

# CS559: Computer Graphics

Lecture 11: Antialiasing & Visibility, Intro to OpenGL

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Spring 2010

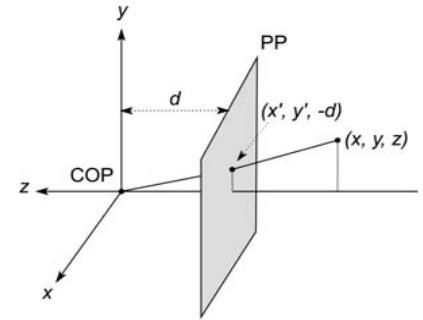
# Annoucement

- Project 2 is out

# Last time: Homogeneous coordinates and perspective projection

Now we can re-write the perspective projection as a matrix equation:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} (d/z)x \\ (d/z)y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z/d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

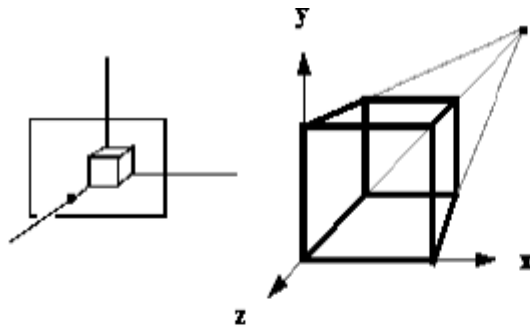


$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

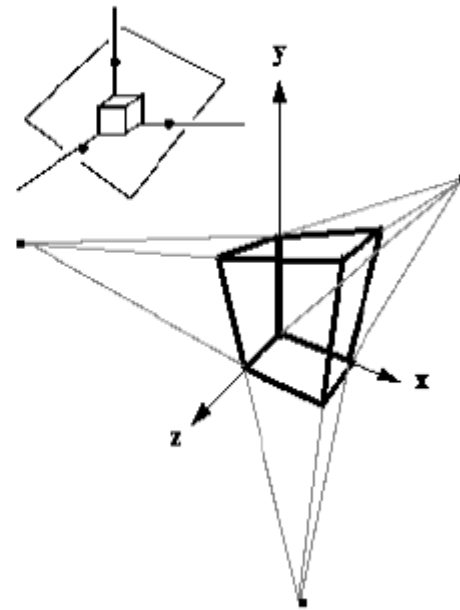
Orthographic projection

$$= \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} = \begin{bmatrix} (d/z)x \\ (d/z)y \\ d \\ 1 \end{bmatrix}$$

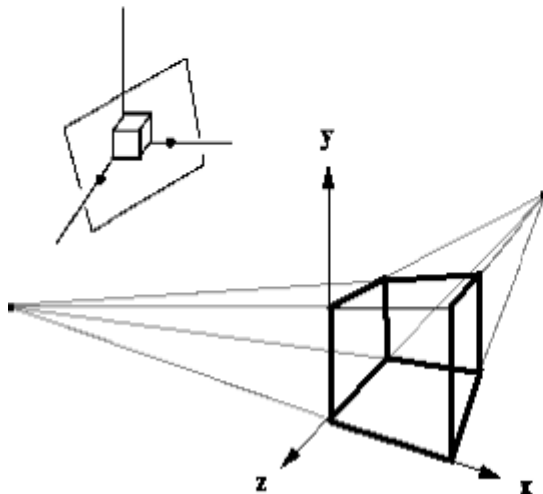
# Vanishing points



*One Point Perspective*  
(z-axis vanishing point)



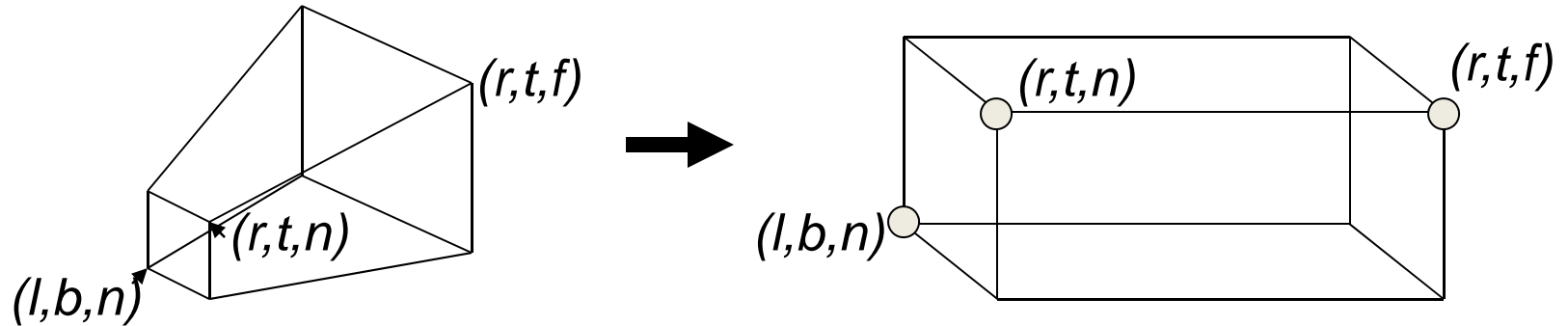
*Three Point Perspective*  
(z, x, and y-axis  
vanishing points)



*Two Point Perspective*  
z, and x-axis vanishing points

# Perspective Projection Matrices

- We want a matrix that will take points in our perspective view volume and transform them into the orthographic view volume
  - This matrix will go in our pipeline before an orthographic projection matrix

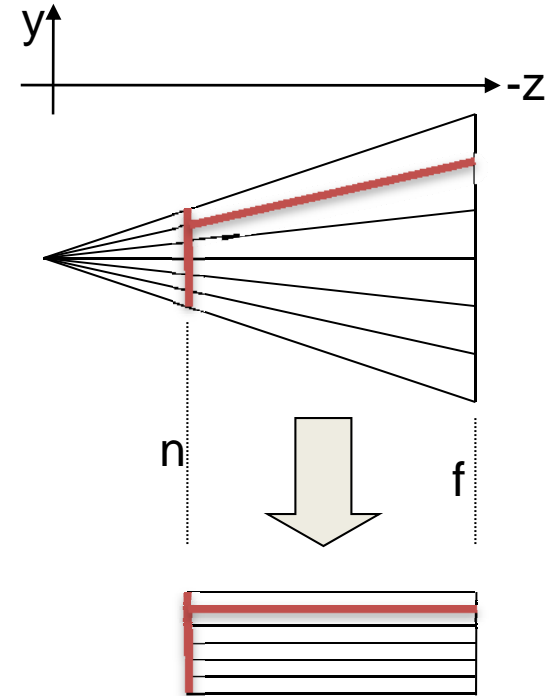


# Properties of this mapping

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/n & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} (n/z)x \\ (n/z)y \\ n \\ 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & (n+f)/n & -f \\ 0 & 0 & 1/n & 0 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = M \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{nx}{z} \\ \frac{ny}{z} \\ f + n - \frac{fn}{z} \\ 1 \end{bmatrix}$$

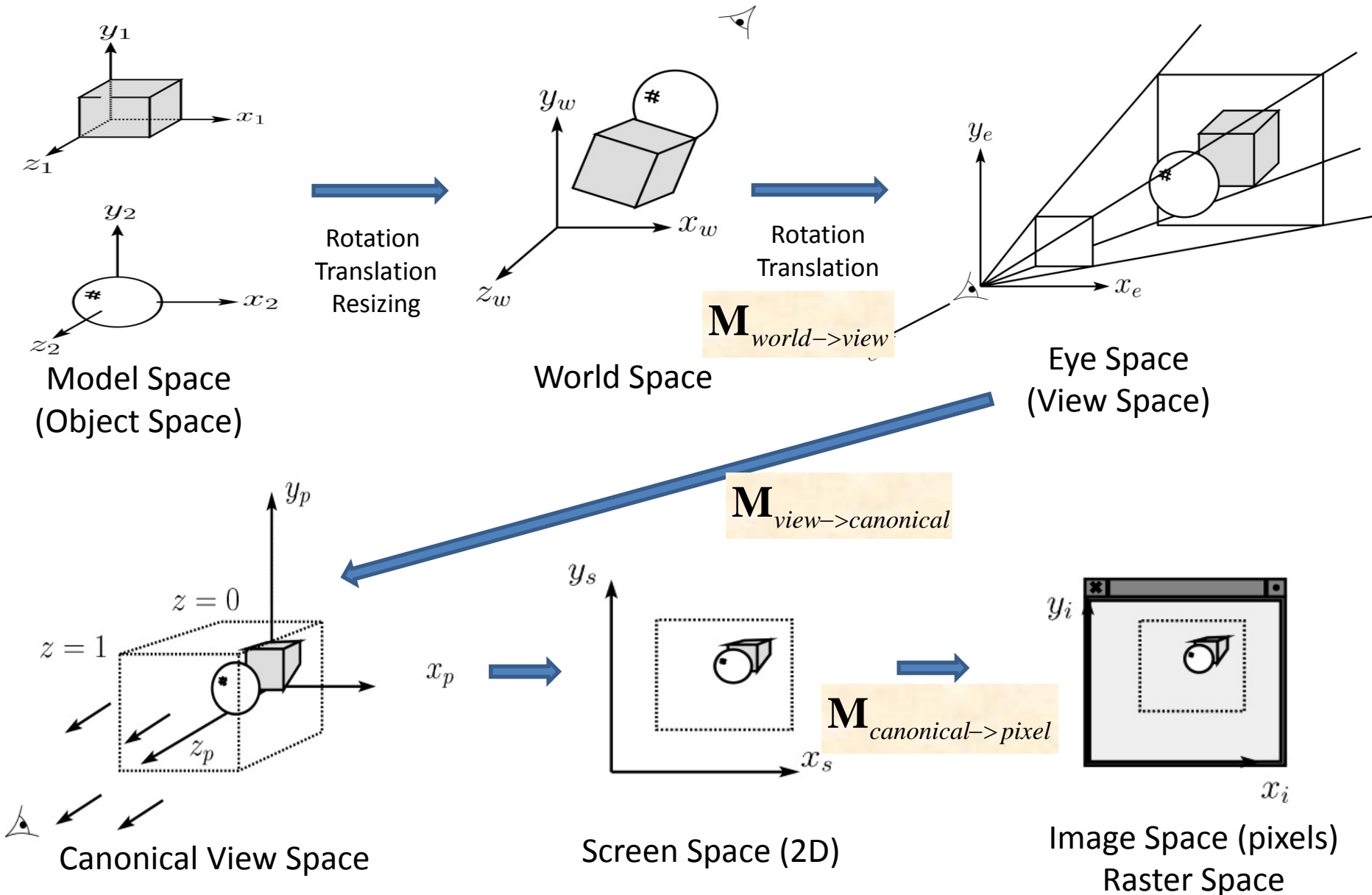


If  $z = n$ ,  $M(x,y,z,1) = [x,y,z,1]$   
near plane is unchanged

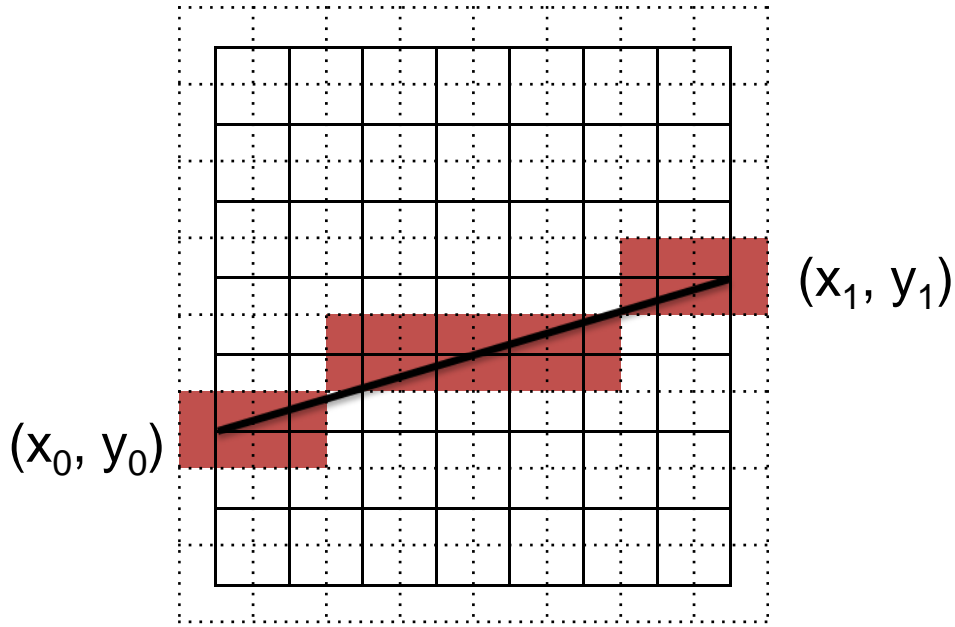
If  $x/z = c_1$ ,  $y/z = c_2$ , then  $x' = n \cdot c_1$ ,  $y' = n \cdot c_2$   
bending converging rays to parallel rays

If  $z_1 < z_2$ ,  $z_1' < z_2'$   
 $z$  ordering is preserved

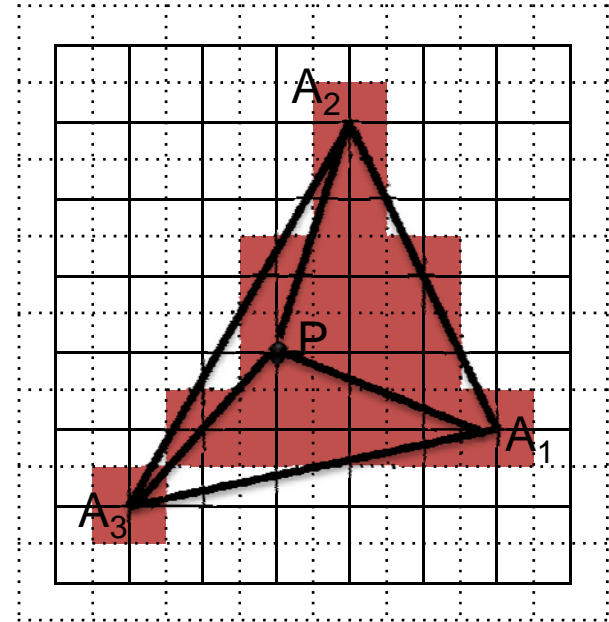
# 3D Geometry Pipeline



# Last time



Line drawing

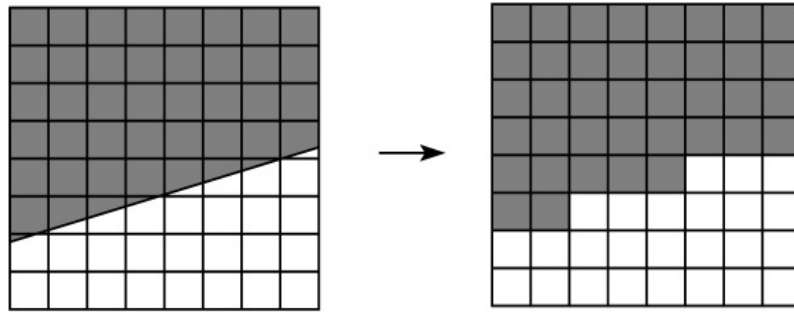


Triangle filling

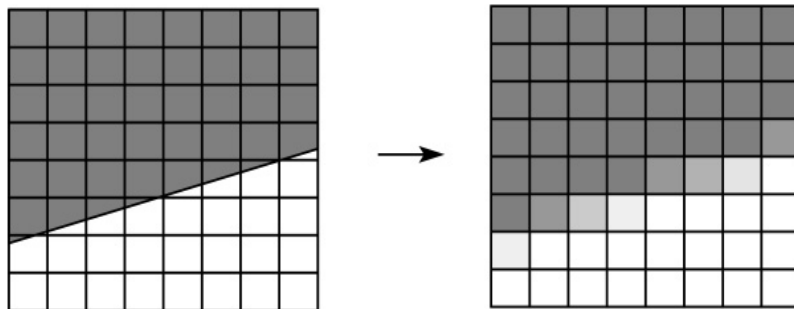


# Aliasing in rendering

- One of the most common rendering artifacts is the “jaggies”. Consider rendering a white polygon against a black background:

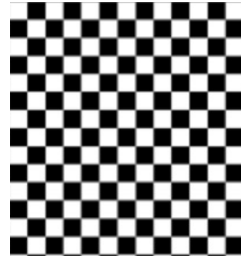


- We would instead like to get a smoother transition:

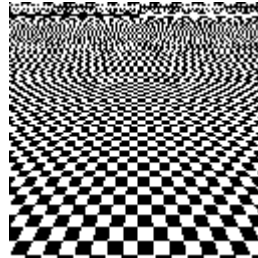


# Other types of Aliasing

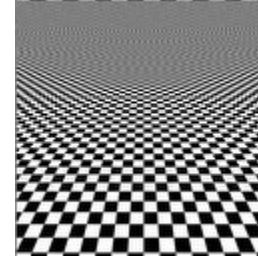
- Image warping



Original



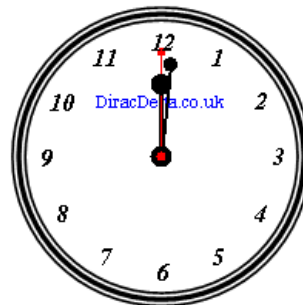
Aliased



Anti-Aliased

Images from answers.com

- Motion Aliasing



- If you were to only **look at the clock every 50 minutes** then the minute hand would appear to rotate anticlockwise.
- The hour hand would still rotate in the correct direction as you have satisfied Nyquist.
- The second hand would jitter around depending on how accurate you were with your observations.

# Anti-aliasing

- **Q:** How do we avoid aliasing artifacts?

## 1. Sampling:

Increase sampling rate -- not practical for fixed resolution display.

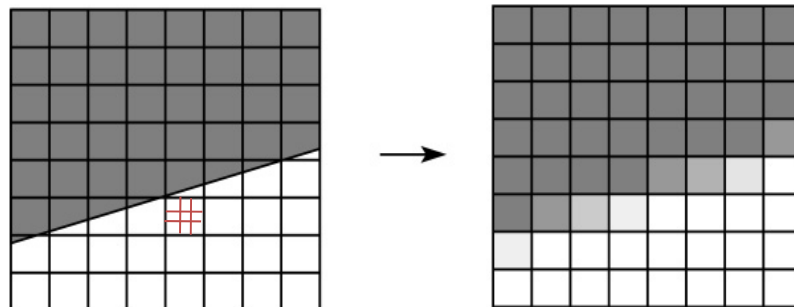
## 2. Pre-filtering:

Smooth out high frequencies analytically. Requires an analytic function.

## 3. Combination:

Supersample and average down.

- **Example - polygon:**



Memory requirement?

# Anti-aliasing

- **Q:** How do we avoid aliasing artifacts?

## 1. Sampling:

Increase sampling rate -- not practical for fixed resolution display.

## 2. Pre-filtering:

Smooth out high frequencies analytically. Requires an analytic function.

# Box filter

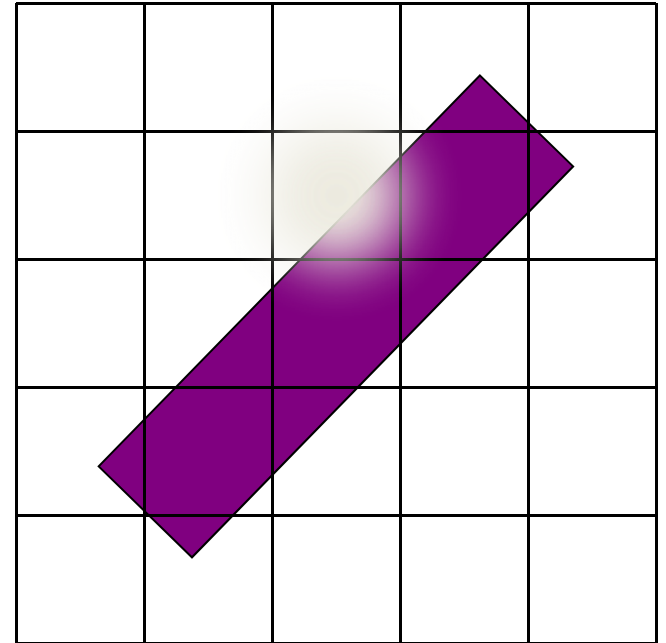
- Consider a line as having thickness (all good drawing programs do this)
- Consider pixels as little squares
- Set brightness according to the proportion of the square covered by the line

A 5x5 grid illustrating the box filter process. A diagonal line with thickness is drawn across the grid. The values in the cells represent the proportion of the square covered by the line. The values are: 0, 0, 0, 1/8, 0 (top row); 0, 0, 1/4, .914, 1/8 (second row); 0, 1/4, .914, 1/4, 0 (third row); 1/8, .914, 1/4, 0, 0 (fourth row); 0, 1/8, 0, 0, 0 (bottom row).

0	0	0	1/8	0
0	0	1/4	.914	1/8
0	1/4	.914	1/4	0
1/8	.914	1/4	0	0
0	1/8	0	0	0

# In general:

- Place the “filter” at each pixel, and integrate product of pixel and line
- Common filters are Gaussians



# Anti-aliasing

- **Q:** How do we avoid aliasing artifacts?

## 1. Sampling:

Increase sampling rate -- not practical for fixed resolution display.

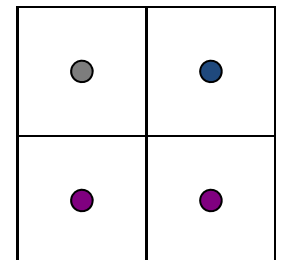
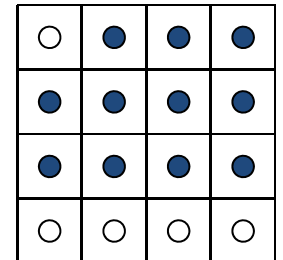
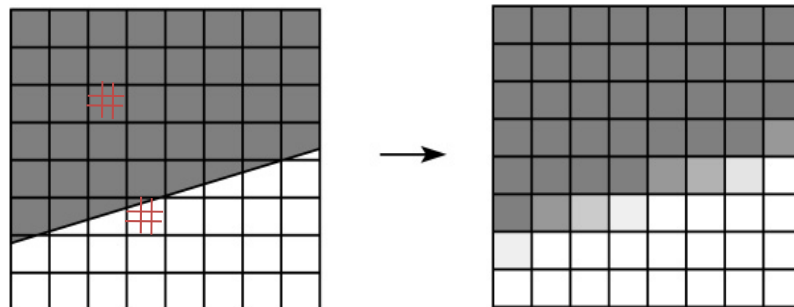
## 2. Pre-filtering:

Smooth out high frequencies analytically. Requires an analytic function.

## 3. Combination:

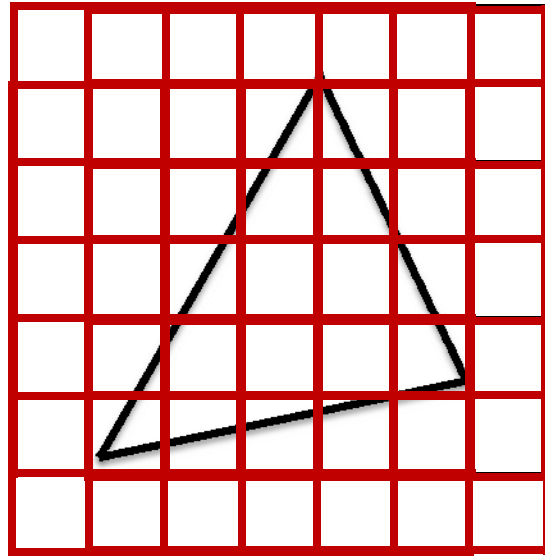
Supersample and average down.

- **Example - polygon:**



Memory requirement?

# Implementing antialiasing

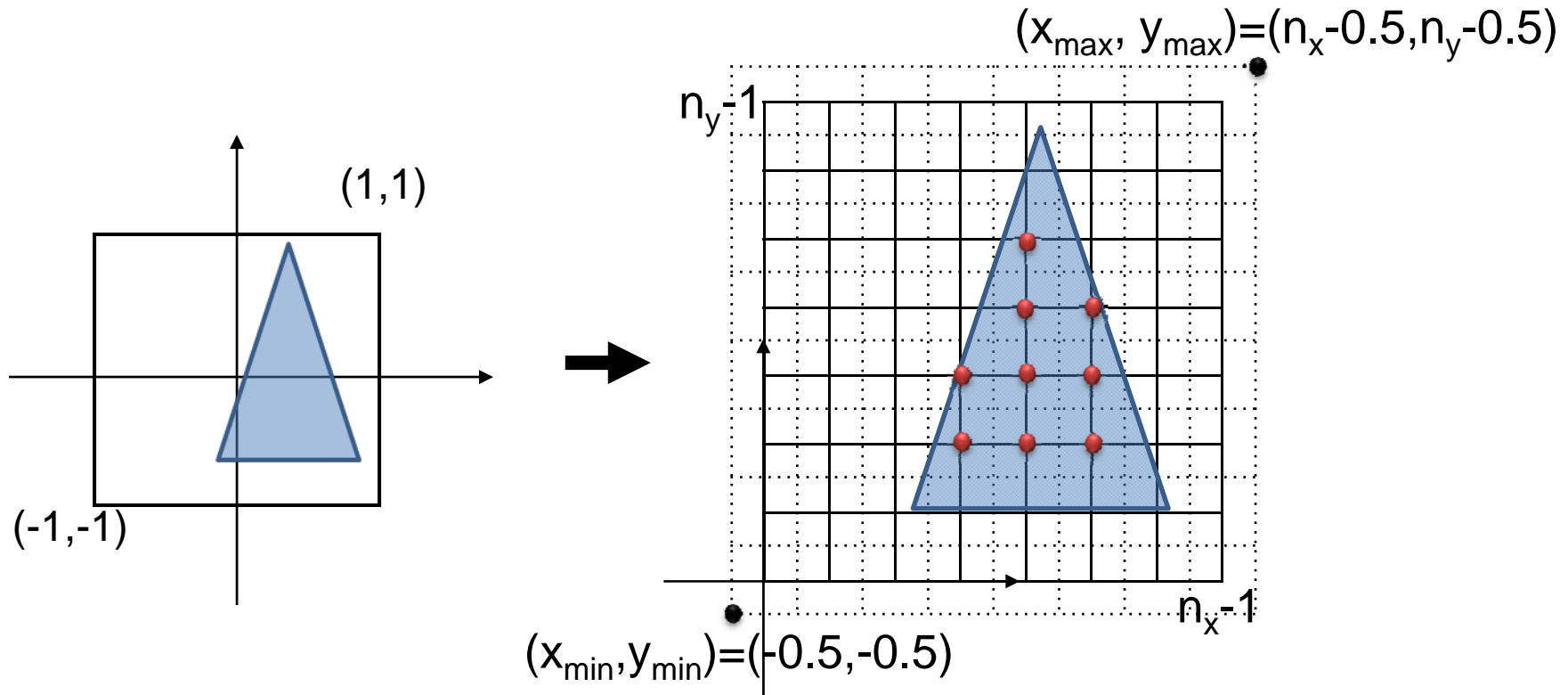


Assuming this is a 2X supersampling grid, how to achieve anti-aliasing without using 4X memory?

Rasterize shifted versions of the triangle on the original grid, accumulate the color, and divide the final image by the number of shifts



# Canonical $\rightarrow$ Window Transform



$$\begin{bmatrix} x_{pixel} \\ y_{pixel} \\ z_{pixel} \\ 1 \end{bmatrix} = \begin{bmatrix} (x_{\max} - x_{\min})/2 & 0 & 0 & (x_{\max} + x_{\min})/2 \\ 0 & (y_{\max} - y_{\min})/2 & 0 & (y_{\max} + y_{\min})/2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{canonical} \\ y_{canonical} \\ z_{canonical} \\ 1 \end{bmatrix}$$

$\mathbf{M}_{canonical \rightarrow pixel}$

# Polygon anti-aliasing

Without antialiasing



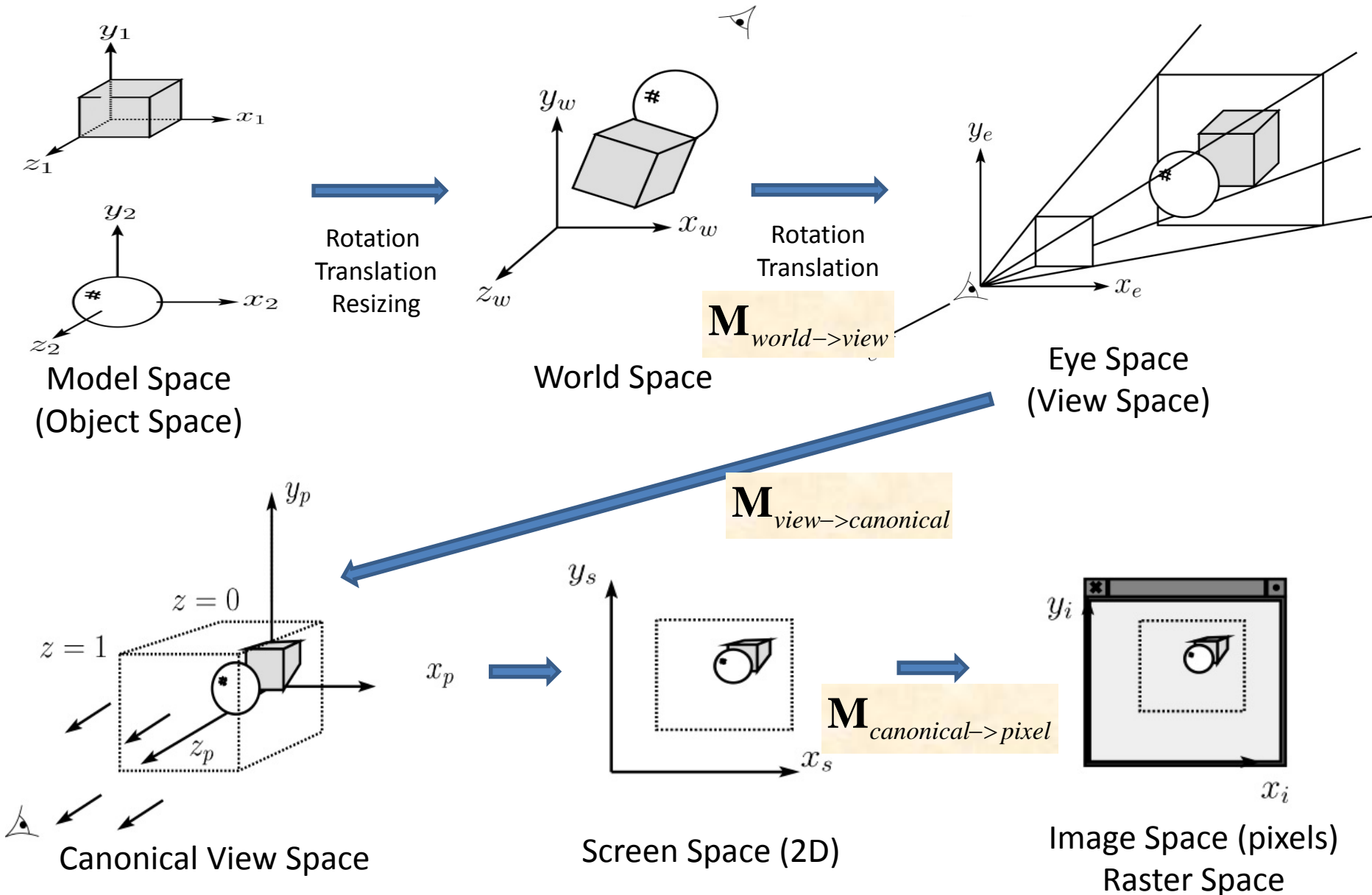
With antialiasing



*Magnification*



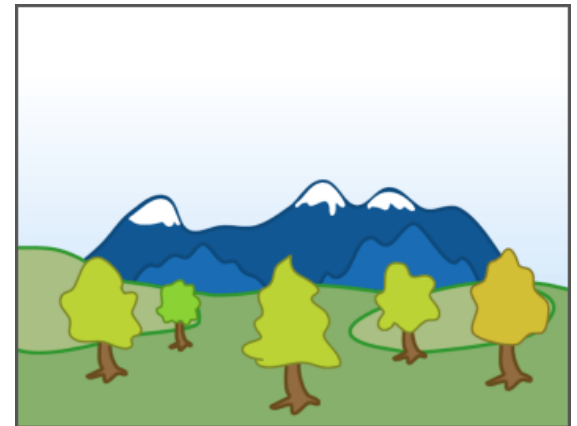
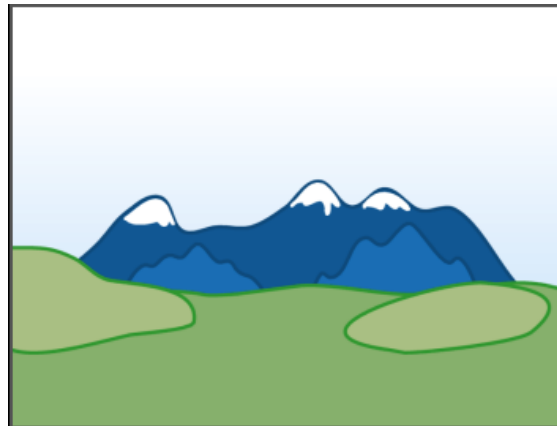
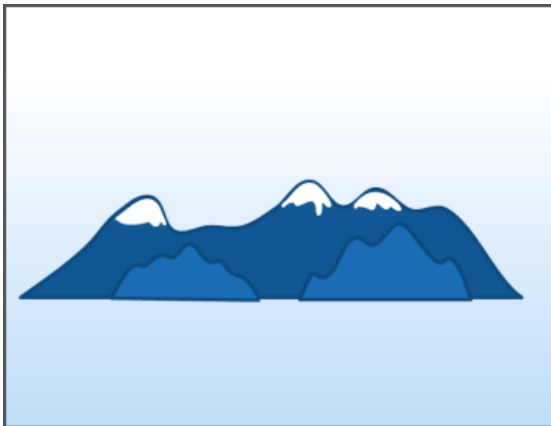
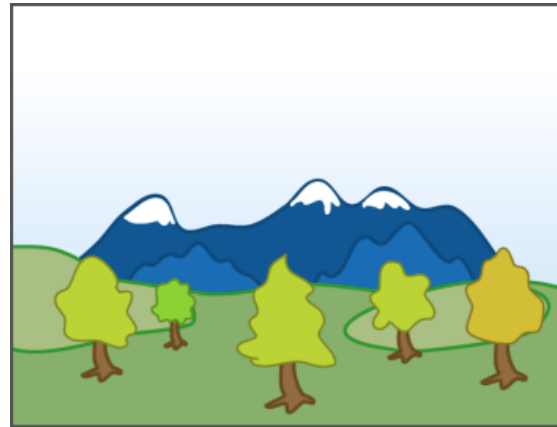
# 3D Geometry Pipeline



# Visibility

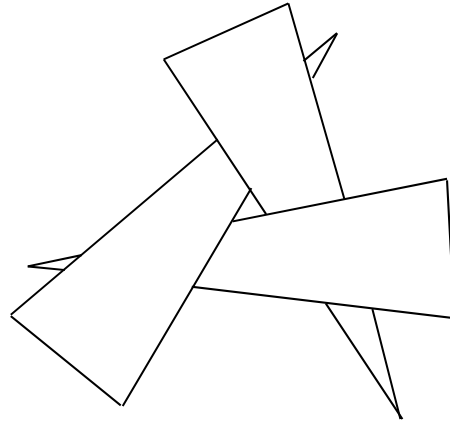
- Given a set of polygons, which is visible at each pixel? (in front, etc.). Also called *hidden surface removal*
- Very large number of different algorithms known.  
Two main classes:
  - Object precision
    - computations that operate on primitives
      - triangle A occludes triangle B
  - Image precision
    - computations at the pixel level
      - pixel P sees point Q

# Painter's Algorithm



Draw objects in a back-to-front order

# Painter's algorithm



Failure case

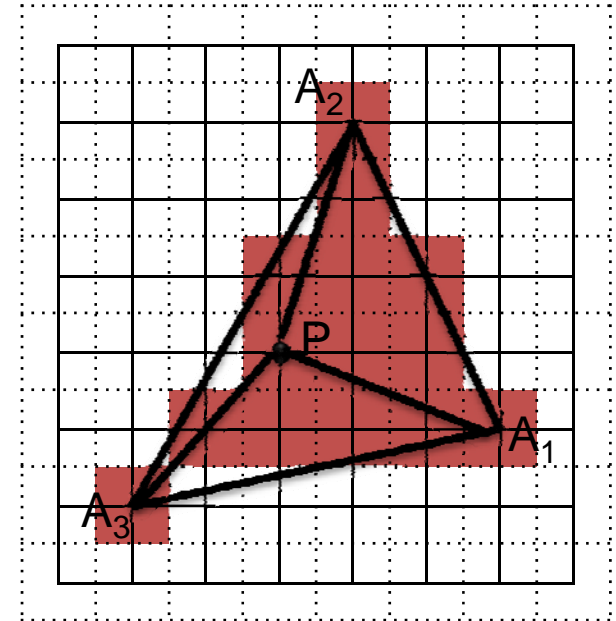
# Z-buffer (image precision)

- The **Z-buffer** or **depth buffer** algorithm [Catmull, 1974] is probably the simplest and most widely used.
- For each pixel on screen, have at least two buffers
  - Color buffer stores the current color of each pixel
    - The thing to ultimately display
  - Z-buffer stores at each pixel the depth of the **nearest thing seen so far**
    - Also called the depth buffer

# Z-buffer

- Here is pseudocode for the Z-buffer hidden surface algorithm:

```
for each pixel  $(i,j)$  do
  Z-buffer  $[i,j] \leftarrow FAR$ 
  Framebuffer  $[i,j] \leftarrow \langle \text{background color} \rangle$ 
end for
for each polygon A do
  for each pixel in A do
    Compute depth  $z$  and shade  $s$  of A at  $(i,j)$ 
    if  $z > Z\text{-buffer}[i,j]$  then
      Z-buffer  $[i,j] \leftarrow z$ 
      Framebuffer  $[i,j] \leftarrow s$ 
    end if
  end for
end for
end for
```



Triangle filling

How to compute shades/color?

How to compute depth  $z$ ?



# Precision of depth

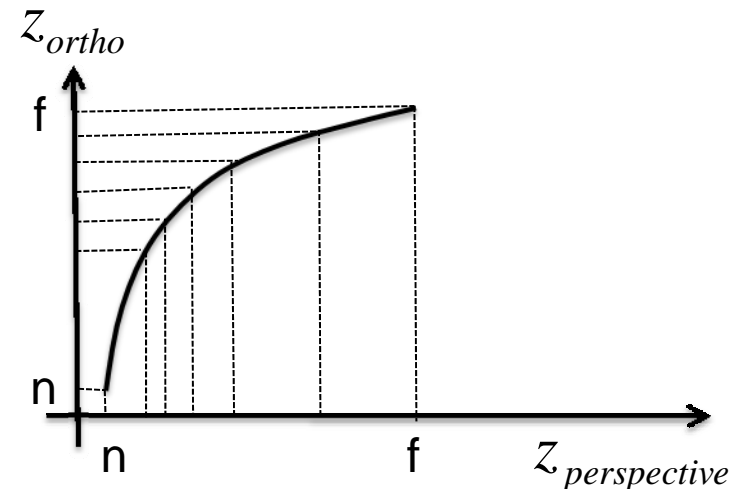
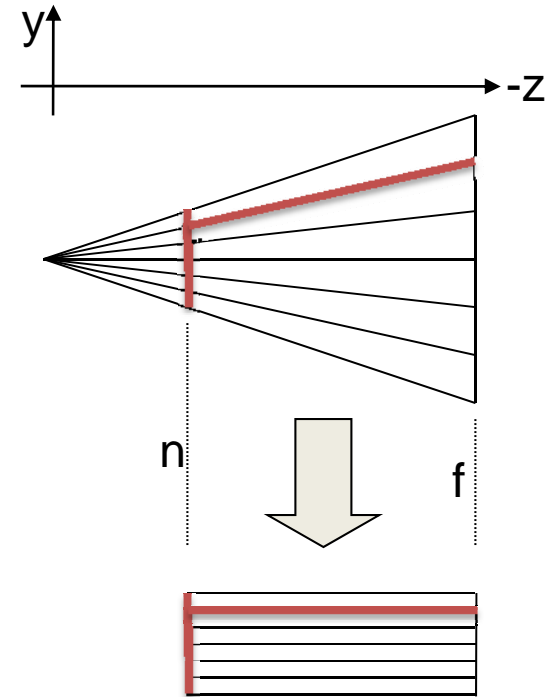
$$z_{ortho} = f + n - \frac{fn}{z_{perspective}}$$

$$\Delta z_{ortho} \approx \frac{fn}{z_{perspective}^2} \Delta z_{perspective}$$

$$\Delta z_{perspective} \approx \frac{z_{perspective}^2}{fn} \Delta z_{ortho}$$

$$\Delta z_{perspective}^{max} \approx \frac{f}{n} \Delta z_{ortho}$$

- Depth resolution not uniform
- More close to near plane, less further away
- Common mistake: set near = 0, far = infity. Don't do this. Can't set near = 0; lose depth resolution.



# Other issues of Z buffer

- Advantages:
  - Simple and now ubiquitous in hardware
    - A z-buffer is part of what makes a graphics card “3D”
  - Computing the required depth values is simple
- Disadvantages:
  - Depth quantization errors can be annoying
  - Can't easily do transparency

$(\alpha_1 I_1 \text{ over } \alpha_2 I_2) \text{ over } \alpha_3 I_3$

$(\alpha_1 I_1 \text{ over } \alpha_3 I_3) \text{ over } \alpha_2 I_2$

# The A-buffer (Image Precision)

- Handles transparent surfaces and filter anti-aliasing
- At each pixel, maintain a pointer to a **list** of polygons sorted by depth

# The A-buffer (Image Precision)

```
for each pixel  $(i,j)$  do
  Z-buffer  $[i,j] \leftarrow FAR$ 
  Framebuffer $[i,j] \leftarrow \langle \text{background color} \rangle$ 
end for
for each polygon  $A$  do
  for each pixel in  $A$  do
    Compute depth  $z$  and shade  $s$  of  $A$  at  $(i,j)$ 
    if  $z > Z\text{-buffer}[i,j]$  then
      Z-buffer  $[i,j] \leftarrow z$ 
      Framebuffer $[i,j] \leftarrow s$ 
    end if
  end for
end for
```

if polygon is opaque and covers pixel, insert into list, removing all polygons farther away  
if polygon is transparent, insert into list, but don't remove farther polygons

# A-Buffer Composite

For each pixel, we have a list of

$$(\alpha_1, I_1, z_1) (\alpha_2, I_2, z_2) \cdots (\alpha_N, I_N, z_N)$$

$$\textit{composite} \{ (\alpha_1, I_1, z_1) (\alpha_2, I_2, z_2) \cdots (\alpha_N, I_N, z_N) \}$$

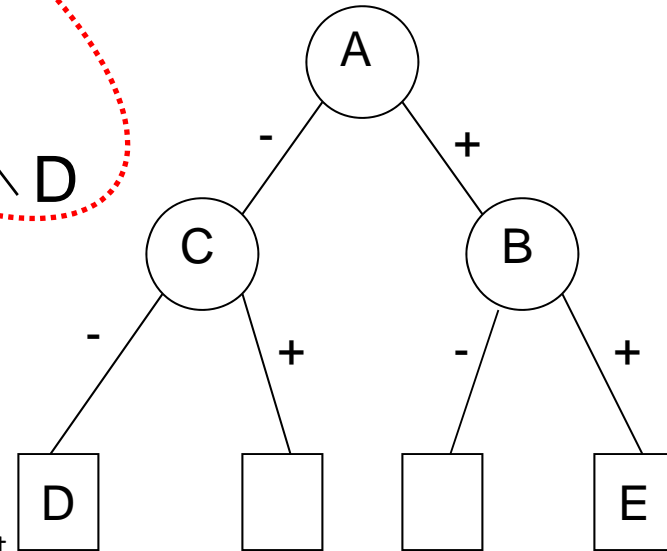
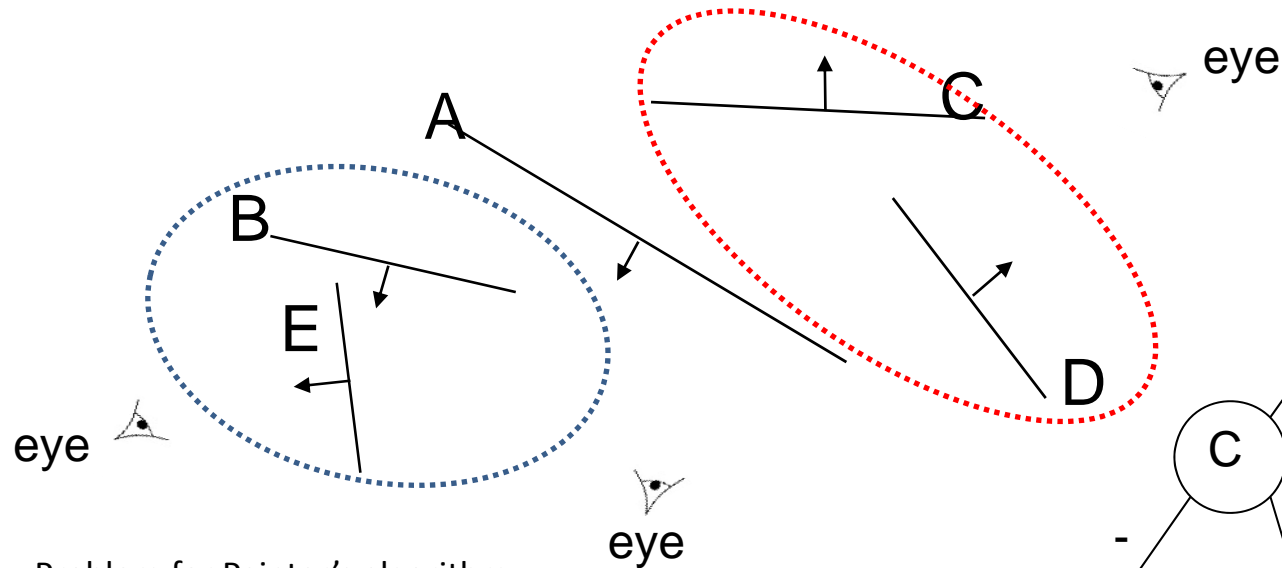
$$= \textit{composite} \{ (\alpha_1, I_1, z_1), \textit{composite} \{ (\alpha_2, I_2, z_2) \cdots (\alpha_N, I_N, z_N) \} \}$$

$$= \alpha_1 I_1 + (1 - \alpha_1) (\alpha_2 I_2 + (1 - \alpha_2) (\alpha_3 I_3 + \cdots \alpha_N I_N))$$

# The A-buffer (2)

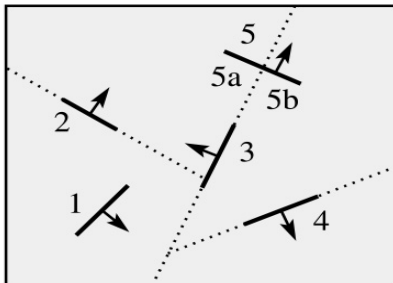
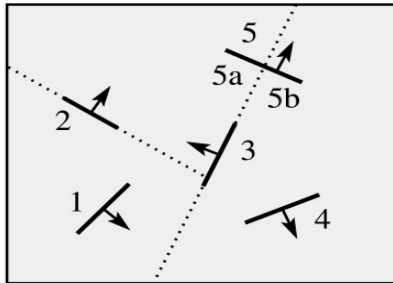
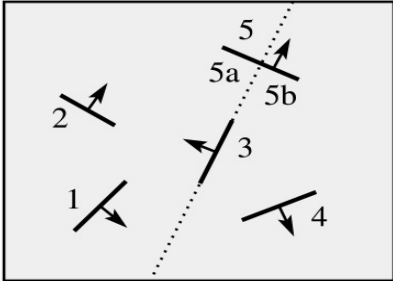
- Advantage:
  - Can do more than Z-buffer
  - Alpha can represent partial coverage as well
- Disadvantages:
  - Not in hardware, and slow in software
  - Still at heart a z-buffer: depth quantization problems
- But, used in high quality rendering tools

# Binary-space partitioning (BSP) trees



- Problem for Painter's algorithm:
  - Order is view dependent
- Idea:
  - Do extra preprocessing to allow quick display from any viewpoint.
- Key observation: A polygon A is painted in correct order if
  - Polygons on far side of A are painted first
  - A is painted next
  - Polygons on near side of A are painted last.
- Solution: build a tree to recursively partition the space and group polygons
- Why it works? What's the assumption?

# BSP tree creation



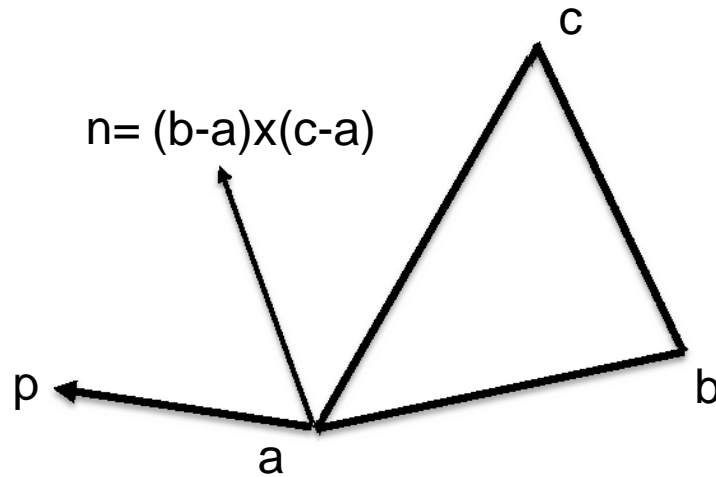
- **procedure** *MakeBSPTree*:
- **takes** *PolygonList L*
- **returns** *BSPTree*
- Choose polygon *A* from *L* to serve as root
- Split all polygons in *L* according to *A*
- $node \leftarrow A$
- $node.neg \leftarrow MakeBSPTree(\text{Polygons on neg. side of } A)$
- $node.pos \leftarrow MakeBSPTree(\text{Polygons on pos. side of } A)$
- **return** *node*
- **end procedure**



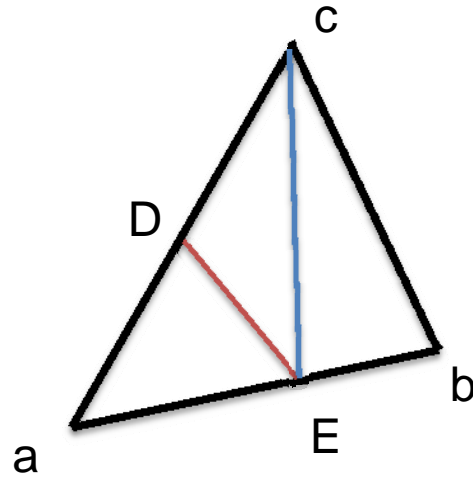
# Plane equation

Plane equation:  $f(p) = n^T(p-a)$

Positive side  $f(p) > 0$   
Negative side  $f(p) < 0$



# Split Triangles



$abc \Rightarrow aED, Ebc, EcD$

# BSP tree display

- **procedure** *DisplayBSPTree*:
- **Takes** *BSPTree T*
- **if** *T* is empty **then return**
- **if** viewer is in front (on pos. side) of *T.node*
- *DisplayBSPTree(T. \_\_\_\_\_)*
- *Draw T.node*
- *DisplayBSPTree(T. \_\_\_\_\_)*
- **else**
- *DisplayBSPTree(T. \_\_\_\_\_)*
- *Draw T.node*
- *DisplayBSPTree(T. \_\_\_\_\_)*
- **end if**
- **end procedure**

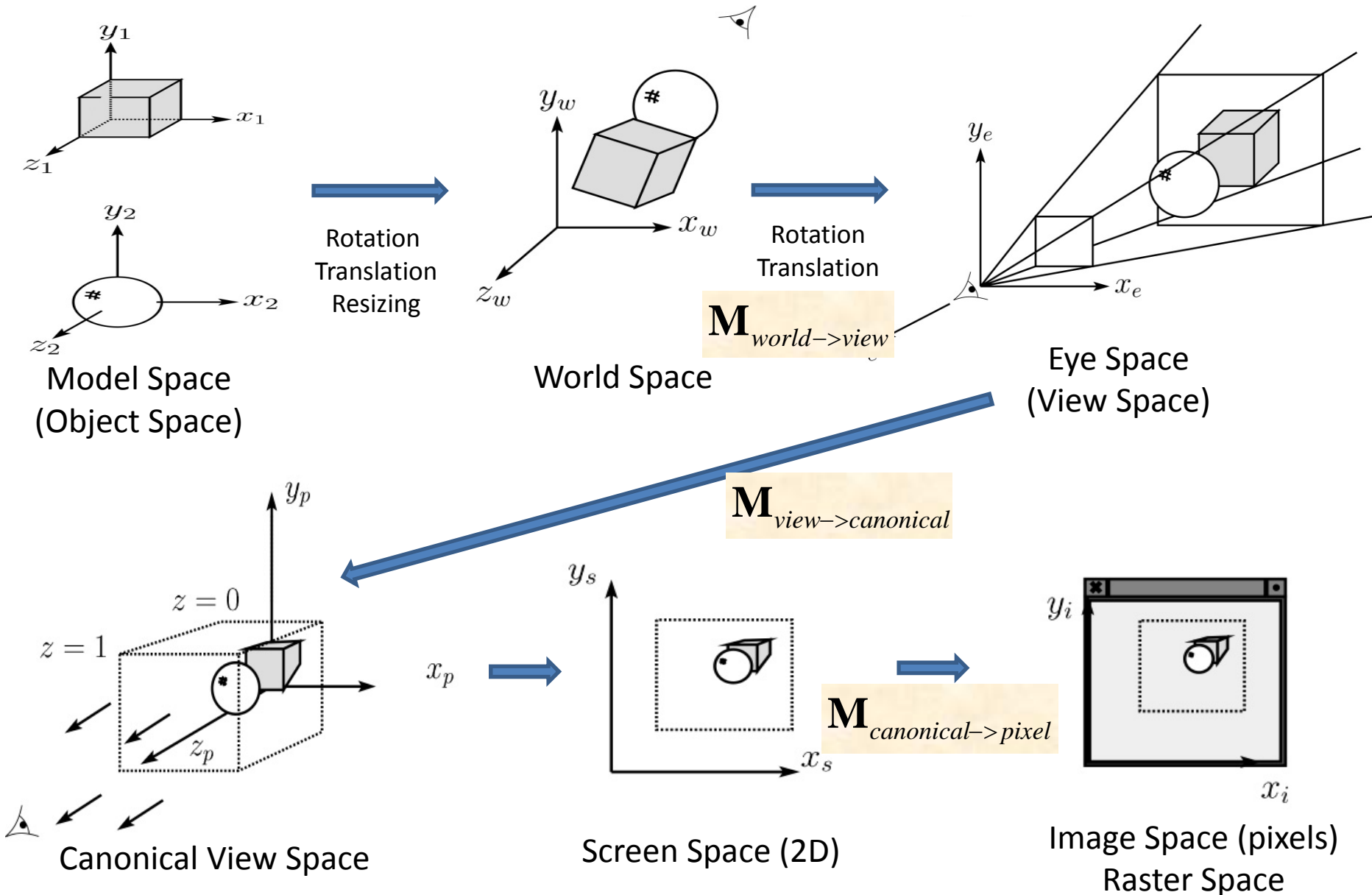
# Performance Notes

- Does how well the tree is balanced matter?
  - No
- Does the number of triangles matter?
  - Yes
- Performance is improved when fewer polygons are split --- in practice, best of  $\sim 5$  random splitting polygons are chosen.
- BSP is created in world coordinates. No projective matrices are applied before building tree.

# BSP-Tree Rendering (2)

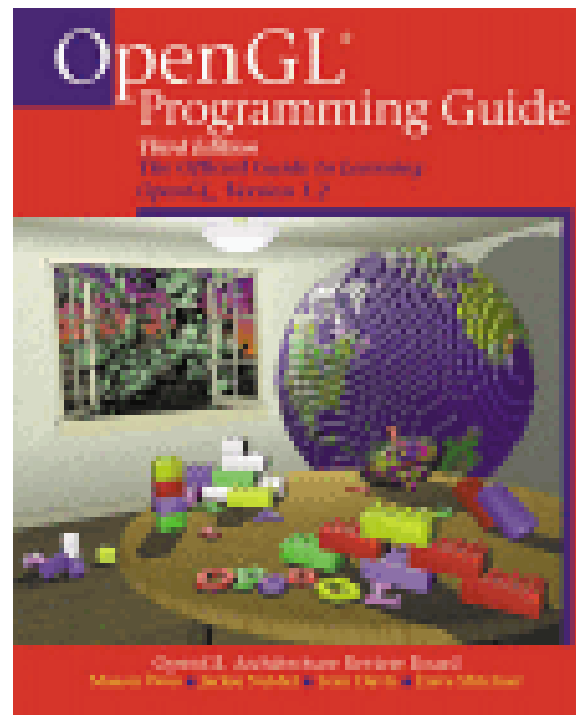
- Advantages:
  - One tree works for any viewing point
  - transparency works
    - Have back to front ordering for compositing
  - Can also render front to back, and avoid drawing back polygons that cannot contribute to the view
    - Major innovation in *Quake*
- Disadvantages:
  - Can be many small pieces of polygon

# 3D Geometry Pipeline

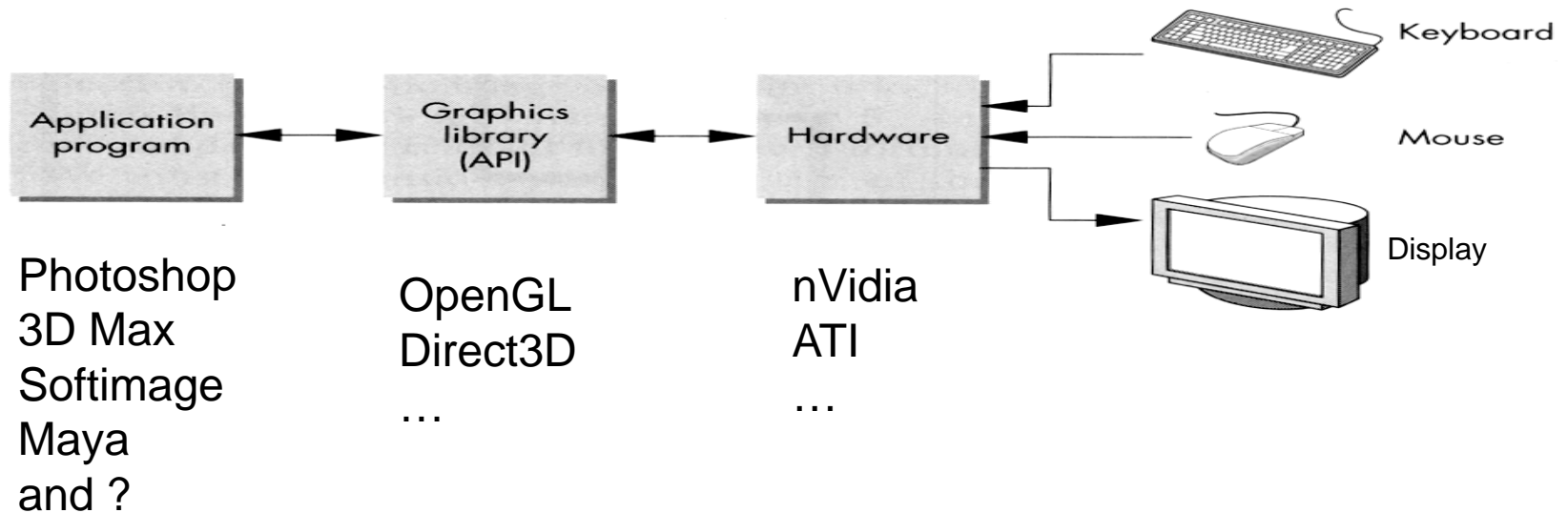


# OpenGL

- We have been focused on math description
- We'll move on practical graphics programming for a week



# Modern graphics systems



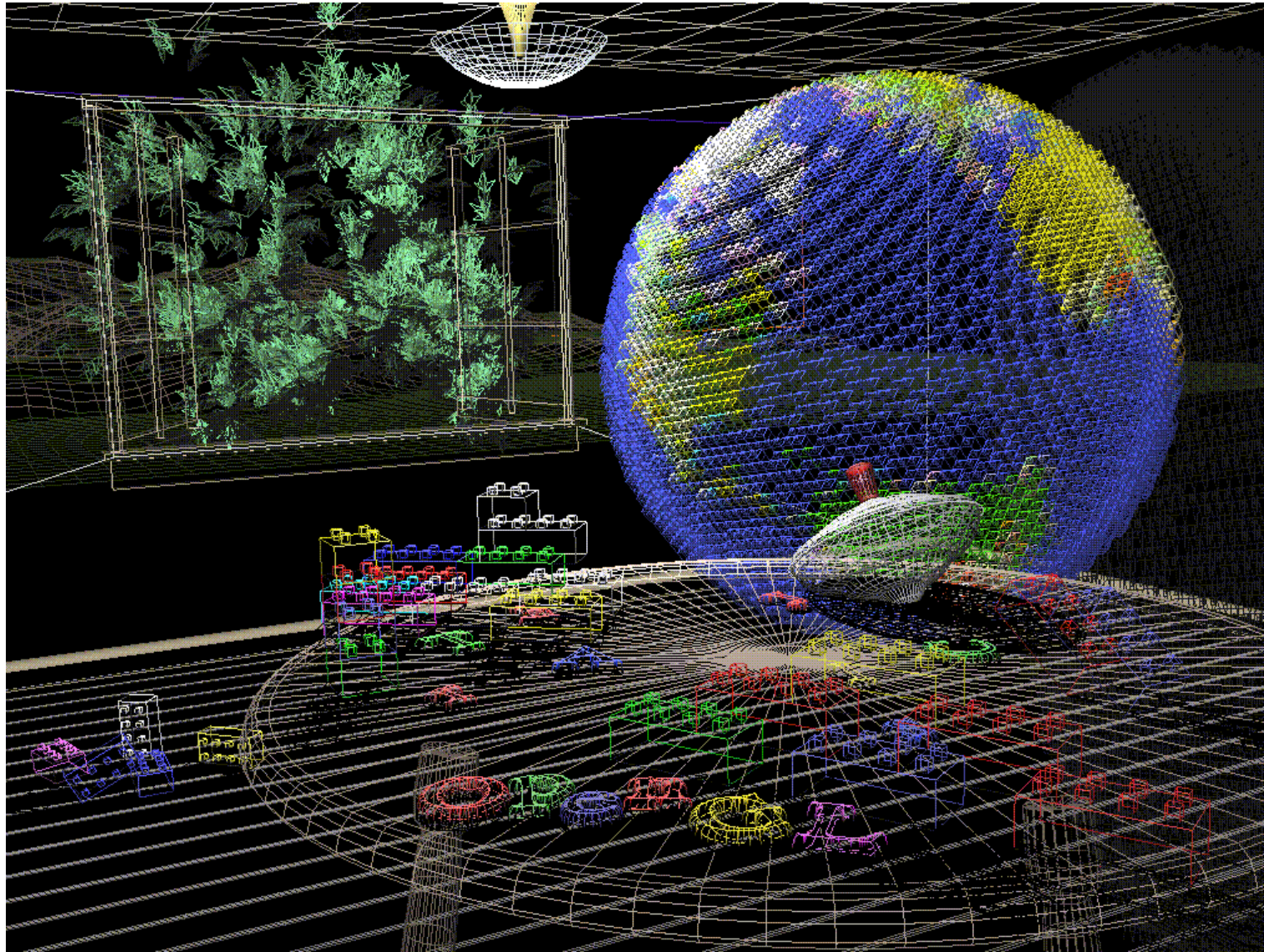
**Your homework**



# OpenGL

- A software interface to graphics hardware.
  - 700 distinct commands
    - 650 in the core OpenGL
    - 50 in the utility library
  - Specify objects, viewing, lighting, surface material
- Hardware independent interface
  - No commands for windowing tasks
  - No high level object models
    - You need to specify geometric primitives
      - Points, lines, polygons.

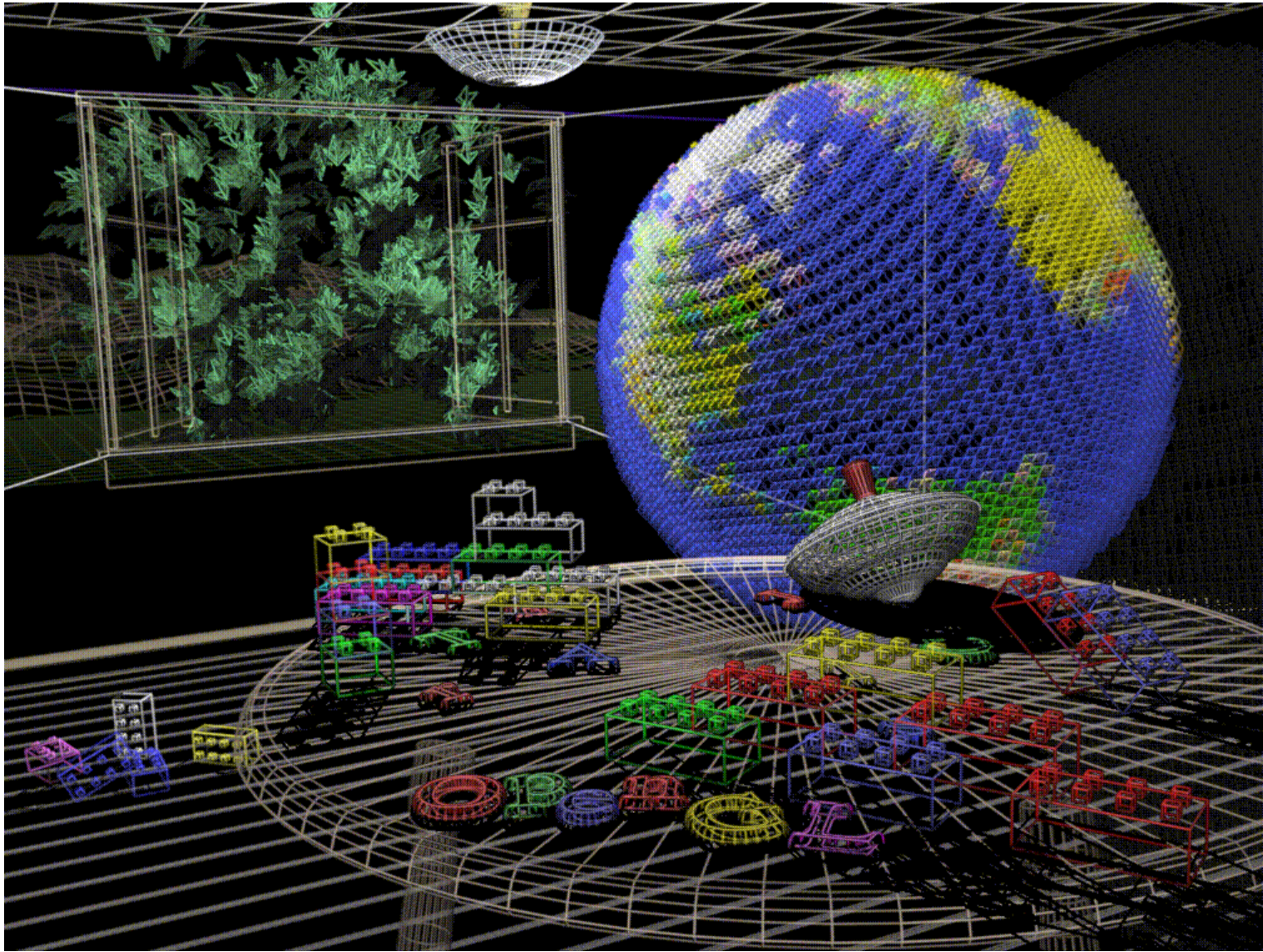
# What can OpenGL do?



wireframe



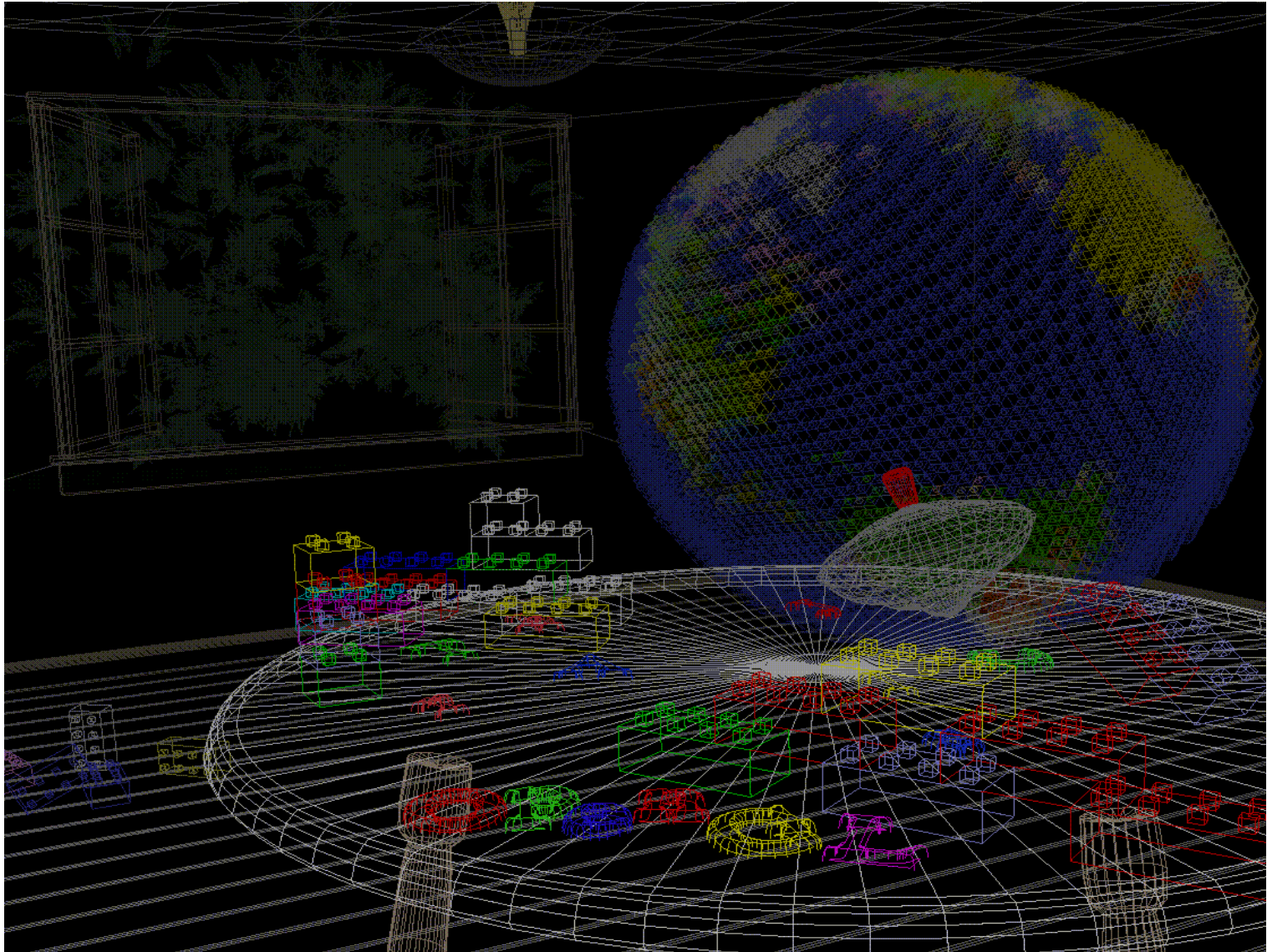
# What can OpenGL do?



Antialised lines

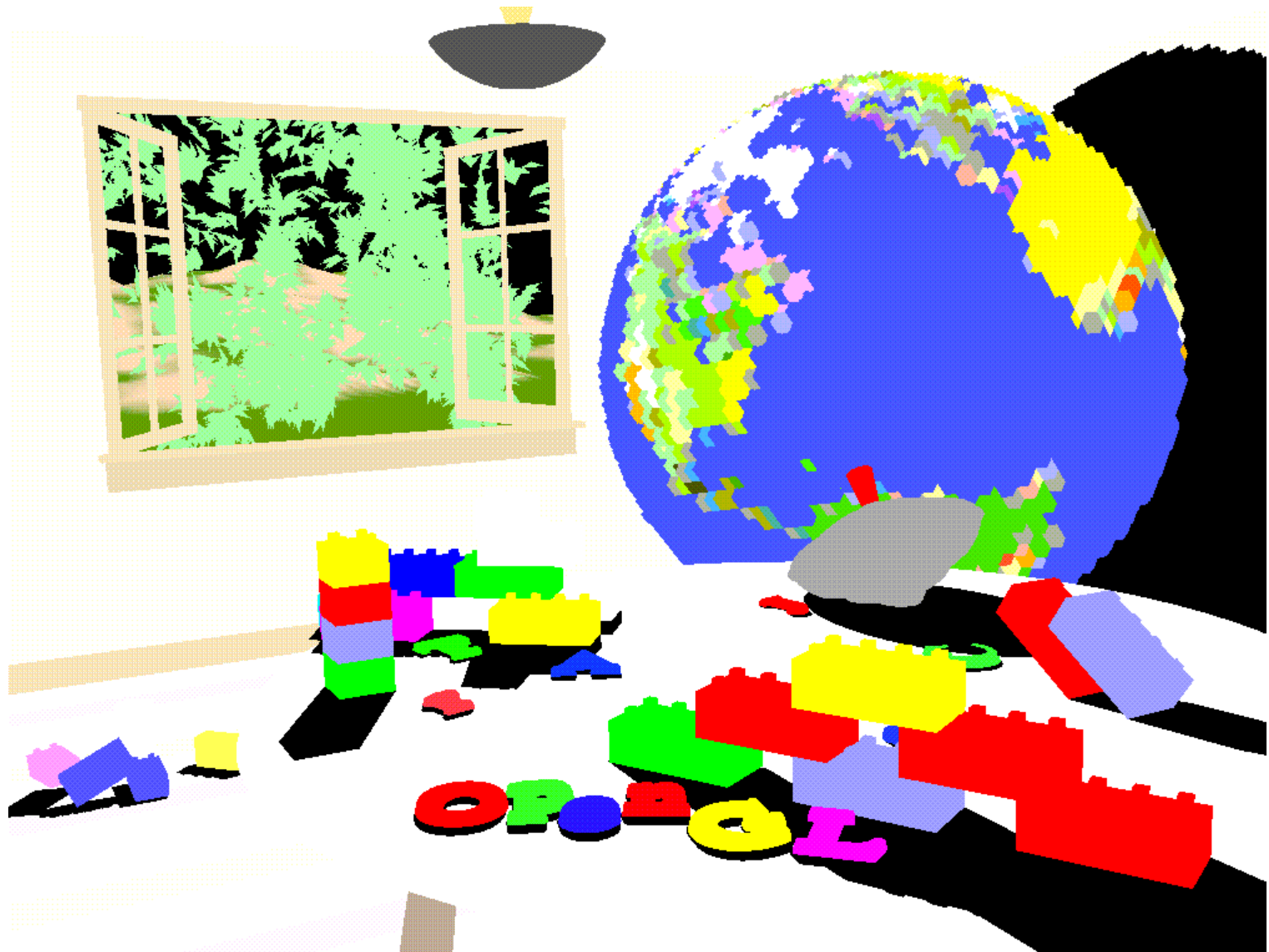


# What can OpenGL do?



Depth cue using fog

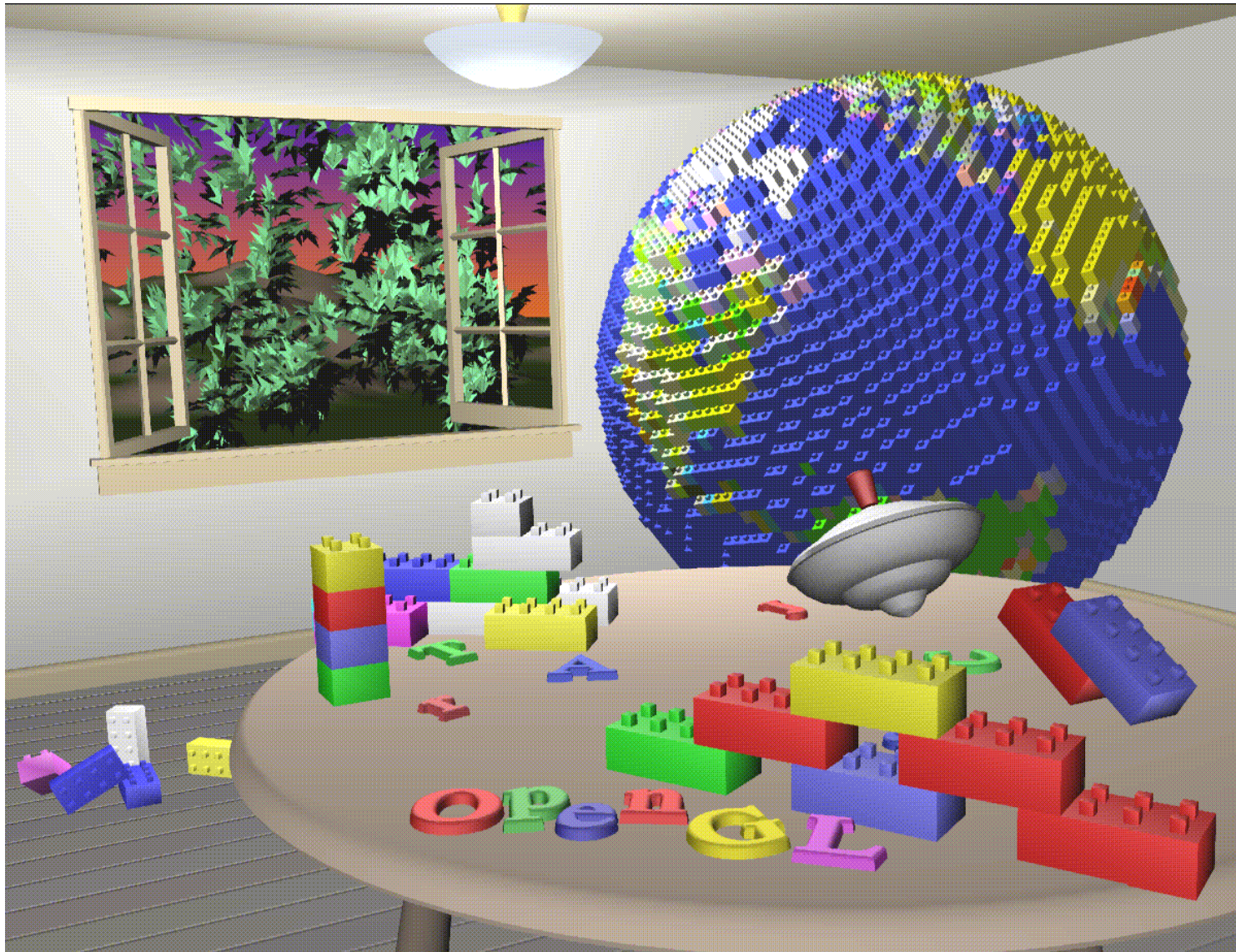
# What can OpenGL do?



Flat-shaded polygons



# What can OpenGL do?



Lighting and smooth-shaded polygons



# What can OpenGL do?



Texturemap and shadow



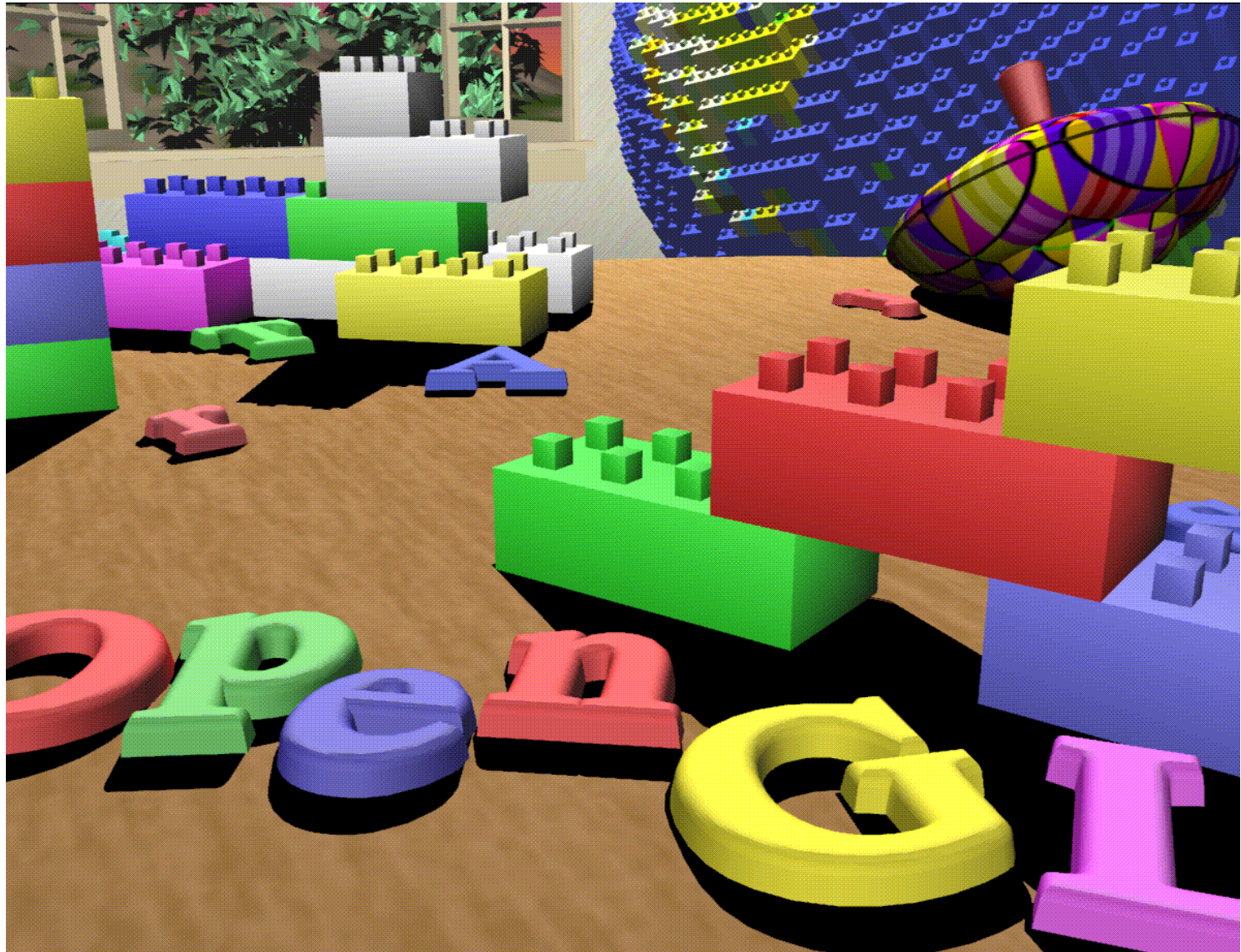
# What can OpenGL do?



Motion blur



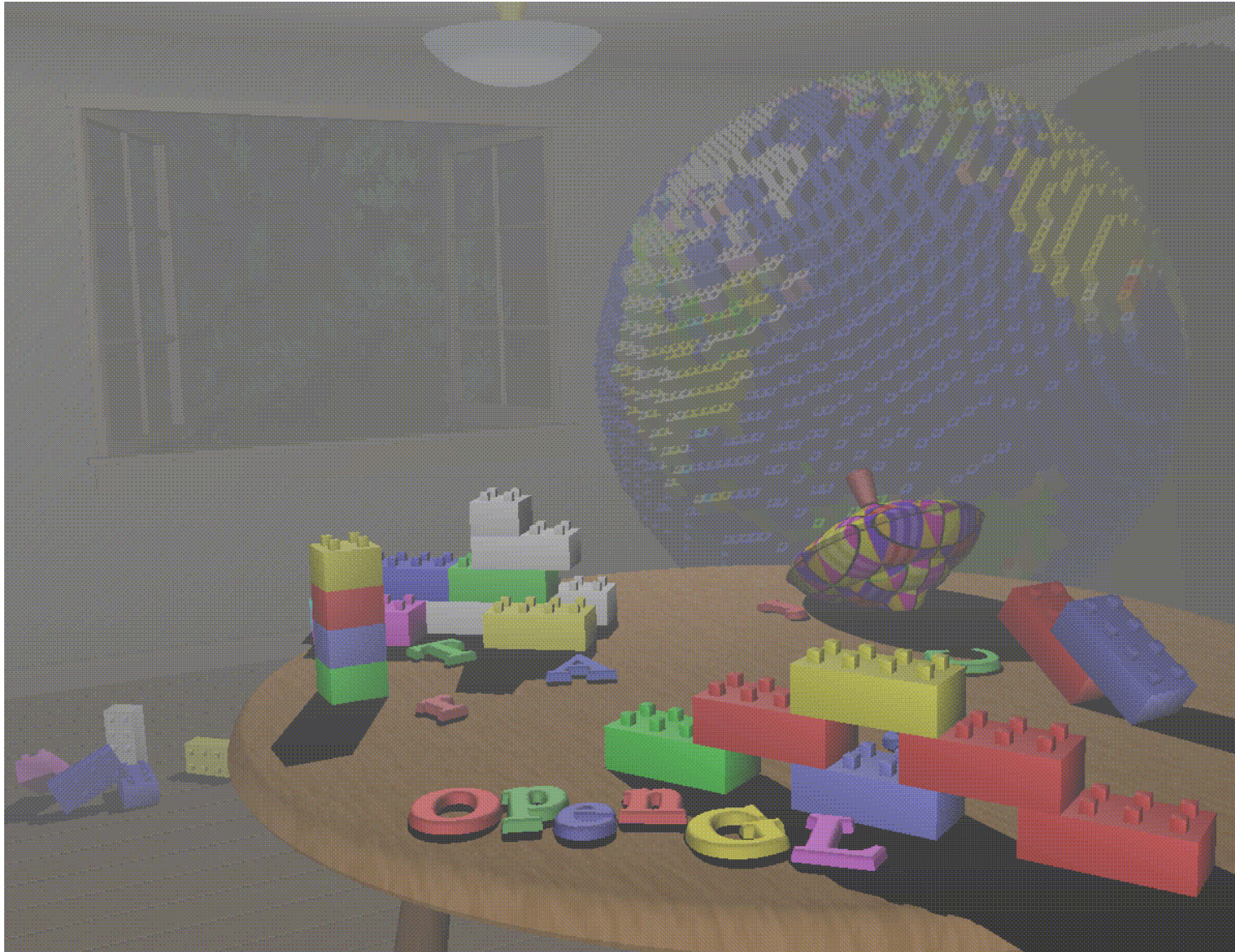
# What can OpenGL do?



View point change

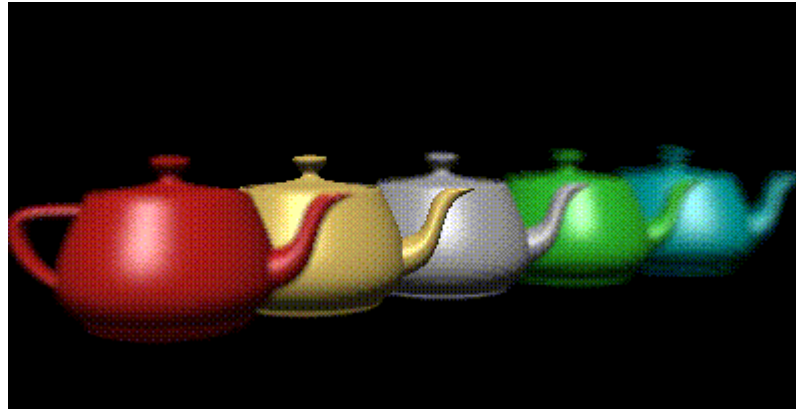


# What can OpenGL do?



Smoke

# What can OpenGL do?



Depth of field

# Hello, world

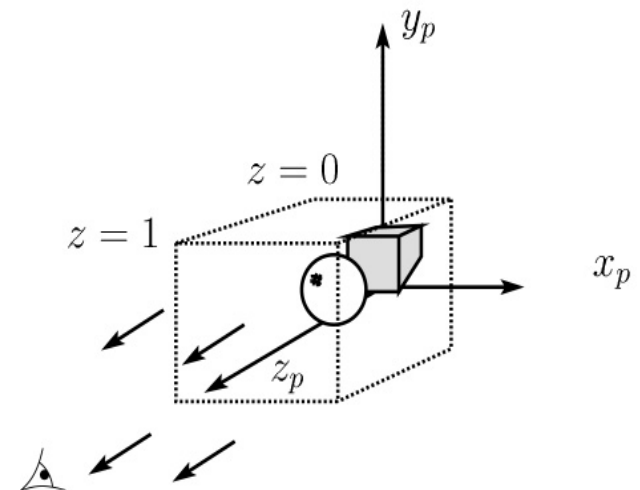
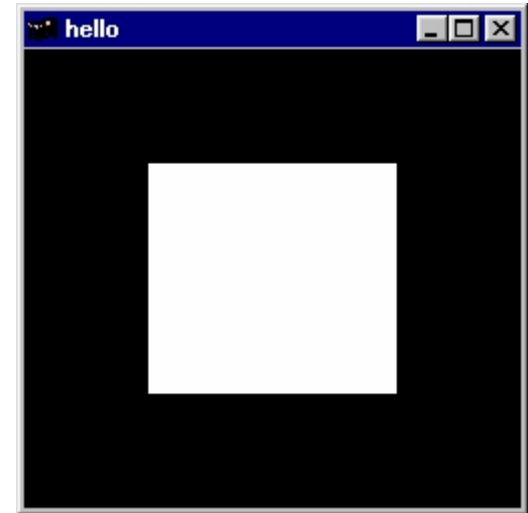
```
#include <whateverYouNeed.h>

main() {

    OpenAWindowPlease();

    glClearColor(0.0, 0.0, 0.0, 0.0);
    glClear(GL_COLOR_BUFFER_BIT);
    glColor3f(1.0, 1.0, 1.0);
    glOrtho(-1.0, 1.0, -1.0, 1.0, -1.0, 1.0);
    glBegin(GL_POLYGON);
        glVertex2f(-0.5, -0.5);
        glVertex2f(-0.5, 0.5);
        glVertex2f(0.5, 0.5);
        glVertex2f(0.5, -0.5);
    glEnd();
    glFlush();

    KeepTheWindowOnTheScreenForAwhile();
}
```



# OpenGL syntax

`gl` prefix for all commands

`GL_` for constants

```
glColor3f(1.0, 1.0, 1.0);
```

```
glColor3d(1.0, 1.0, 1.0); glColor3s(1, 1, 1); glColor3i(1, 1, 1); .....
```

Suffix	Data Type	Typical Corresponding C-Language Type	OpenGL Type Definition
b	8-bit integer	signed char	GLbyte
s	16-bit integer	short	GLshort
i	32-bit integer	long	GLint, GLsizei
f	32-bit floating-point	float	GLfloat, GLclampf
d	64-bit floating-point	double	GLdouble, GLclampd
ub	8-bit unsigned integer	unsigned char	GLubyte, GLboolean
us	16-bit unsigned integer	unsigned short	GLushort
ui	32-bit unsigned integer	unsigned long	GLuint, GLenum, GLbitfield

# OpenGL syntax

```
glVertex2i(1, 1);
```



```
glVertex2f(1.0, 1.0);
```

```
glColor3f(1.0, 0.0, 0.0);
```



```
glColor3ub(255, 0, 0);
```

```
glColor3f(1.0, 0.0, 0.0);
```

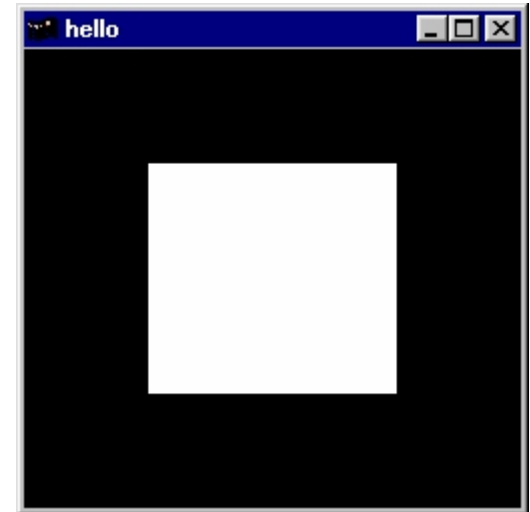


```
float color_array[] = {1.0, 0.0, 0.0};  
glColor3fv(color_array);
```

# Windows management GLUT lib

```
#include <GL/gl.h>
#include <GL/glut.h>

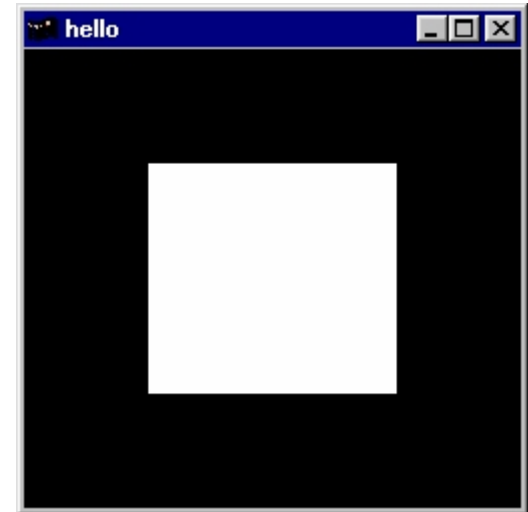
int main(int argc, char** argv){
    glutInit(&argc, argv);
    glutInitDisplayMode (GLUT_SINGLE | GLUT_RGB);
    glutInitWindowSize (250, 250);
    glutInitWindowPosition (100, 100);
    glutCreateWindow ("hello");
    init ();
}
```



# Windows management GLUT lib

```
#include <GL/gl.h>
#include <GL/glut.h>

int main(int argc, char** argv){
    glutInit(&argc, argv);
    glutInitDisplayMode (GLUT_SINGLE | GLUT_RGB);
    glutInitWindowSize (250, 250);
    glutInitWindowPosition (100, 100);
    glutCreateWindow ("hello");
    init ();
    glutDisplayFunc(display);
}
```



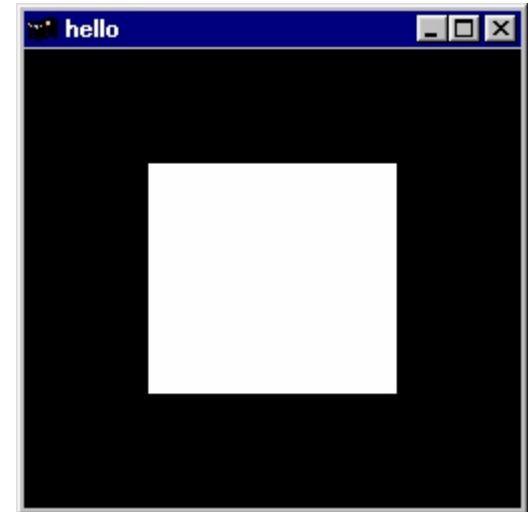
```
void init (void) {
    glClearColor (0.0, 0.0, 0.0, 0.0);
    glMatrixMode(GL_PROJECTION);
    glLoadIdentity();
    glOrtho(0.0, 1.0, 0.0, 1.0, -1.0, 1.0);
}
```



# Windows management GLUT lib

```
#include <GL/gl.h>
#include <GL/glut.h>

int main(int argc, char** argv){
    glutInit(&argc, argv);
    glutInitDisplayMode (GLUT_SINGLE | GLUT_RGB);
    glutInitWindowSize (250, 250);
    glutInitWindowPosition (100, 100);
    glutCreateWindow ("hello");
    init ();
    glutDisplayFunc(display);
    glutMainLoop();
    return 0;
}
```



```
void display(void){
    glClear (GL_COLOR_BUFFER_BIT);
    glColor3f (1.0, 1.0, 1.0);
    glBegin(GL_POLYGON);
    glVertex2f(-0.5, -0.5);
    glVertex2f(-0.5, 0.5);
    glVertex2f(0.5, 0.5);
    glVertex2f(0.5, -0.5);
    glEnd();
    glFlush ();
}
```

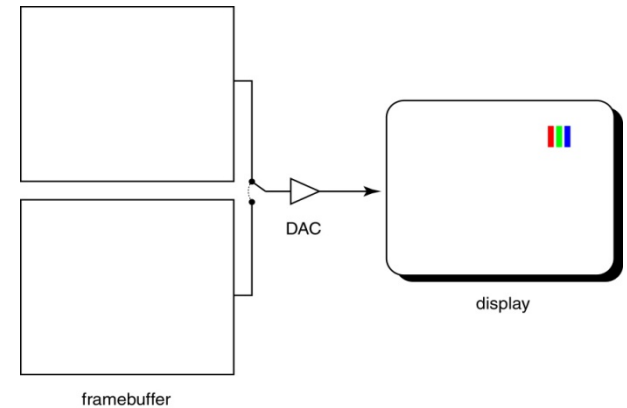
# Animation



```
open_window();  
for (i = 0; i < 1000000; i++) {  
    clear_the_window();  
    draw_frame(i);  
    wait_until_a_24th_of_a_second_is_over();  
}
```

**Q:** What happens when you write to the framebuffer while it is being displayed on the monitor?

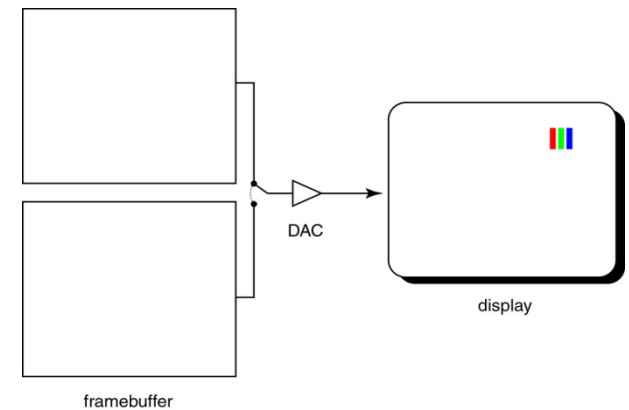
# Animation



```
open_window();  
for (i = 0; i < 1000000; i++) {  
    clear_the_window();  
    draw_frame(i);  
    wait_until_a_24th_of_a_second_is_over();  
}
```

**Q:** What happens when you write to the framebuffer while it is being displayed on the monitor?

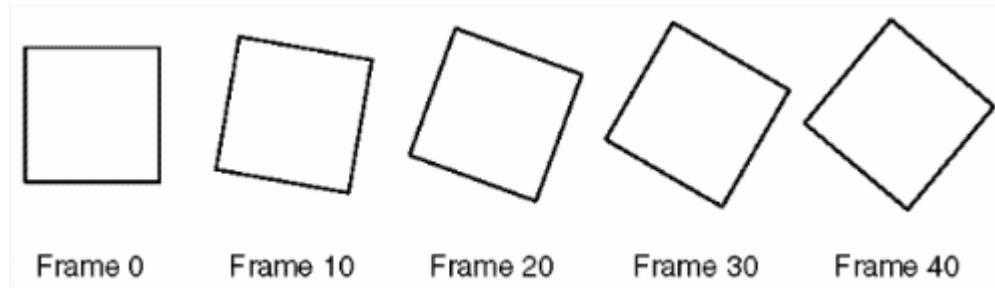
# Animation



```
open_window();  
for (i = 0; i < 1000000; i++) {  
    clear_the_window();  
    draw_frame(i);  
    wait_until_a_24th_of_a_second_is_over();  
    swap_the_buffers();  
}
```

**Q:** What happens when you write to the framebuffer while it is being displayed on the monitor?

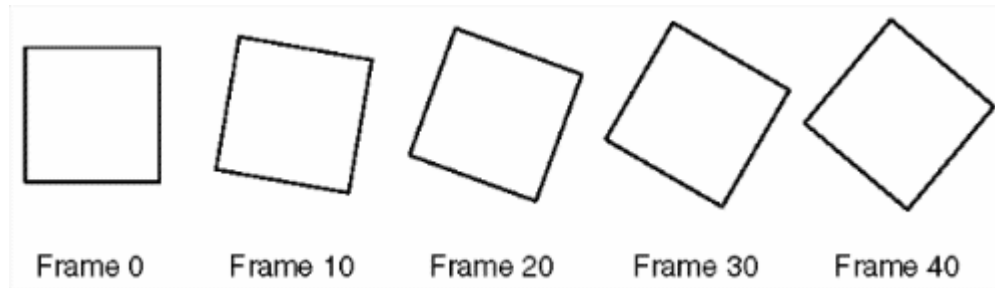
# Animation Example



```
int main(int argc, char** argv){  
    glutInit(&argc, argv);  
    glutInitDisplayMode (GLUT_DOUBLE | GLUT_RGB);  
    glutInitWindowSize (250, 250);  
    glutInitWindowPosition (100, 100);  
    glutCreateWindow (argv[0]);  
    init ();  
    glutDisplayFunc(display);  
  
}
```

```
void init(void) {  
    glClearColor (0.0, 0.0, 0.0, 0.0);  
    glShadeModel (GL_FLAT);  
}
```

# Animation Example



```
static GLfloat spin = 0.0;

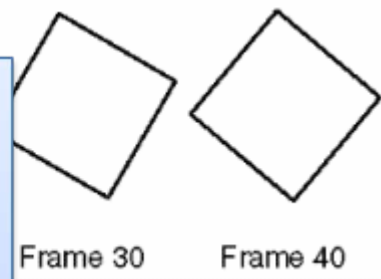
void display(void){
    glClear(GL_COLOR_BUFFER_BIT);
    glPushMatrix();
    glRotatef(spin, 0.0, 0.0, 1.0);
    glColor3f(1.0, 1.0, 1.0);
    glRectf(-25.0, -25.0, 25.0, 25.0);
    glPopMatrix();
    glutSwapBuffers();
}
```

```
int main(int argc, char** argv){
    glutInit(&argc, argv);
    glutInitDisplayMode (GLUT_DOUBLE | GLUT_RGB);
    glutInitWindowSize (250, 250);
    glutInitWindowPosition (100, 100);
    glutCreateWindow (argv[0]);
    init ();
    glutDisplayFunc(display);
    glutMouseFunc(mouse);
}
```

# Animation Example

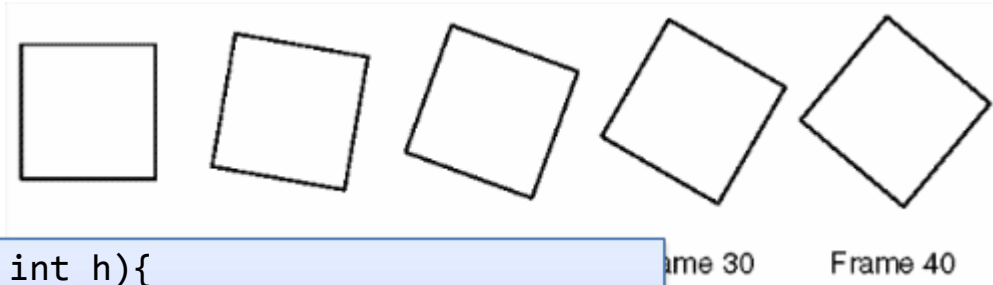
```
void mouse(int button, int state, int x, int y) {  
    switch (button) {  
        case GLUT_LEFT_BUTTON:  
            if (state == GLUT_DOWN)  
                glutIdleFunc(spinDisplay);  
            break;  
        case GLUT_MIDDLE_BUTTON:  
            if (state == GLUT_DOWN)  
                glutIdleFunc(NULL);  
            break;  
        default:  
            break;  
    }  
}
```

```
spinDisplay(void){  
    spin = spin + 2.0;  
    if (spin > 360.0) spin -= 360.0;  
    glutPostRedisplay();  
}
```



```
int argc, char** argv){  
    &argc, argv);  
    displayMode (GLUT_DOUBLE | GLUT_RGB);  
    windowSize (250, 250);  
    windowPosition (100, 100);  
    createWindow (argv[0]);  
    glutDisplayFunc(display);  
    glutMouseFunc(mouse);  
    glutReshapeFunc(reshape);  
}
```

# Animation Example



```
void reshape(int w, int h){  
    glViewport (0, 0, (GLsizei) w, (GLsizei) h);  
    glMatrixMode(GL_PROJECTION);  
    glLoadIdentity();  
    glOrtho(-50.0, 50.0, -50.0, 50.0, -1.0, 1.0);  
    glMatrixMode(GL_MODELVIEW);  
    glLoadIdentity();  
}
```

```
int main(int argc, char** argv){  
    glutInit(&argc, argv);  
    glutInitMode (GLUT_DOUBLE | GLUT_RGB);  
    glutWindowSize (250, 250);  
    glutWindowPosition (100, 100);  
    glutCreateWindow (argv[0]);
```

```
    glutInitWindowPosition (100, 100);  
    glutInitWindowSize (250, 250);  
    glutInitWindowPosition (100, 100);  
    glutInitWindowSize (250, 250);  
    glutCreateWindow (argv[0]);  
    init ();  
    glutDisplayFunc(display);  
    glutMouseFunc(mouse);  
    glutReshapeFunc(reshape);  
    glutMainLoop();    return 0;  
}
```