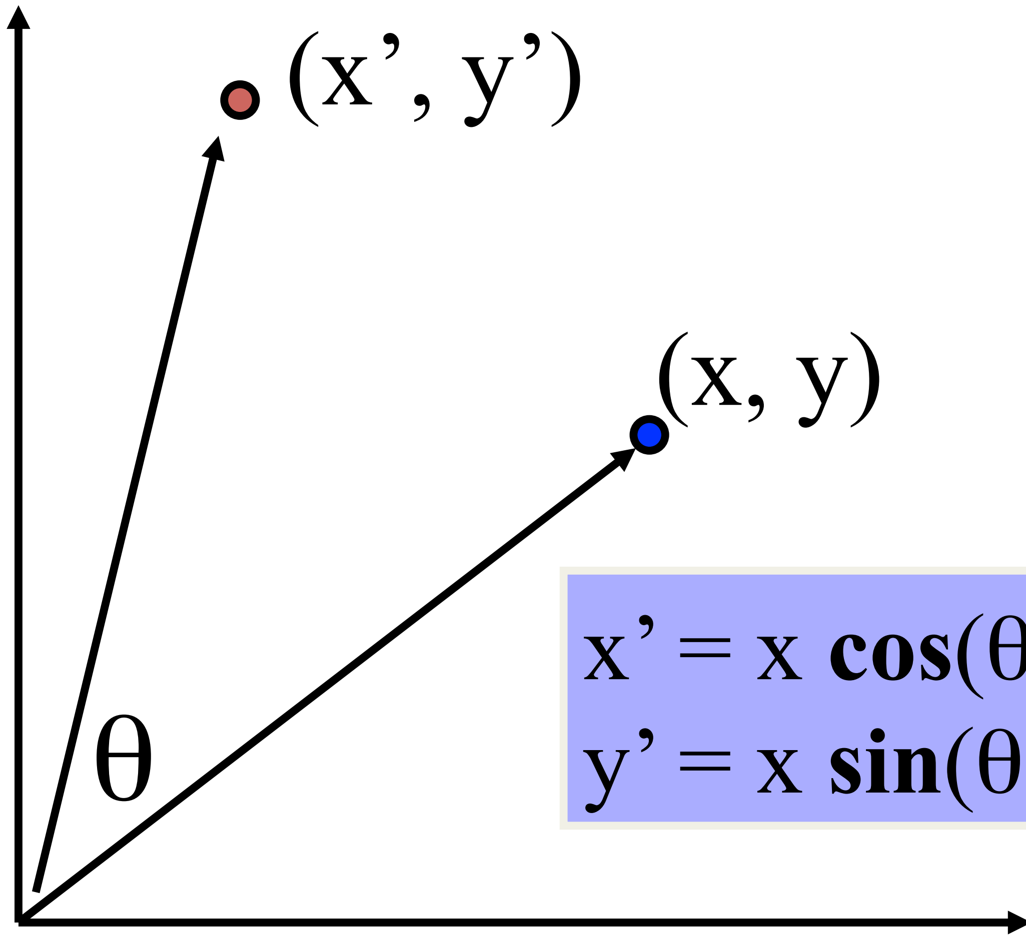
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2x \\ 0.5y \end{bmatrix}$$



x × 2,
y × 0.5



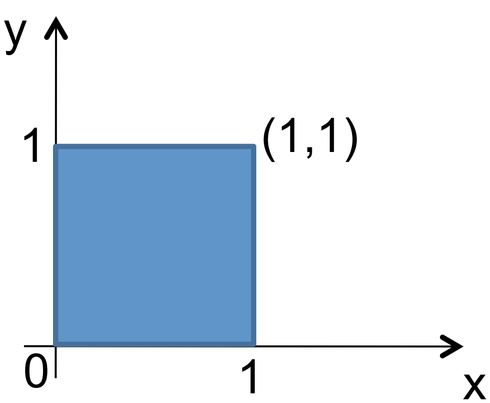
2-D Rotation

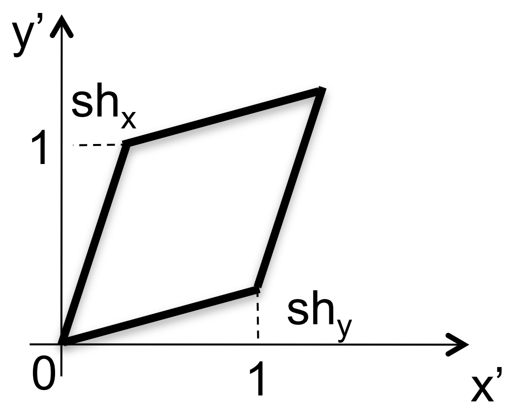


$$x' = x \,\mathbf{cos}(\theta) - y \,\mathbf{sin}(\theta)$$
$$y' = x \,\mathbf{sin}(\theta) + y \,\mathbf{cos}(\theta)$$

2x2 Matrices

- What types of transformations can be represented with a 2x2 matrix?





2D Shear?

$$x' = x + sh_x * y$$

$$y' = sh_y * x + y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & sh_x \\ sh_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2x2 Matrices

- What types of transformations can be represented with a 2x2 matrix?

2D Mirror about Y axis?

$$x' = -x$$
$$y' = y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Mirror over (0,0)?

$$x' = -x$$
$$y' = -y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

All 2D Linear Transformations

- Linear transformations are combinations of …
  - Scale,
  - Rotation,
  - Shear, and
  - Mirror

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Any 2D transform can be decomposed into the product of a rotation, scale, and a rotation

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = rotate(31.7^\circ) \cdot scale(1.618, 0.618) \cdot rotate(-58.3^\circ)$$

All 2D Linear Transformations

- Linear transformations are combinations of …
  - Scale,
  - Rotation,
  - Shear, and
  - Mirror

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- A symmetric 2D transform can be decomposed into the product of a rotation, scale, and the inverse rotation

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = rotate(31.7^\circ) \cdot scale(2.618, 0.382) \cdot rotate(-31.7^\circ)$$

Today

- More on 2D transformation
- Use it for image warping and morphing

All 2D Linear Transformations

- Linear transformations are combinations of …
  - Scale,
  - Rotation,
  - Shear, and
  - Mirror

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Properties of linear transformations:
  - Origin maps to origin
  - Lines map to lines
  - Parallel lines remain parallel
  - Ratios are preserved
  - Closed under composition

$$\frac{AB}{BC} = \frac{A'B'}{B'C'} \quad \text{if A, B, C are on a line}$$

2x2 Matrices

- What types of transformations can not be represented with a 2x2 matrix?

2D Translation?

$$x' = x + t_x$$
$$y' = y + t_y$$

NO!

Only linear 2D transformations can be represented with a 2x2 matrix

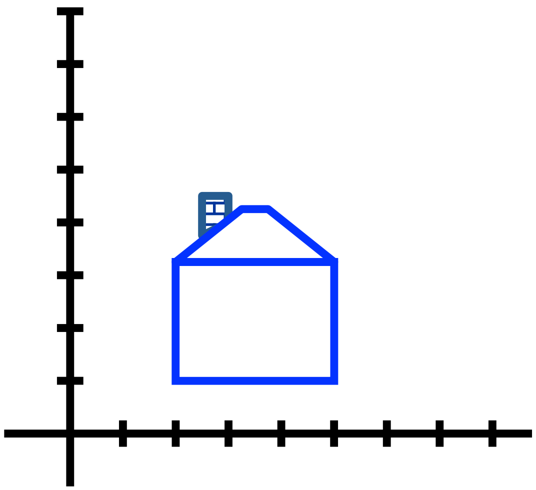
Translation

- Example of translation

Homogeneous Coordinates

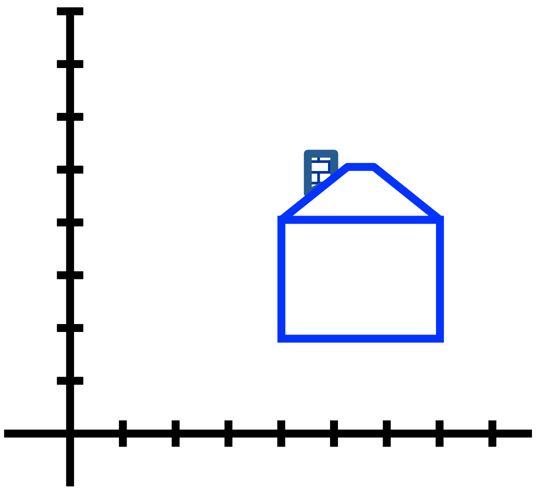
$$\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$



$t_x = 2$

$t_y = 1$



Homogeneous coordinates

- Why do we need it?
  - Can express all linear transformation as special cases

Homogeneous coordinates

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  - Can express all linear transformation as special cases
  - Easy to compute a composite transformation that involve several translations and linear transformation

Homogeneous coordinates

- Why do we need it?
  - Can express all linear transformation as special cases
  - Easy to compute a composite transformation that involve several translations and linear transformation
  - More to come

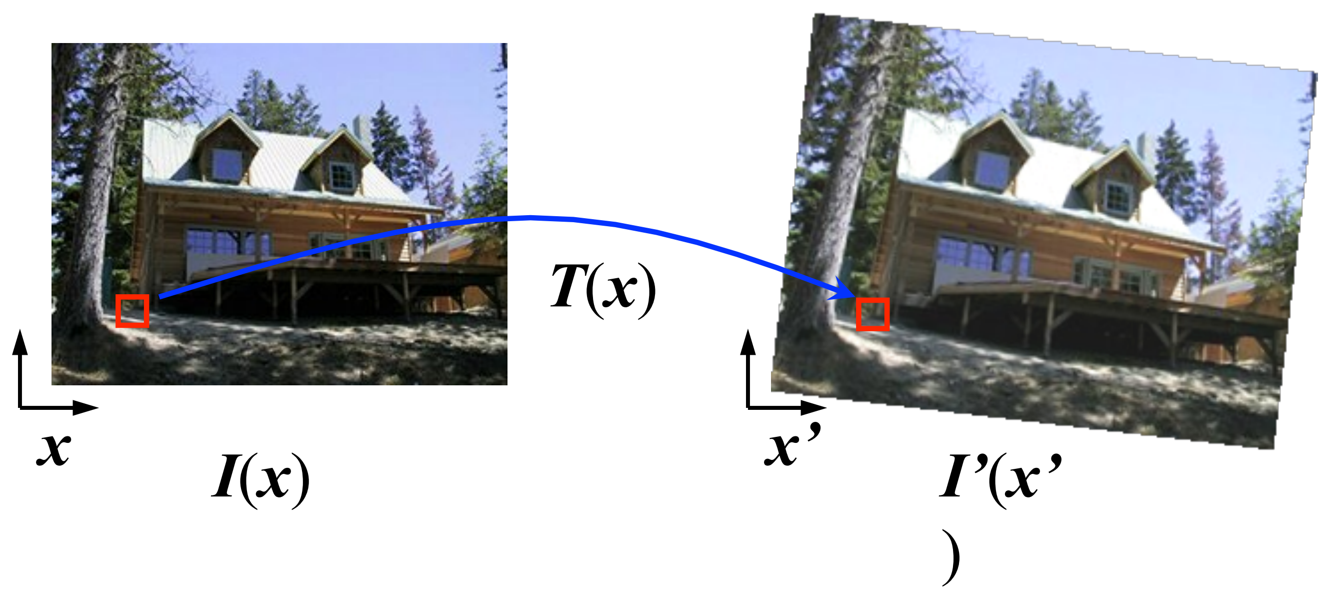
Affine Transformations

- Affine transformations are combinations of …
  - Linear transformations, and
  - Translations
- Properties of affine transformations:
  - Origin does not necessarily map to origin
  - Lines map to lines
  - Parallel lines remain parallel
  - Ratios are preserved
  - Closed under composition
  - Models change of basis

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

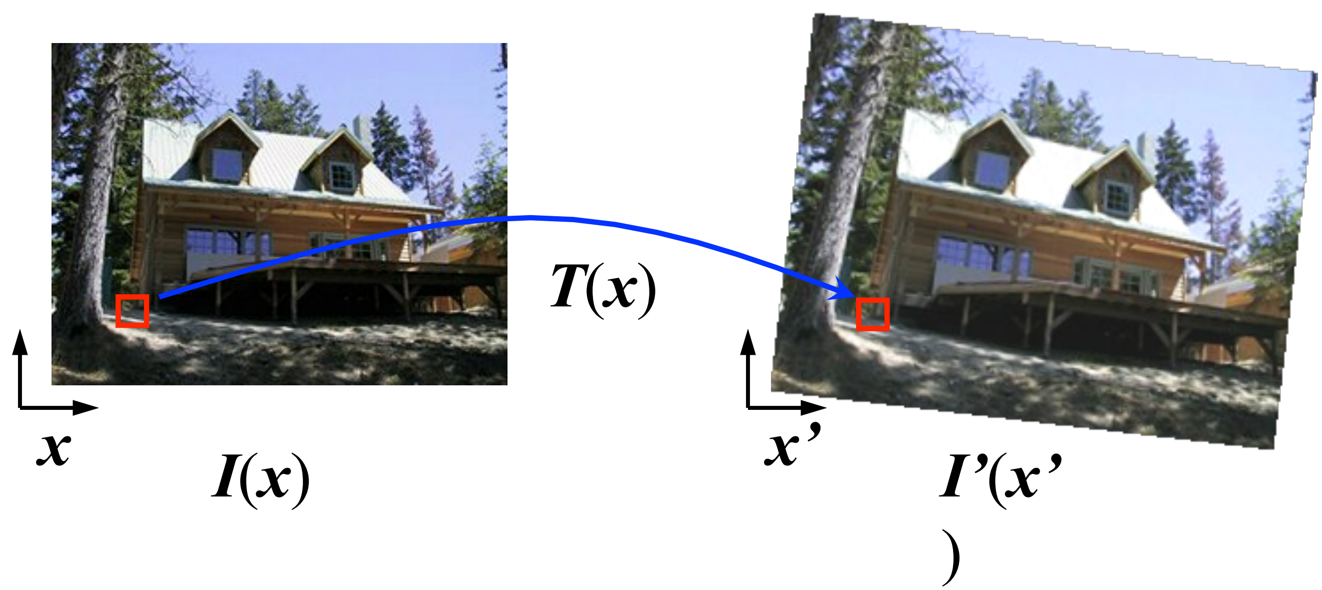
Image warping

- Given a coordinate transform $\boldsymbol{x'} = \boldsymbol{T}(\boldsymbol{x})$ and a source image $\boldsymbol{I}(\boldsymbol{x})$, how do we compute a transformed image $\boldsymbol{I'}(\boldsymbol{x'}) = \boldsymbol{I}(\boldsymbol{T}(\boldsymbol{x}))$?



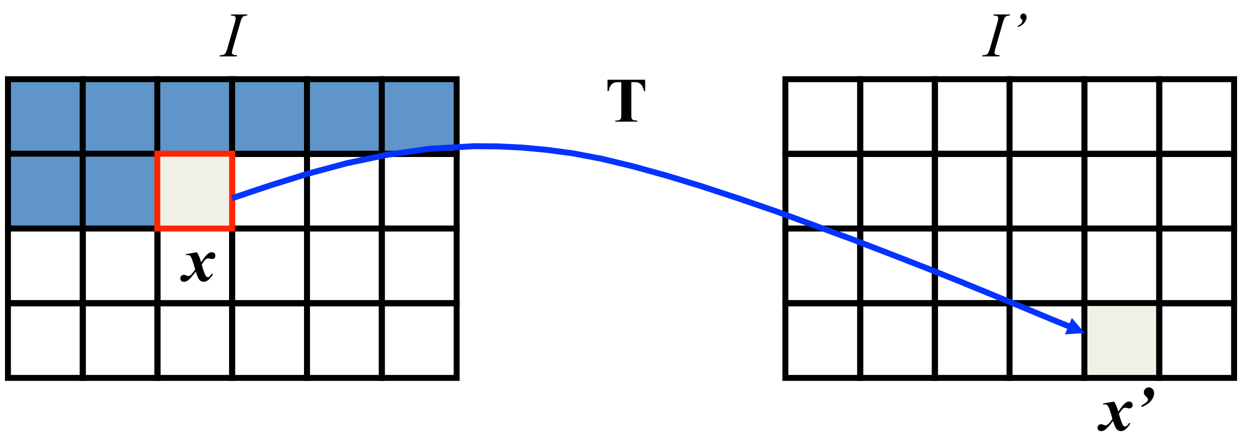
Forward warping

- Send each pixel ***I***(***x***) to its corresponding location ***x'*** = ***T***(***x***) in ***I'***(***x'***)



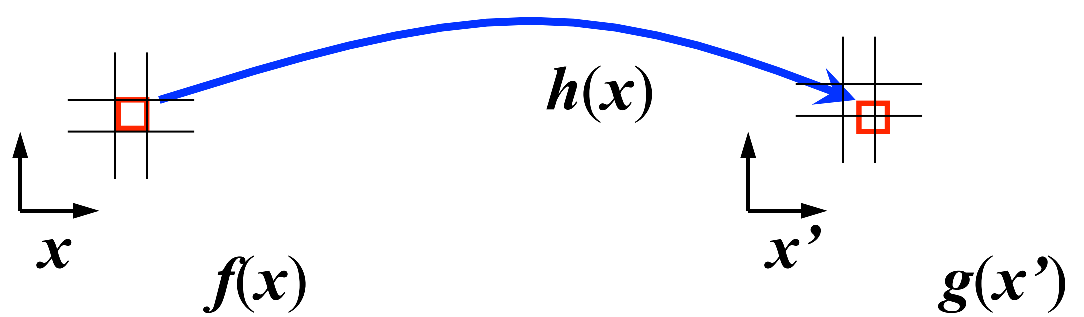
Forward warping

```
fwarp(I, I', T)
{
  for (y=0; y<I.height; y++)
    for (x=0; x<I.width; x++) {
      (x',y')=T(x,y);
      I'(x',y')=I(x,y);
    }
}
```



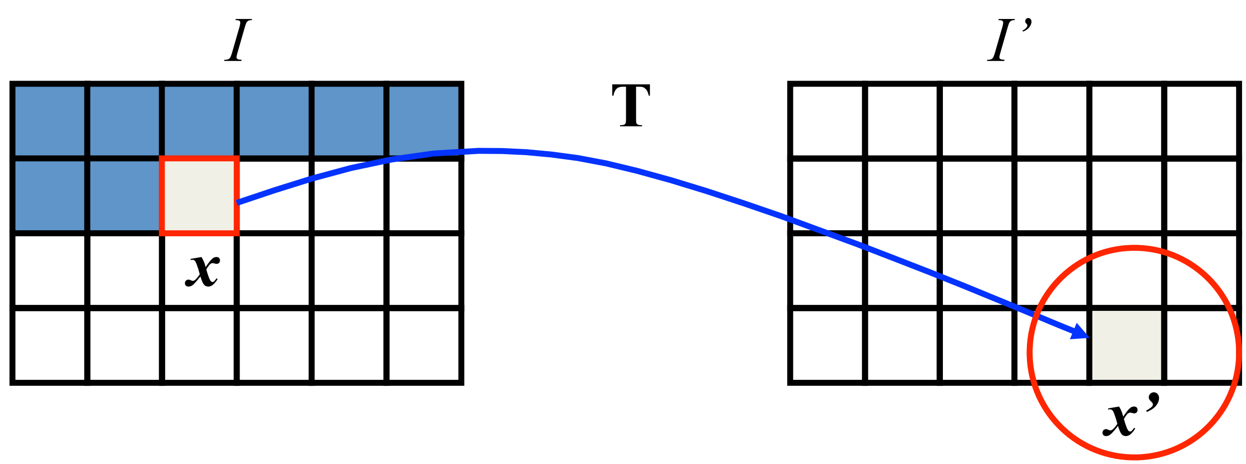
Forward warping

- Send each pixel $I(\boldsymbol{x})$ to its corresponding location $\boldsymbol{x}' = \boldsymbol{T}(\boldsymbol{x})$ in $\boldsymbol{I}'(\boldsymbol{x}')$
  - What if pixel lands "between" two pixels?
  - Will be there holes?
  - Answer: add "contribution" to several pixels, normalize later (*splatting*)



Forward warping

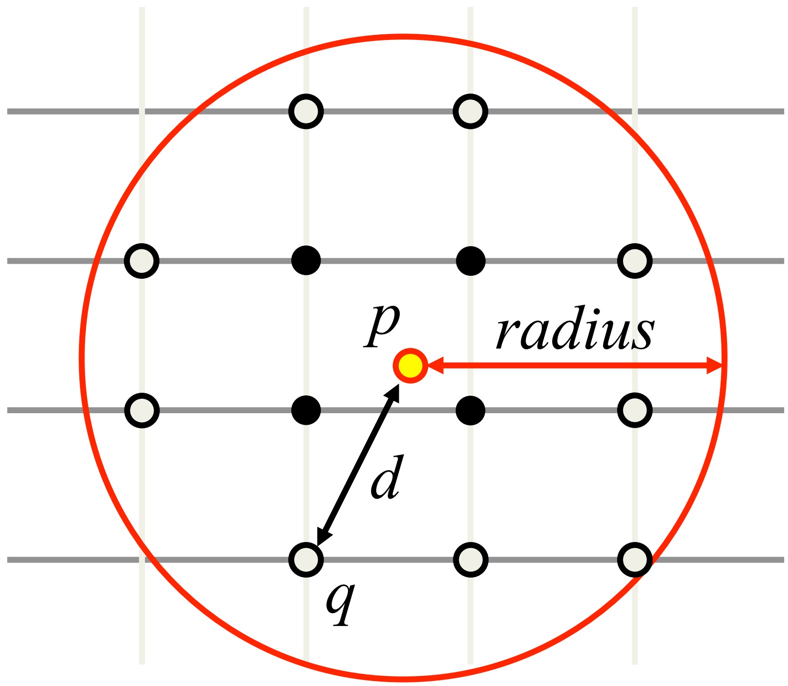
```
fwarp(I, I', T)
{
  for (y=0; y<I.height; y++)
    for (x=0; x<I.width; x++) {
      (x',y')=T(x,y);
      Splatting(I',x',y',I(x,y),kernel);
    }
}
```



Splatting

- Computed weighted sum of contributed colors using a kernel function, where weights are normalized values of filter kernel k, such as Gauss

May get a blurry image!



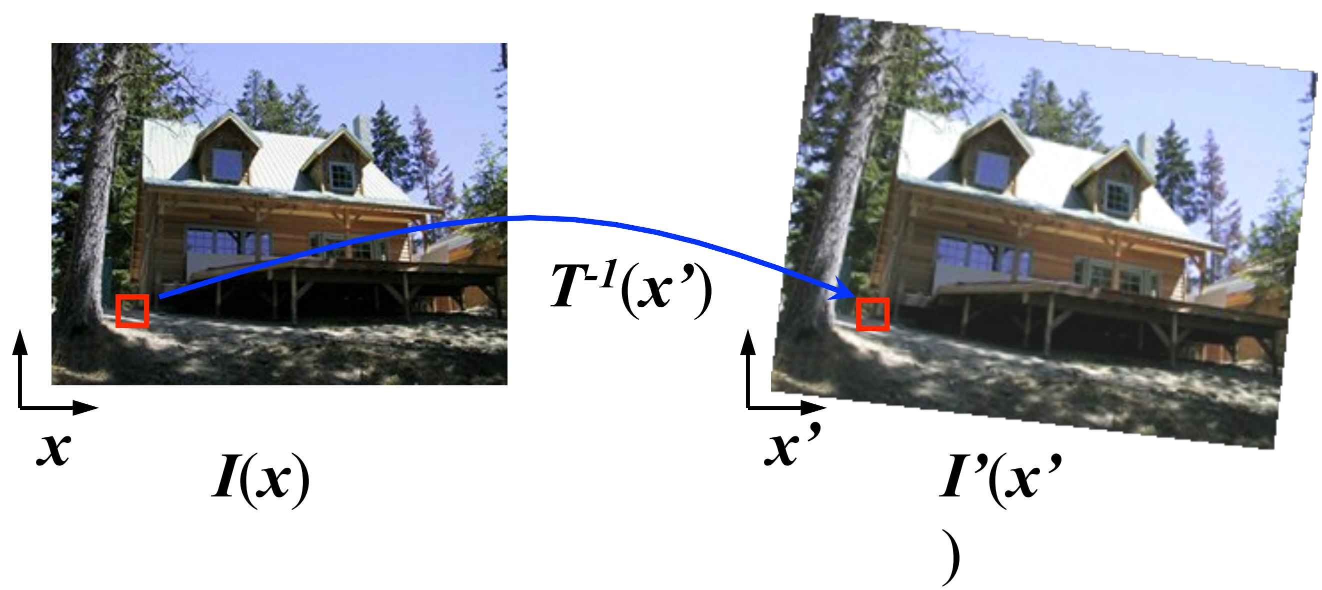
Destination Image

```
for all q
  q.color = 0;
  q.weight = 0;

for all p from source image
  for all q's dist < radius
    d = dist(p, q);
    w = kernel(d);
    q.color += w*p;
    q.weight += w;

for all q
  q.Color /= q.weight;
```

$$= e^{-\frac{d^2}{2\sigma^2}}$$

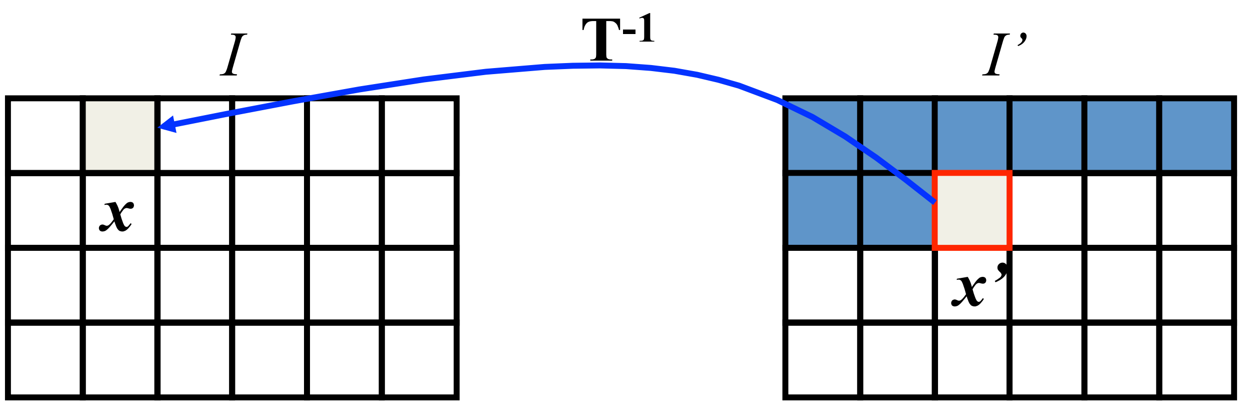
Inverse warping

- Get each pixel $\boldsymbol{I'}(\boldsymbol{x'})$ from its corresponding location $\boldsymbol{x} = \boldsymbol{T^{-1}}(\boldsymbol{x'})$ in $\boldsymbol{I}(\boldsymbol{x})$



Inverse warping

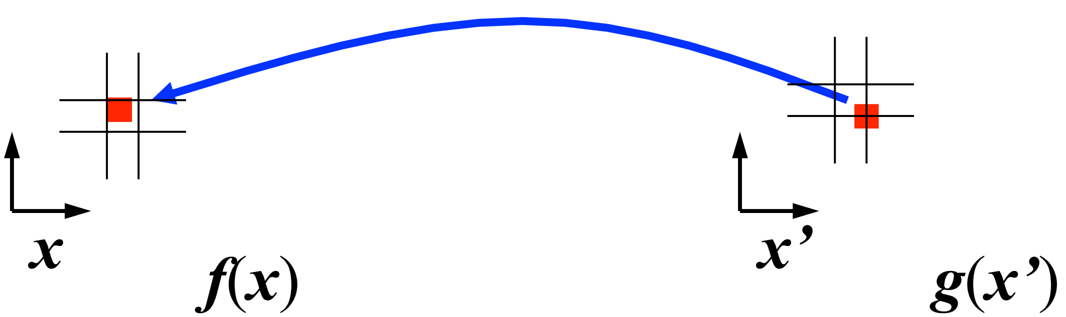
```
iwarp(I, I', T)
{
  for (y=0; y<I'.height; y++)
    for (x=0; x<I'.width; x++) {
      (x,y)=T^-1(x',y');
      I'(x',y')=I(x,y);
    }
}
```



Inverse warping

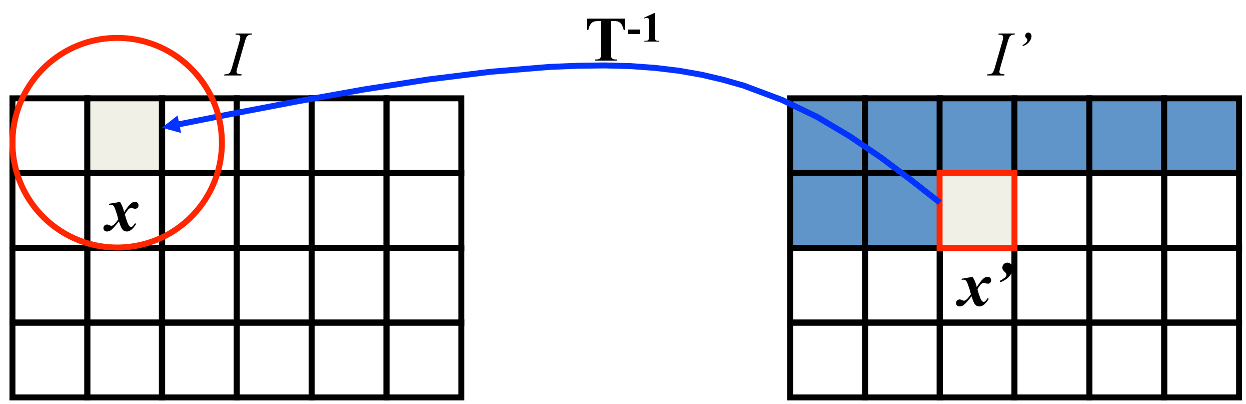
- Get each pixel $\boldsymbol{I'}(\boldsymbol{x'})$ from its corresponding location $\boldsymbol{x} = \boldsymbol{T}^{-1}(\boldsymbol{x'})$ in $\boldsymbol{I}(\boldsymbol{x})$

  - What if pixel comes from "between" two pixels?
  - Answer: *resample* color value from *interpolated* source image



Inverse warping

```
iwarp(I, I', T)
{
  for (y=0; y<I'.height; y++)
    for (x=0; x<I'.width; x++) {
      (x,y)=T^-1(x',y');
      I'(x',y')=Reconstruct(I,x,y,kernel);
    }
}
```

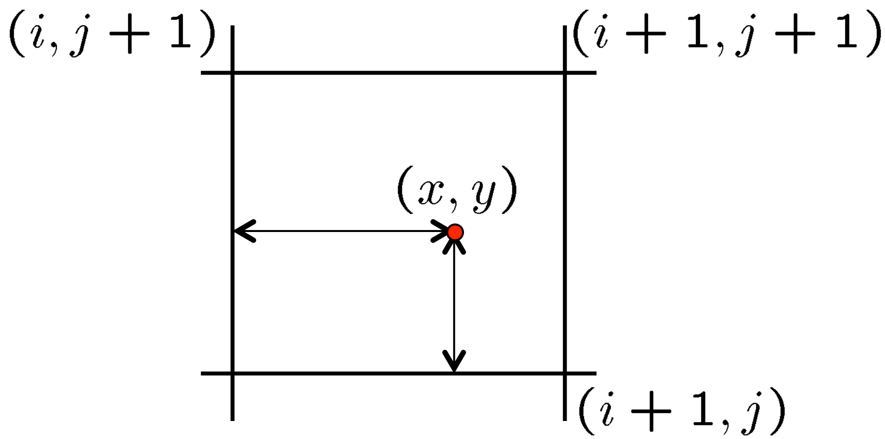


Reconstruction (interpolation)

- Possible reconstruction filters (kernels):
  - nearest neighbor
  - bilinear
  - bicubic
  - sinc

Bilinear interpolation (tent filter)

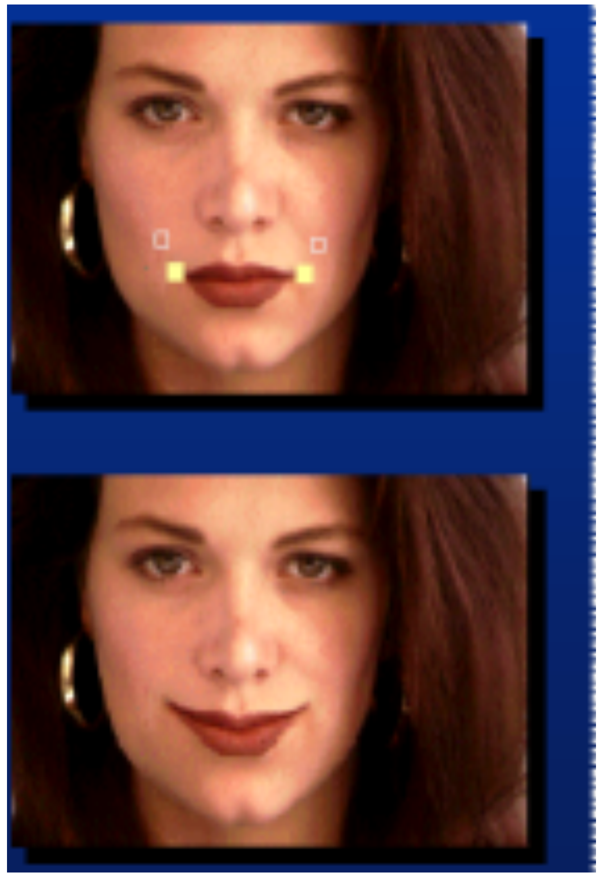
- A simple method for resampling images



$$
\begin{aligned}
f(x,y) = \quad & (1-a)(1-b) && f[i,j] \\
& +a(1-b) && f[i+1,j] \\
& +ab && f[i+1,j+1] \\
& +(1-a)b && f[i,j+1]
\end{aligned}
$$

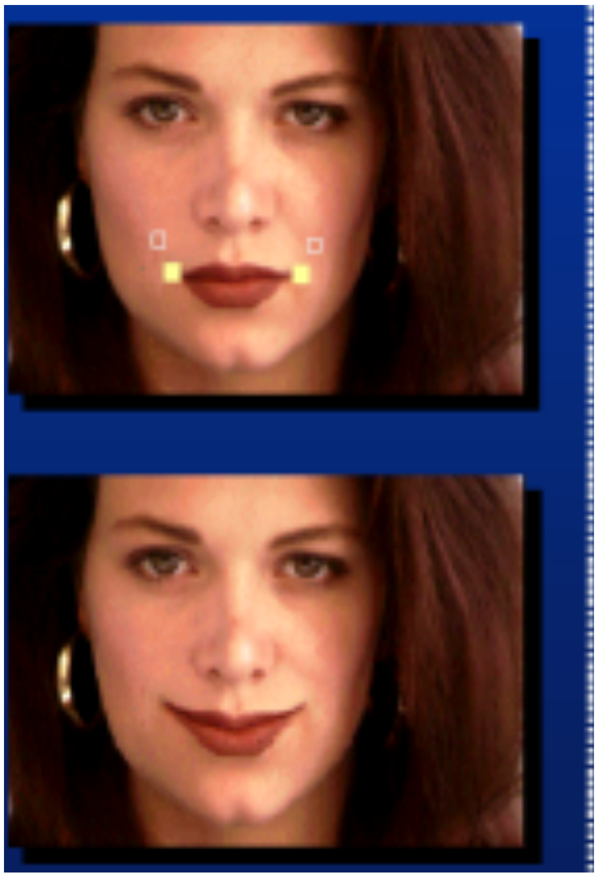
What might be the problem of bilinear interpolation?

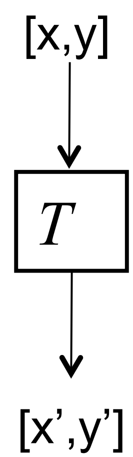
Non-parametric image warping



Non-parametric image warping

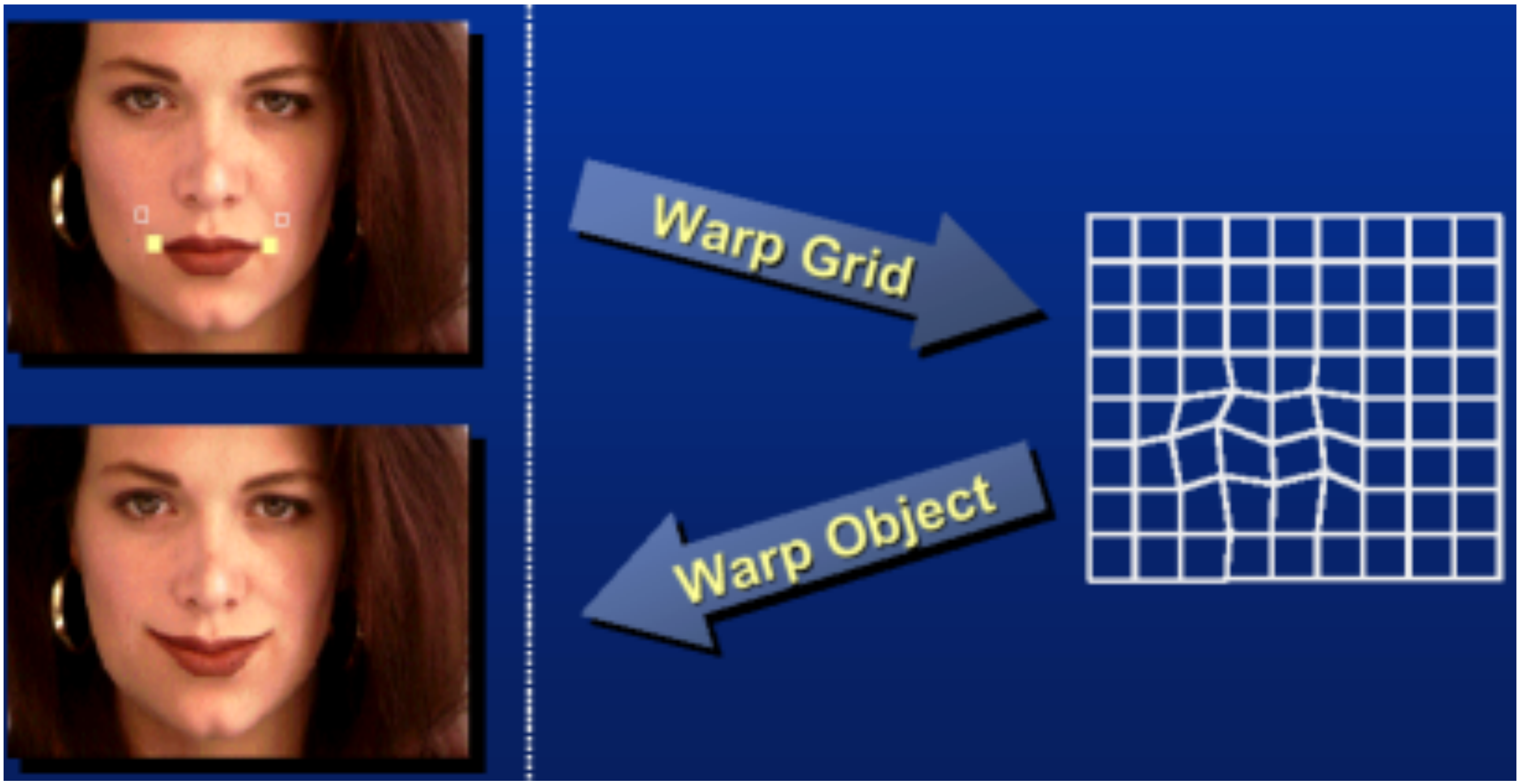
- Specify a more detailed warp function





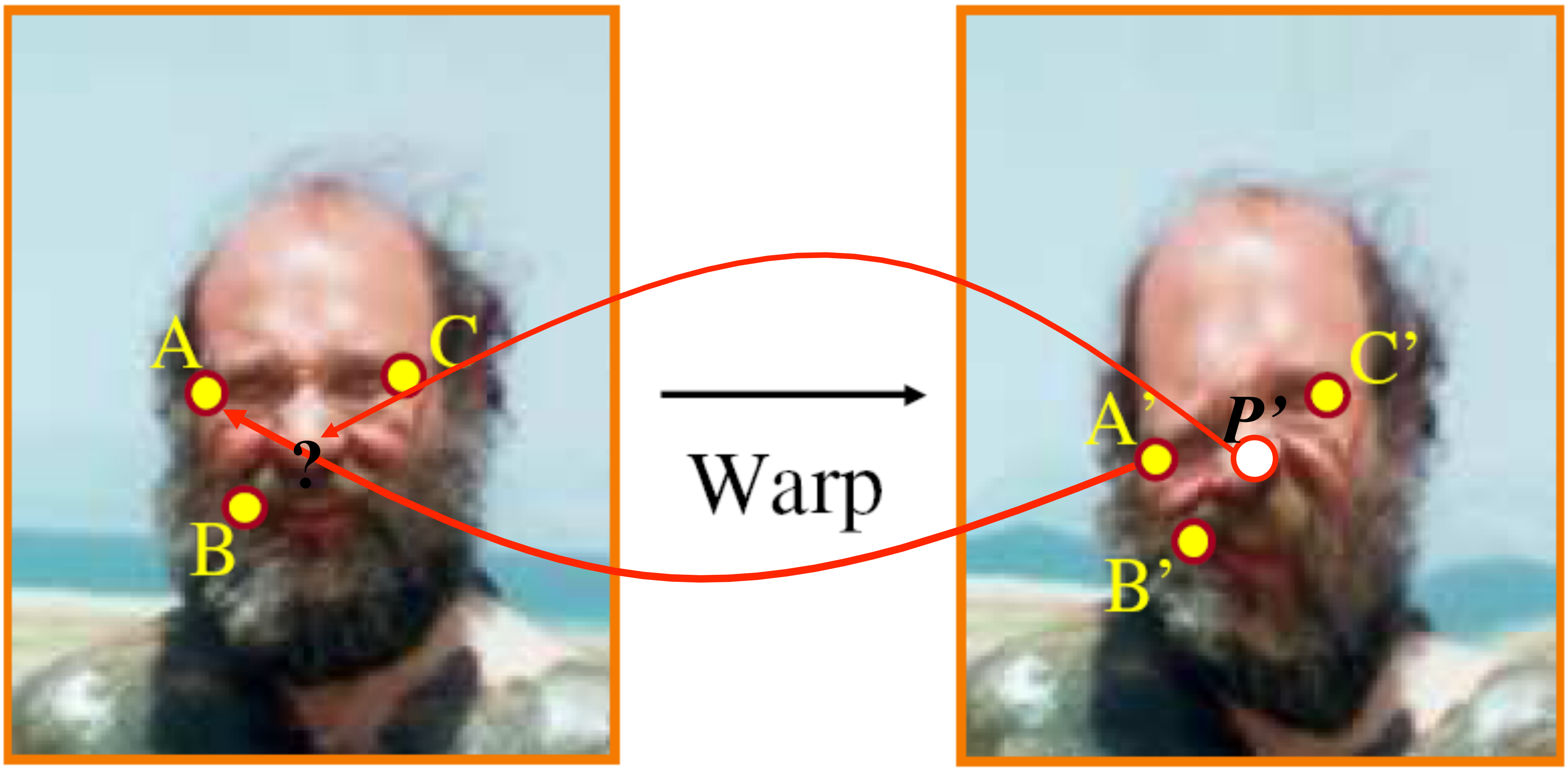
Non-parametric image warping

- Specify a more detailed warp function
- Tabulate pixel motion (lookup table)

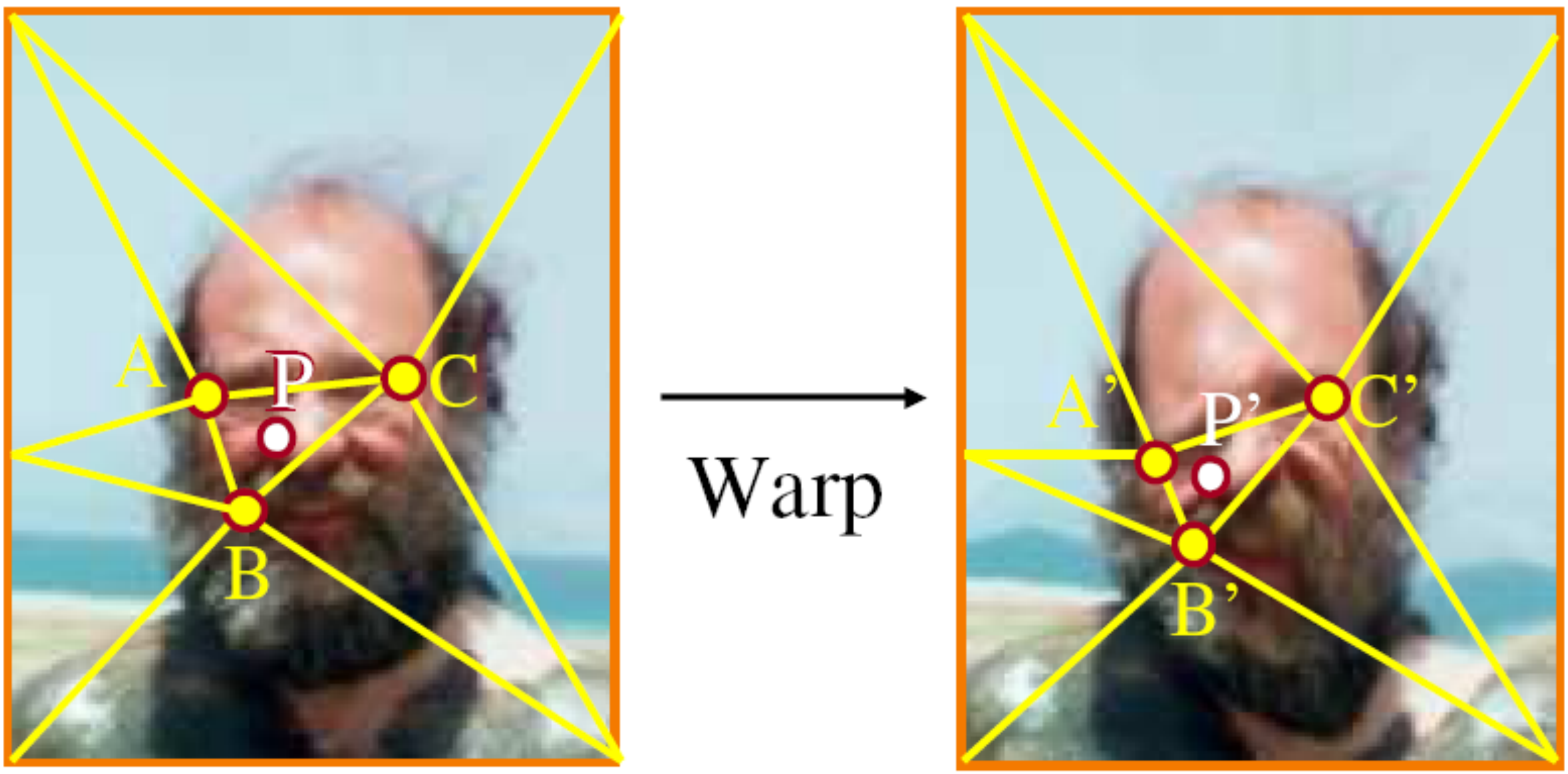


Non-parametric image warping

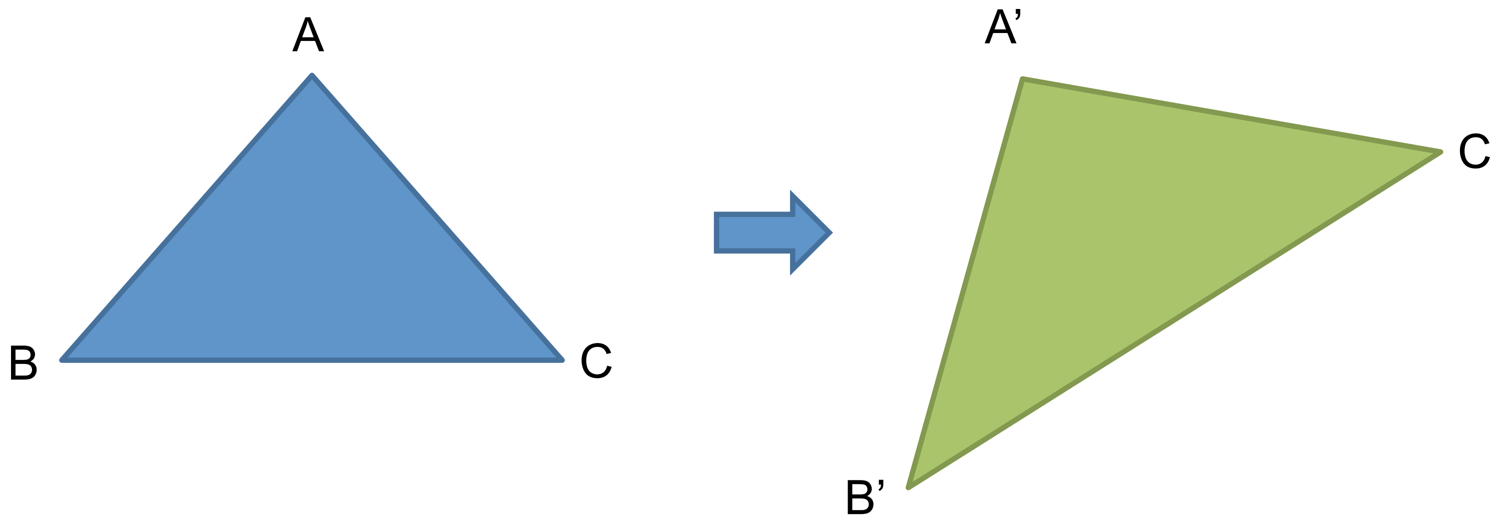
- Mappings implied by correspondences
- Inverse warping



Non-parametric image warping



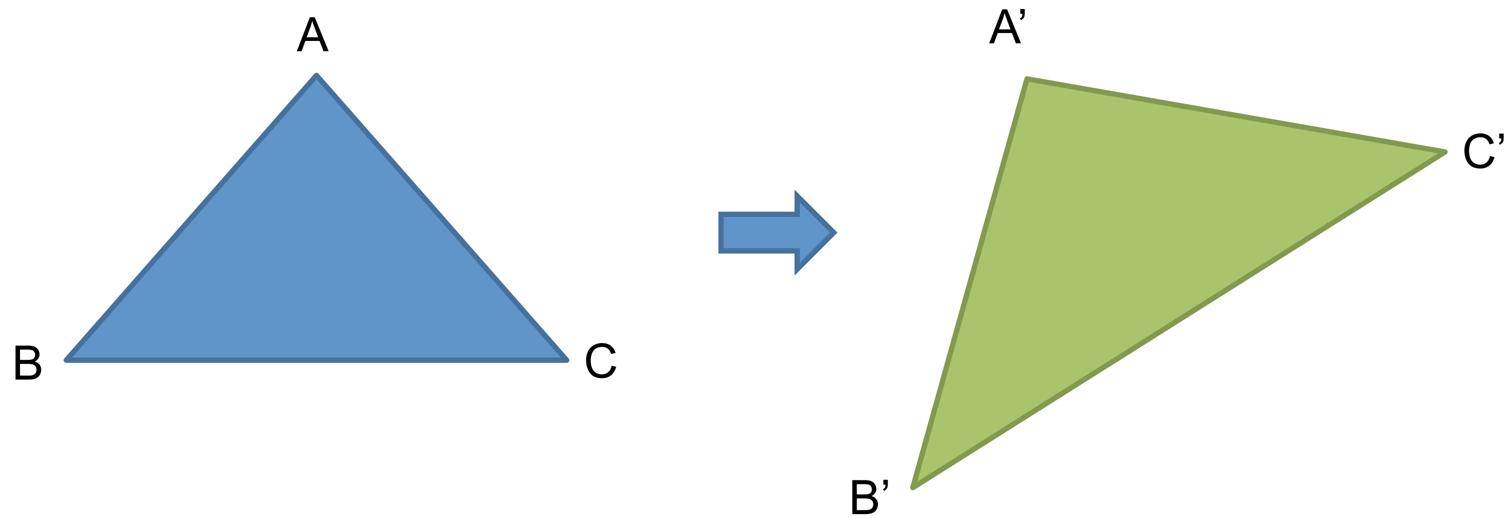
Warping between two triangles



- Idea: find an affine that transforms ABC to A'B'C'

$$\begin{bmatrix} a & b & e \\ c & d & f \\ 0 & 0 & 1 \end{bmatrix}$$

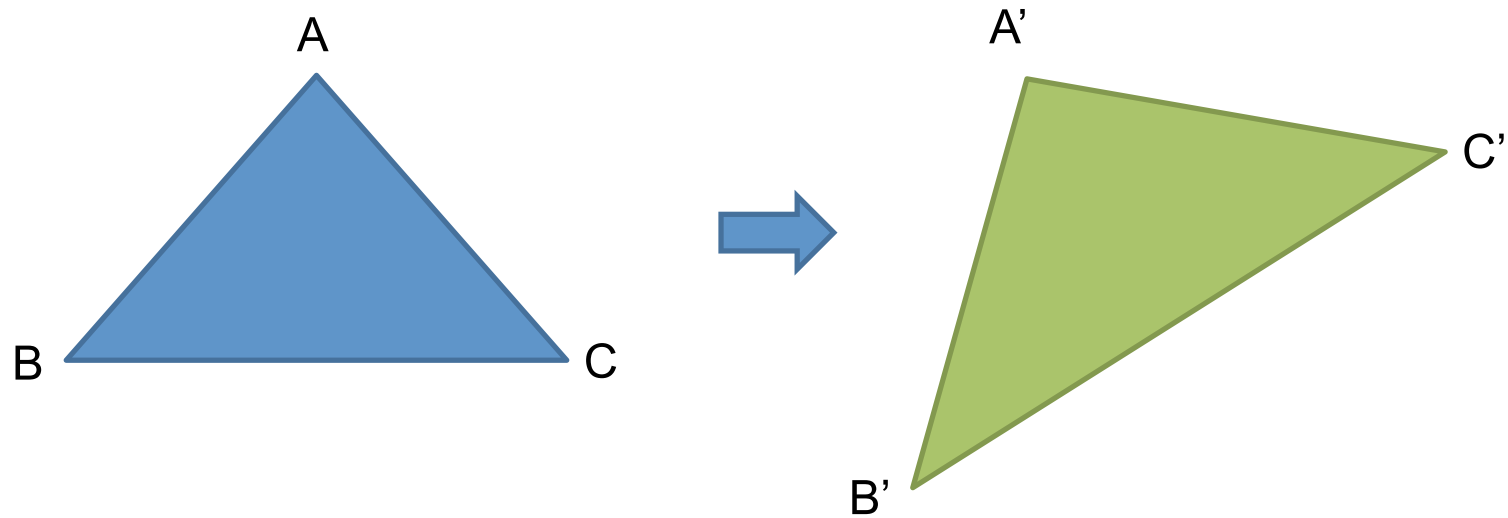
Warping between two triangles



- Idea: find an affine that transforms ABC to A'B'C'
- 6 unknowns, 6 equations

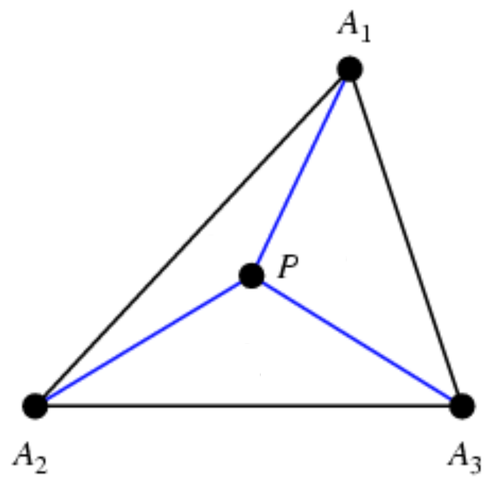
$$\begin{bmatrix} a & b & e \\ c & d & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_A \\ y_A \\ 1 \end{bmatrix} = \begin{bmatrix} x_{A'} \\ y_{A'} \\ 1 \end{bmatrix} \quad \begin{bmatrix} a & b & e \\ c & d & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_B \\ y_B \\ 1 \end{bmatrix} = \begin{bmatrix} x_{B'} \\ y_{B'} \\ 1 \end{bmatrix} \quad \begin{bmatrix} a & b & e \\ c & d & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_C \\ y_C \\ 1 \end{bmatrix} = \begin{bmatrix} x_{C'} \\ y_{C'} \\ 1 \end{bmatrix}$$

Warping between two triangles



- Idea: find an affine that transforms ABC to A'B'C'
- 6 unknowns, 6 equations
- A more direct way

Barycentric coordinates



- Idea: represent P using A1,A2,A3

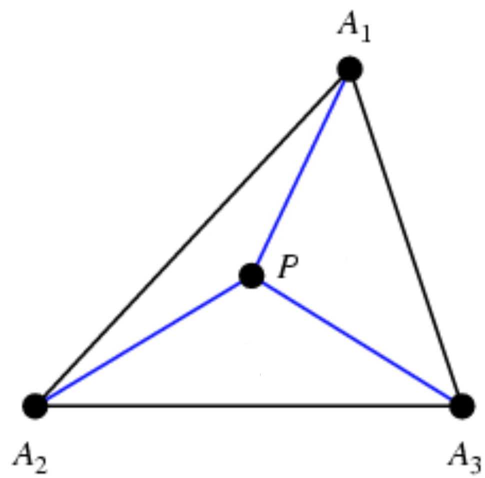
$$P - A_1 = \beta \cdot (A_2 - A_1) + \gamma \cdot (A_3 - A_1)$$

$$P = (1 - \beta - \gamma) \cdot A_1 + \beta \cdot A_2 + \gamma \cdot A_3$$

$$P = t_1 \cdot A_1 + t_2 \cdot A_2 + t_3 \cdot A_3$$

$$t_1 + t_2 + t_3 = 1$$

Barycentric coordinates



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$$P = (1 - \beta - \gamma) \cdot A_1 + \beta \cdot A_2 + \gamma \cdot A_3$$

$$\begin{bmatrix} x_P \\ y_P \end{bmatrix} = t_1 \cdot \begin{bmatrix} x_{A_1} \\ y_{A_1} \end{bmatrix} + t_2 \cdot \begin{bmatrix} x_{A_2} \\ y_{A_2} \end{bmatrix} + t_3 \cdot \begin{bmatrix} x_{A_3} \\ y_{A_3} \end{bmatrix}$$

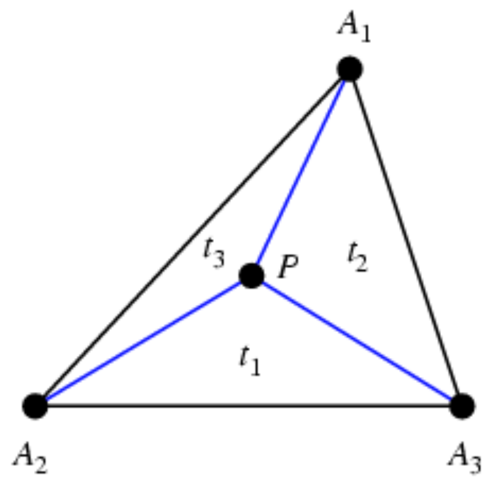
$$t_1 + t_2 + t_3 = 1$$

$$t_1 = \frac{area(PA_2A_3)}{area(A_1A_2A_3)}$$

$$t_2 = \frac{area(PA_3A_1)}{area(A_1A_2A_3)}$$

$$t_3 = \frac{area(PA_1A_2)}{area(A_1A_2A_3)}$$

Barycentric coordinates



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$$P = (1 - \beta - \gamma) \cdot A_1 + \beta \cdot A_2 + \gamma \cdot A_3$$

$$P = t_1 \cdot A_1 + t_2 \cdot A_2 + t_3 \cdot A_3$$

$$t_1 + t_2 + t_3 = 1$$

$$t_1 = \frac{area(PA_2A_3)}{area(A_1A_2A_3)}$$

$$t_2 = \frac{area(PA_3A_1)}{area(A_1A_2A_3)}$$

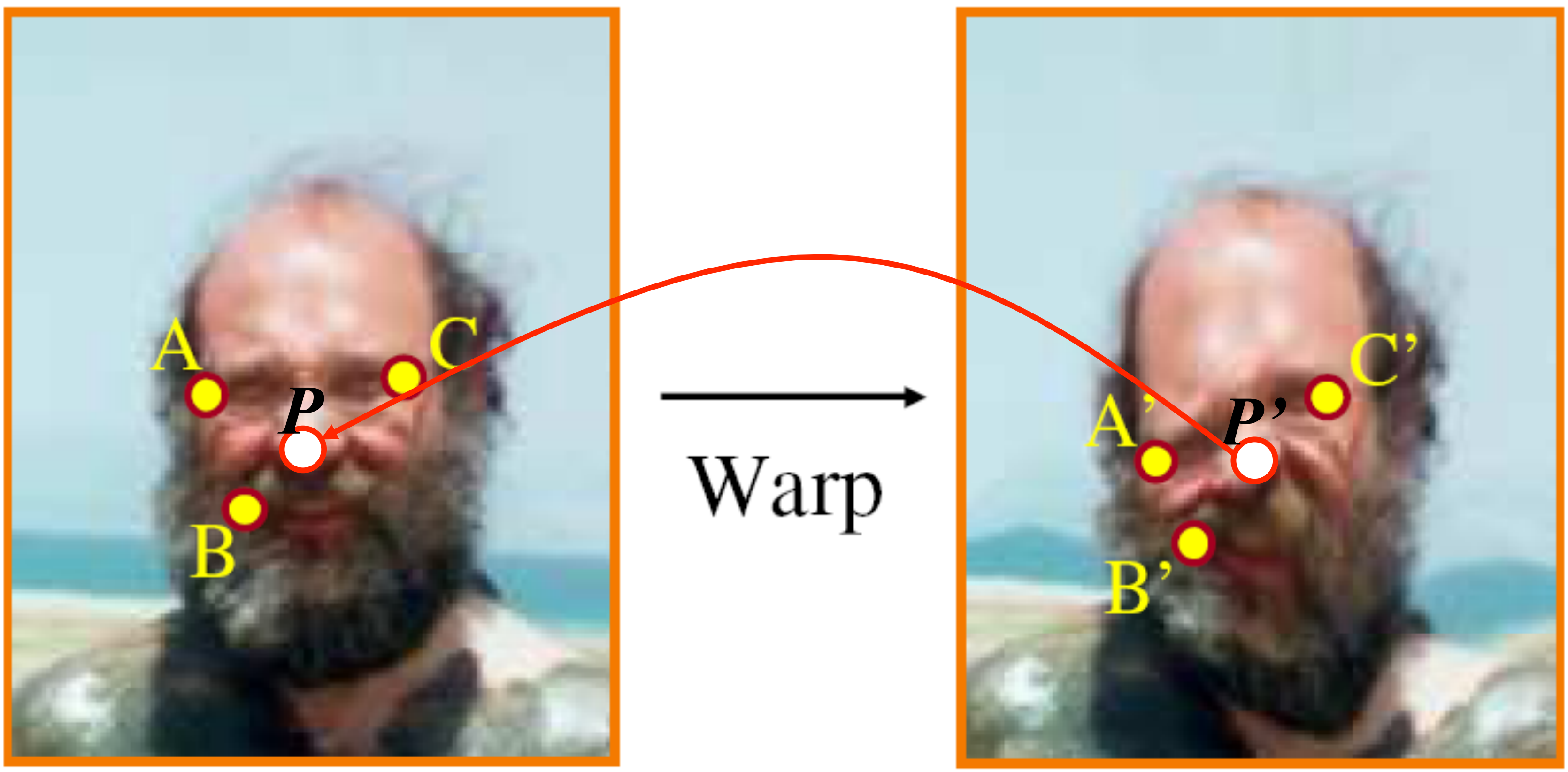
$$t_3 = \frac{area(PA_1A_2)}{area(A_1A_2A_3)}$$

Non-parametric image warping

$$P = w_A A + w_B B + w_C C$$

$$P' = w_A A' + w_B B' + w_C C'$$

Barycentric coordinate



Turns out to be equivalent to affine transform