CS559: Computer Graphics

Lecture 15: B-Spline, Lighting, and Shading

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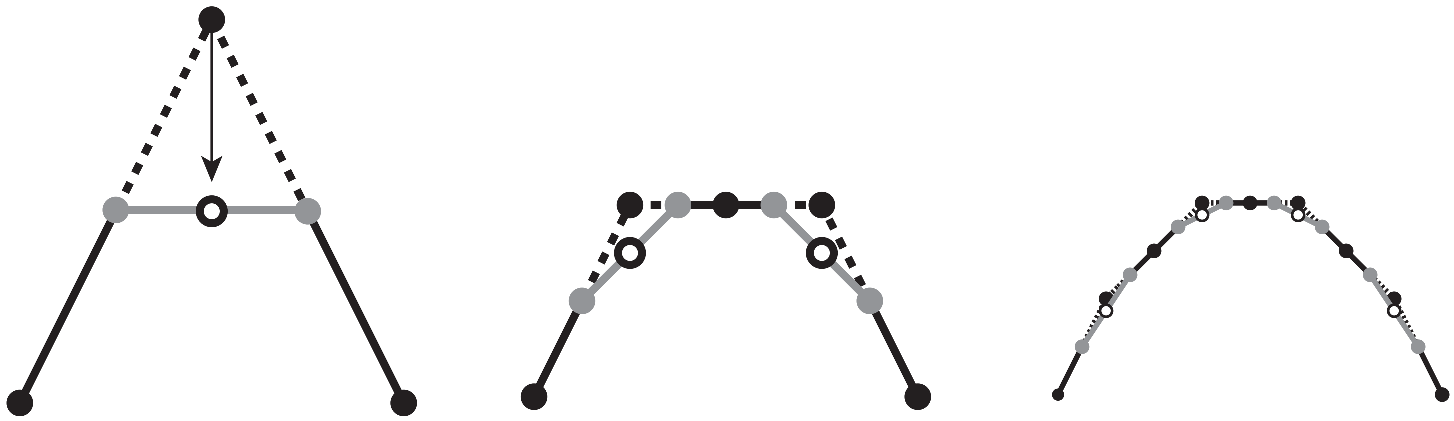
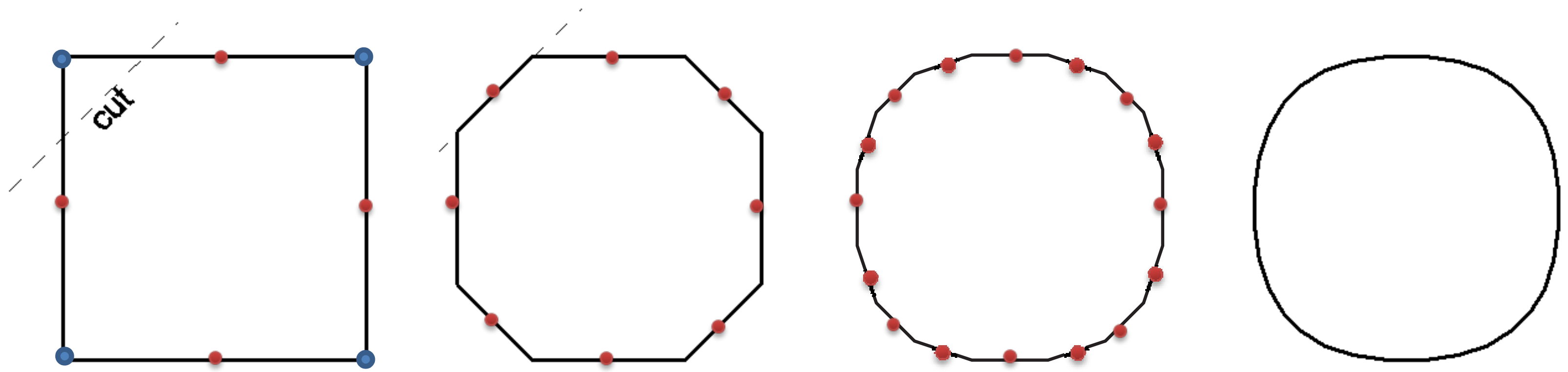
Bezier Curve Subdivision

- Why is subdivision useful?
  - Collision/intersection detection
    - Recursive search
  - Good for curve editing and approximation

Open Curve Approxmiation



Closed Curve Approximation



	Interpolate control points	Has local control	C2 continuity
Natural cubics	Yes	No	Yes
Hermite cubics	Yes	Yes	No
Cardinal Cubics	Yes	Yes	No
Bezier Cubics	Yes	Yes	No

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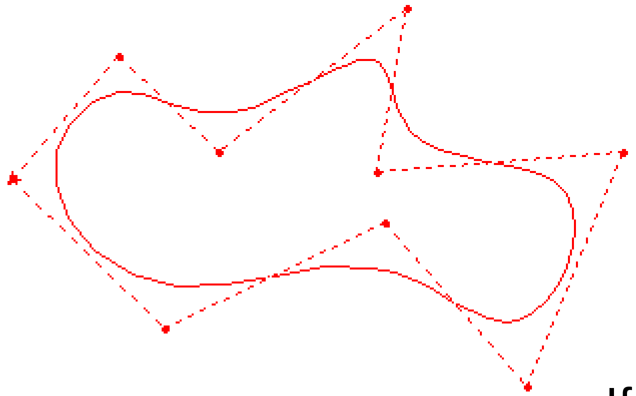
Bspline Curves	No	Yes	Yes

Bsplines

- Given p1,…pn, define a curve that approximates the curve.

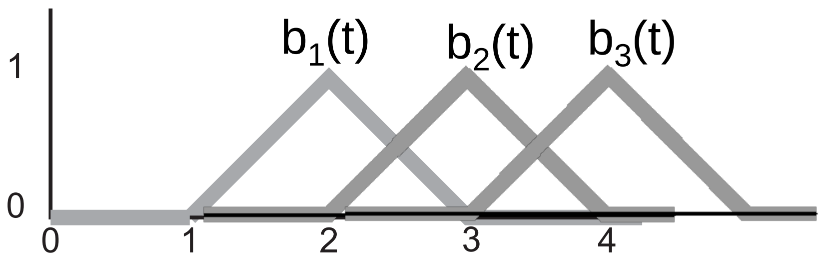


$$\mathbf{f}(t) = \sum_{i=1}^{n} b_i(t)\mathbf{p}_i$$

If $b_i(t)$ is very smooth, so will be **f**

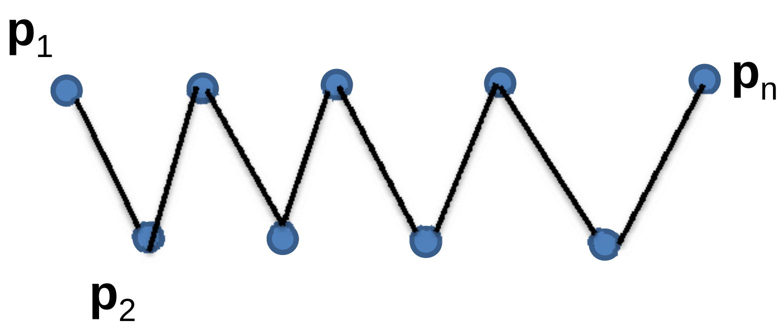
If $b_i(t)$ has local support, **f** will have local control

Uniform Linear B-splines



$$b_{i,2}(t) = b_{0,2}(t-i)$$

$$b_{0,2}(u) = \begin{cases} u & u \in [0,1) \\ 2-u & u \in [1,2) \\ 0 & \text{otherwise} \end{cases}$$



$$\mathbf{f}(t) = \sum_{i=1}^{n} b_i(t)\mathbf{p}_i$$

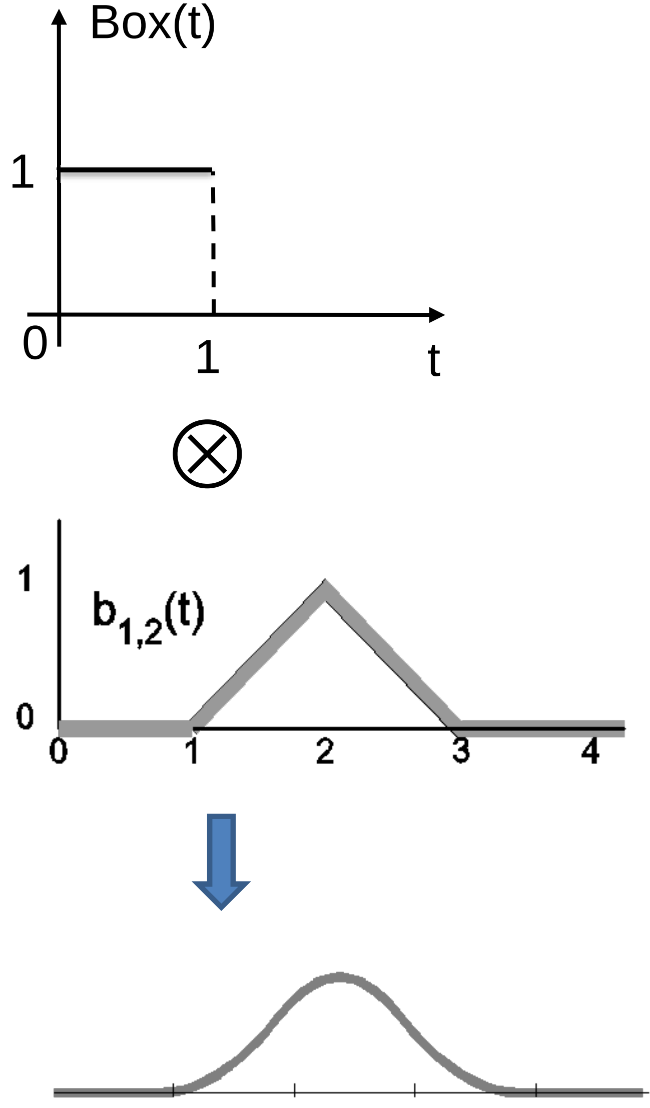
How can we make the curve smooth?

- Convolution/filtering

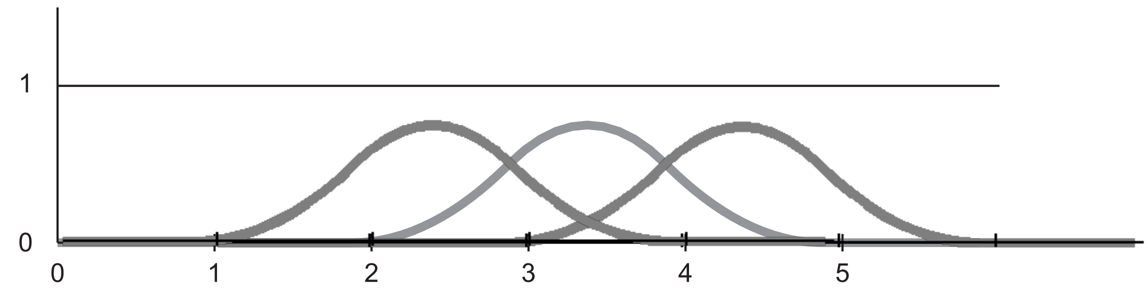
$$\mathbf{f}(t) = \sum_{i=1}^{n} b_i(t)\mathbf{p}_i$$

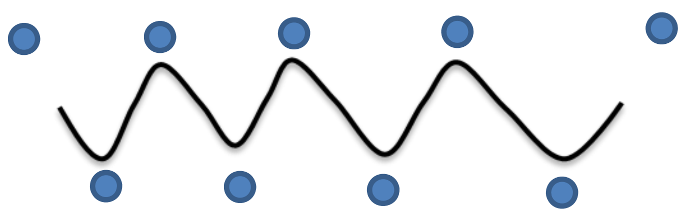
$$\mathbf{f}(t) \otimes \mathrm{Box}(t) = \left(\sum_{i=1}^{n} b_i(t)\mathbf{p}_i \right) \otimes \mathrm{Box}(t)$$

$$= \left(\sum_{i=1}^{n} \left(b_i(t) \otimes \mathrm{Box}(t) \right) \mathbf{p}_i \right)$$



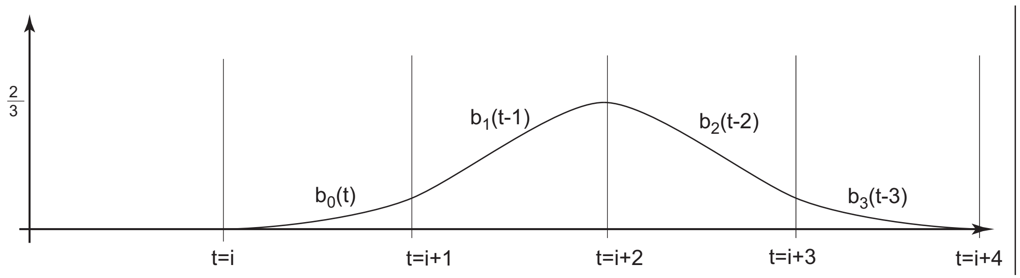
Uniform Quadratic B-splines

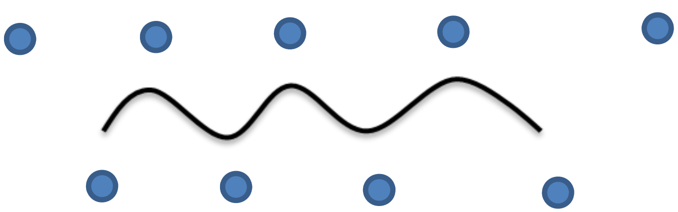




$$\mathbf{f}(t) = \sum_{i=1}^{n} b_i(t)\mathbf{p}_i$$

Uniform Cubic Bspline



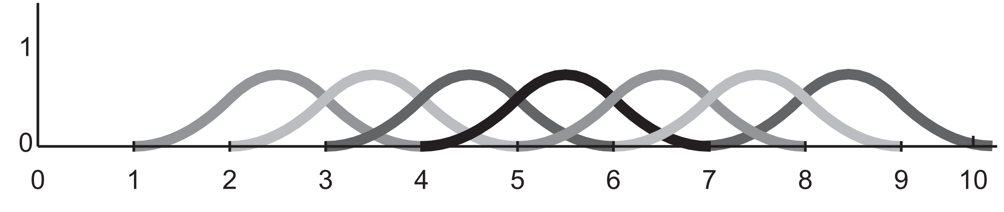


$$\mathbf{f}(t) = \sum_{i=1}^{n} b_i(t)\mathbf{p}_i$$

Uniform B-splines

- Why smoother?
  - Linear = box filter ⊗ box filter
  - Quadric = linear ⊗ box filter
  - Cubic = quadric ⊗ box filter

- Sum = 1 property, translation invariant



- Local control

$$\mathbf{f}(t) = \sum_{i=1}^{n} b_i(t)\mathbf{p}_i$$

- C(k-2) continuity

	Interpolate control points	Has local control	C2 continuity
Natural cubics	Yes	No	Yes
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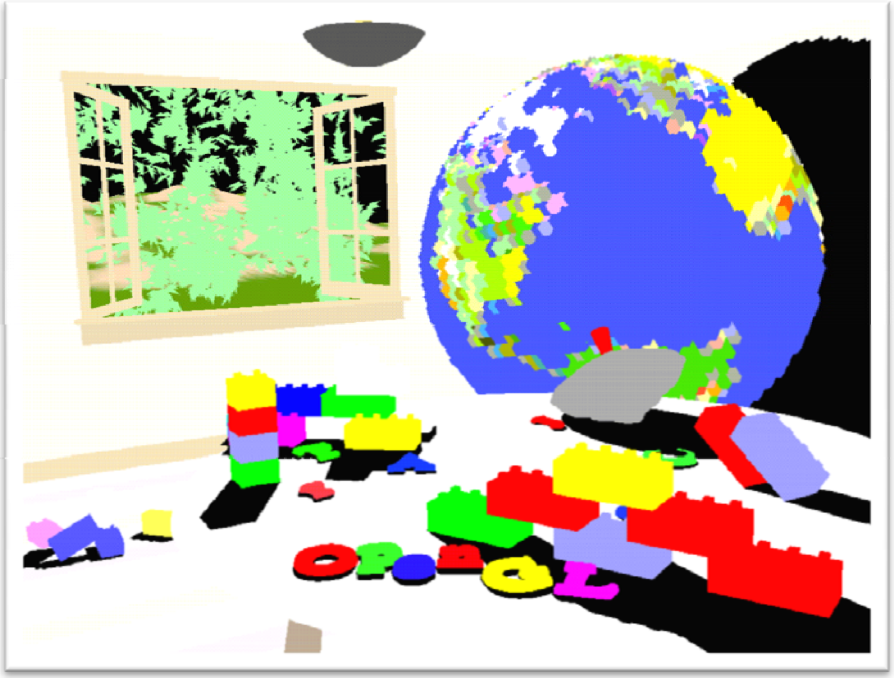
So far…

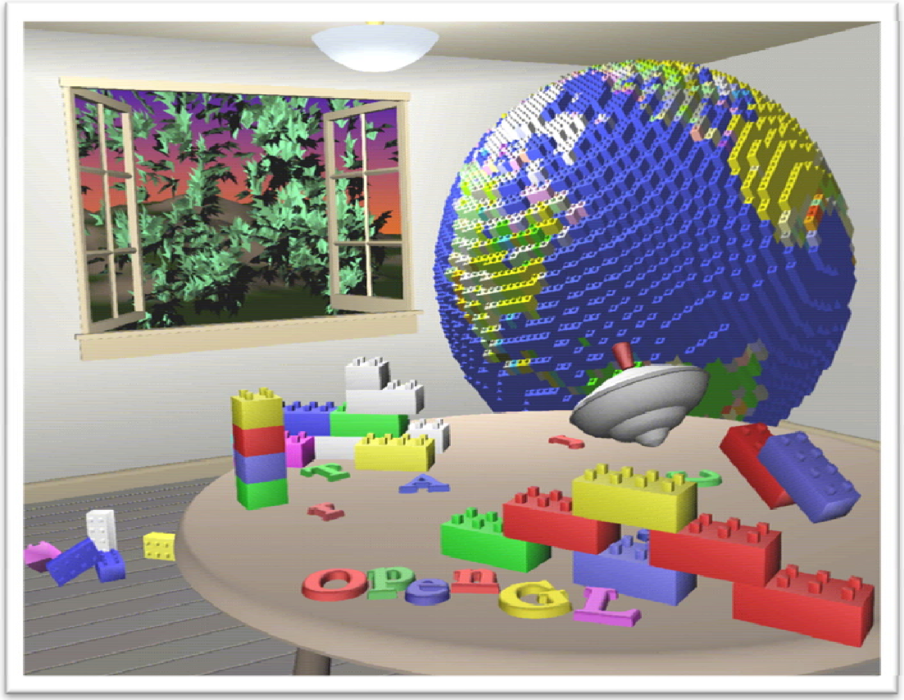
- We’ve talked exclusively about geometry.
  - What is the shape of an object?
    - glBegin() … glEnd()
  - How do I place it in a virtual 3D space?
    - glMatrixMode() …
  - How to change viewpoints
    - gluLookAt()
  - How do I know which pixels it covers?
    - Rasterization
  - How do I know which of the pixels I should actually draw?
    - Z-buffer, BSP

So far

```
glColor(…);
Apply_transforms();
Draw_objects();
```



Flat shaded



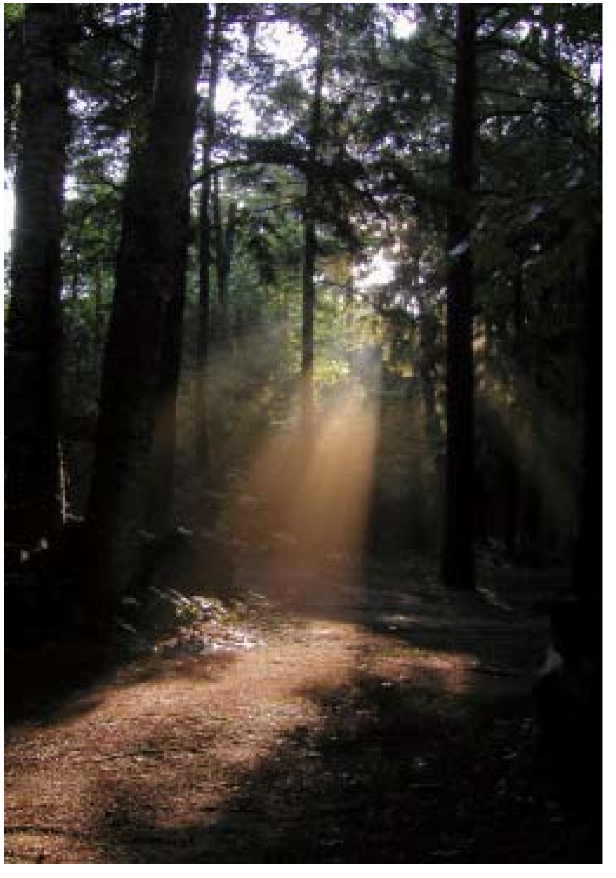
Lit surface

Next…

- Once we know geometry, we have to ask one more important question:
  - To what value do I set each pixel?
- Answering this question is the job of the **shading model**.
- Other names:
  - Lighting model
  - Light reflection model
  - Local illumination model
  - Reflectance model
  - BRDF

An abundance of photons

- Properly determining the right color is *really hard.*



Particle Scattering

An abundance of photons

- Properly determining the right color is *really hard.*

Translucency

An abundance of photons

- Properly determining the right color is *really hard.*

Refraction

An abundance of photons

- Properly determining the right color is *really hard.*

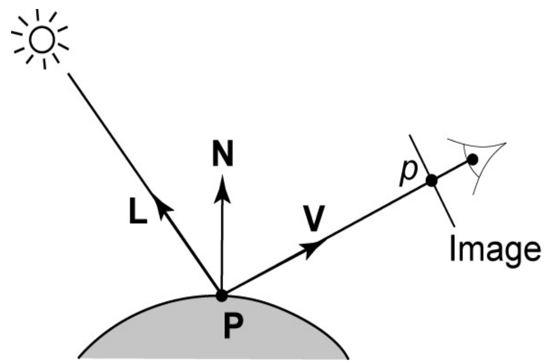
Global Effect

Our problem

- We’re going to build up to an *approximation* of reality called the **Phong illumination model**.
- It has the following characteristics:
  - *not* physically based
  - gives a “first-order” *approximation* to physical light reflection
  - very fast
  - widely used

- In addition, we will assume **local illumination**, i.e., light goes: light source -> surface -> viewer.
- No interreflections, no shadows.

Setup…



$$\|\mathbf{N}\| = \|\mathbf{L}\| = \|\mathbf{V}\| = 1$$

- Given:
  - a point **P** on a surface visible through pixel *p*
  - The normal **N** at **P**
  - The lighting direction, **L**, and intensity, *L* ,at **P**
  - The viewing direction, **V**, at **P**
  - The shading coefficients at **P**
- Compute the color, *I*, of pixel *p*.
- Assume that the direction vectors are normalized:

“Iteration zero”

- The simplest thing you can do is…
- Assign each polygon a single color:

$$I = k_e$$

- where
  - I is the resulting intensity
  - k_e is the **emissivity** or intrinsic shade associated with the object

- This has some special-purpose uses, but not really good for drawing a scene.

"Iteration one"

- Let's make the color at least dependent on the overall quantity of light available in the scene:

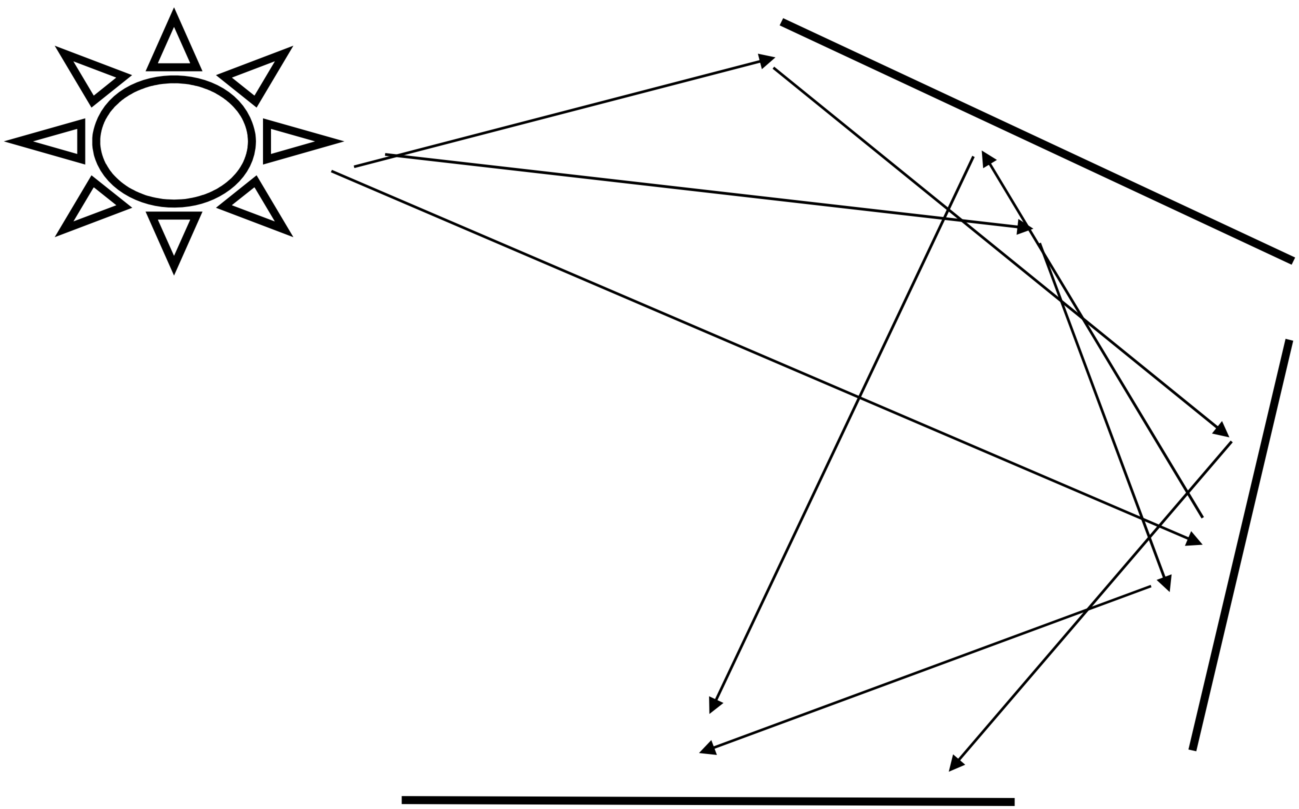
$$I = k_e + k_a L_a$$

  - k_a is the **ambient reflection coefficient**.
    - really the reflectance of ambient light
    - "ambient" light is assumed to be equal in all directions
  - L_a is the **ambient light intensity**.

- Physically, what is "ambient" light?

Ambient Term

- Hack to simulate multiple bounces, scattering of light
- Assume light equally from all directions



Slide from Ravi Ramamoorthi

Wavelength dependence

- Really, k_e, k_a, and L_a are functions over all wavelengths λ.
- Ideally, we would do the calculation on these functions. For the ambient shading equation, we would start with:

$$I(\lambda) = k_a(\lambda) L_a(\lambda)$$

- then we would find good RGB values to represent the spectrum $I(\lambda)$.
- Traditionally, though, k_a and I_a are represented as RGB triples, and the computation is performed on each color channel separately:

$$I_R = k_{a,R} L_{a,R}$$
$$I_G = k_{a,G} L_{a,G}$$
$$I_B = k_{a,B} L_{a,B}$$

Diffuse reflection

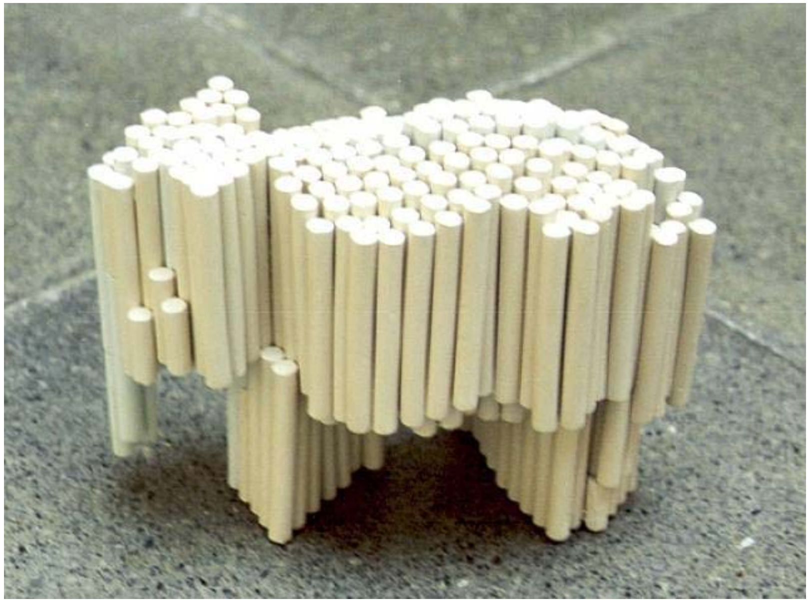
$$I = k_e + k_a L_a$$

- So far, objects are uniformly lit.
  - not the way things really appear
  - in reality, light sources are localized in position or direction

- **Diffuse**, or **Lambertian** reflection will allow reflected intensity to vary with the direction of the light.

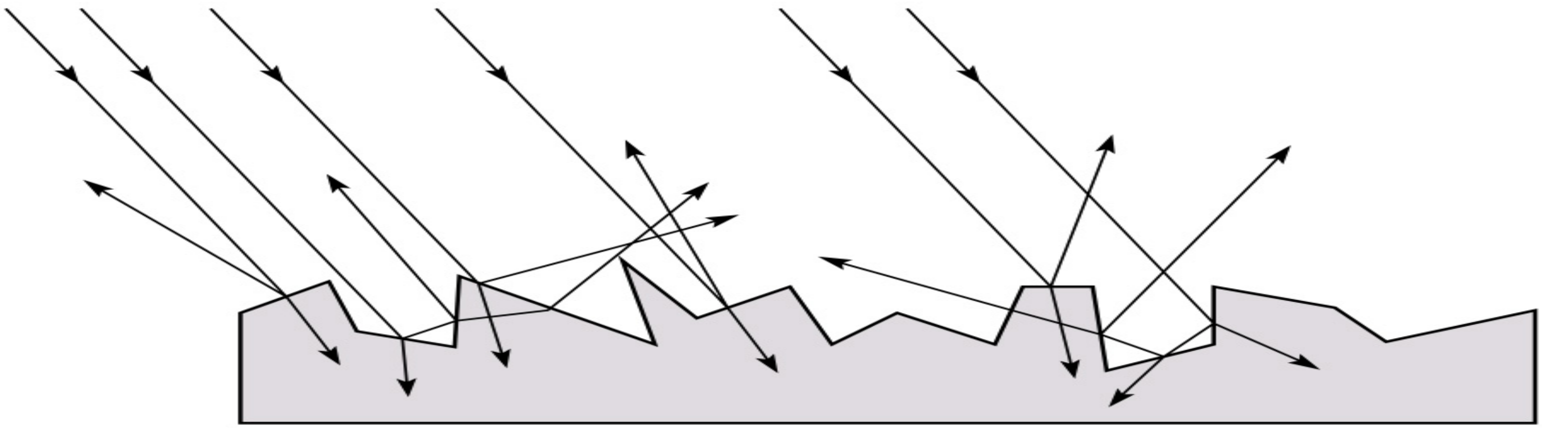
Diffuse reflectors

- Diffuse reflection occurs from dull, matte surfaces, like latex paint, or chalk.
- These **diffuse** or **Lambertian** reflectors reradiate light equally in all directions.

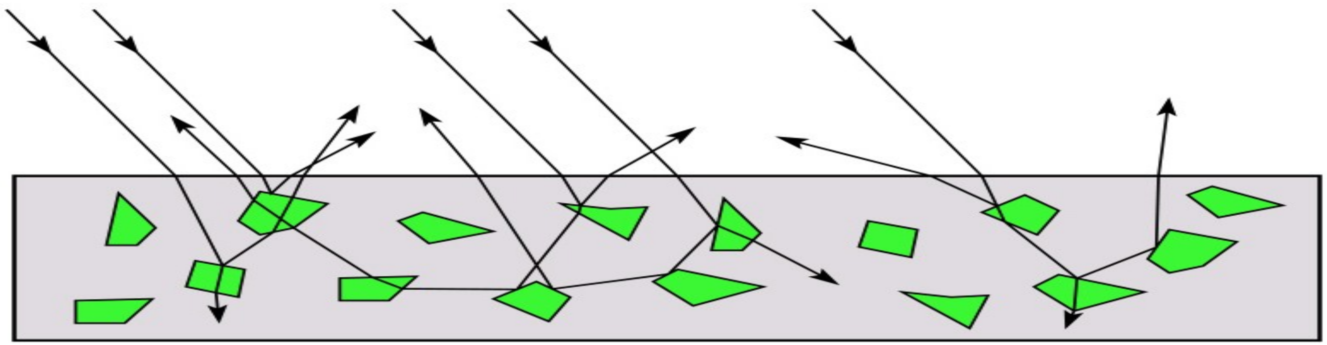
Diffuse reflectors

- Diffuse reflection occurs from dull, matte surfaces, like latex paint, or chalk.
- These **diffuse** or **Lambertian** reflectors reradiate light equally in all directions.
- Picture a rough surface with lots of tiny **microfacets**.



Diffuse reflectors

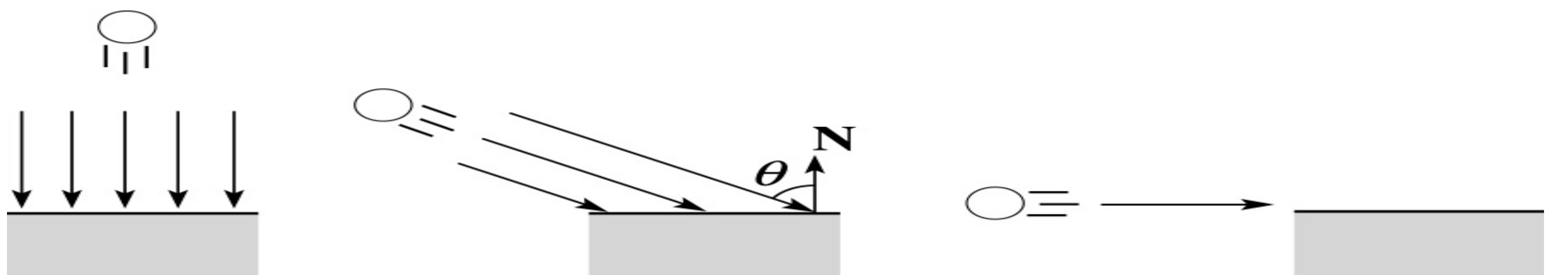
- …or picture a surface with little pigment particles embedded beneath the surface (neglect reflection at the surface for the moment):



- The microfacets and pigments distribute light rays in all directions.
- Embedded pigments are responsible for the coloration of diffusely reflected light in plastics and paints.
- Note: the figures above are intuitive, but not strictly (physically) correct.

Diffuse reflectors, cont.

- The reflected intensity from a diffuse surface does not depend on the direction of the viewer. The incoming light, though, does depend on the direction of the light source:



“Iteration two”

- The incoming energy is proportional to _ $\cos\theta$ __, giving the diffuse reflection equations:

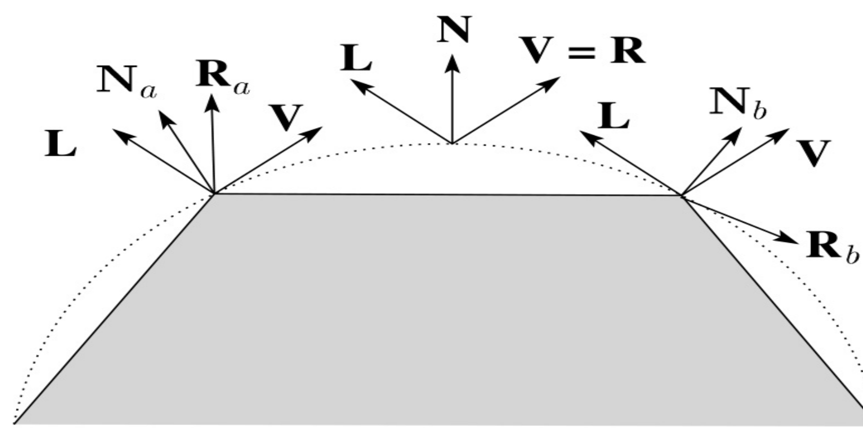
$$I = k_e + k_a L_a + k_d L \cdot (\mathbf{L} \cdot \mathbf{N})$$

$$= k_e + k_a L_a + k_d L \cdot \max(0, \mathbf{L} \cdot \mathbf{N})$$

- where:
  - k_d is the **diffuse reflection coefficient**
  - L_d is the intensity of the light source
  - **N** is the normal to the surface (unit vector)
  - **L** is the direction to the light source (unit vector)

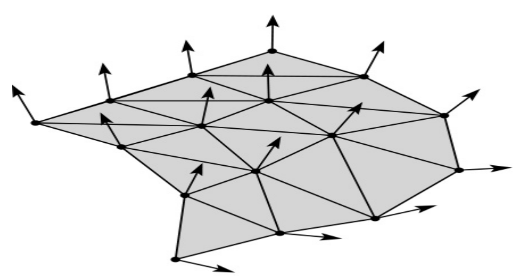
Gouraud interpolation artifacts

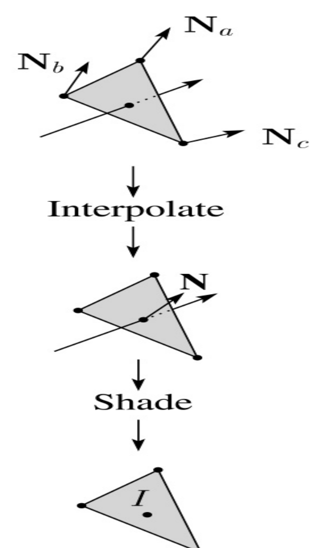
- Gouraud interpolation has significant limitations.
  - If the polygonal approximation is too coarse, we can miss specular highlights.



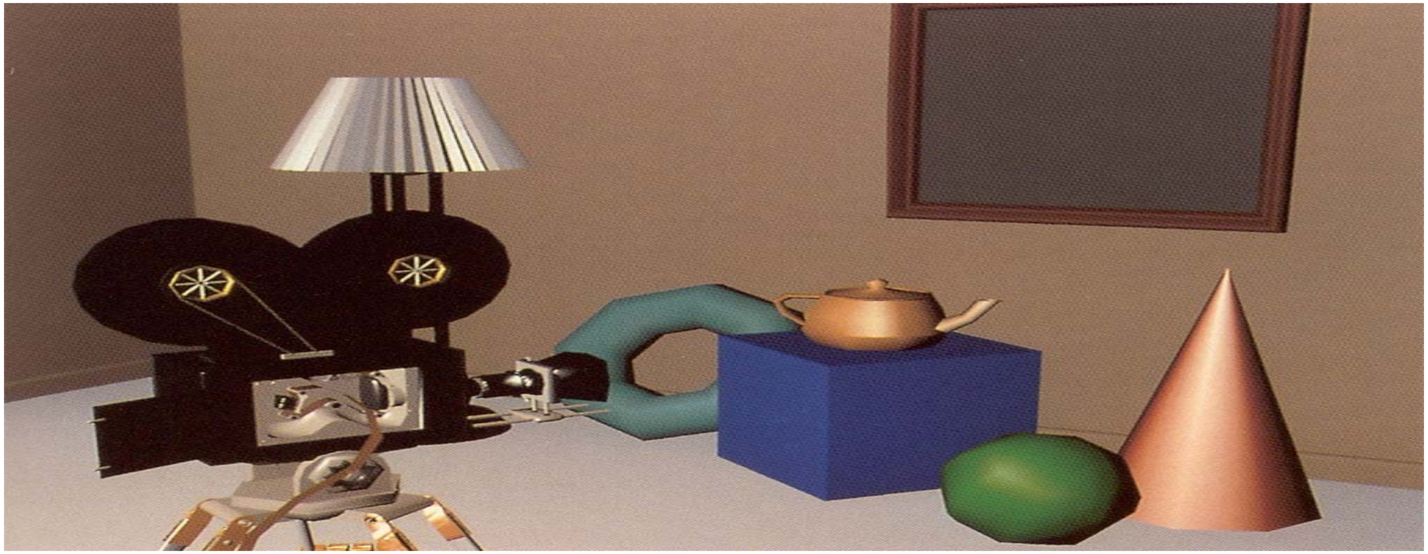
Phong interpolation

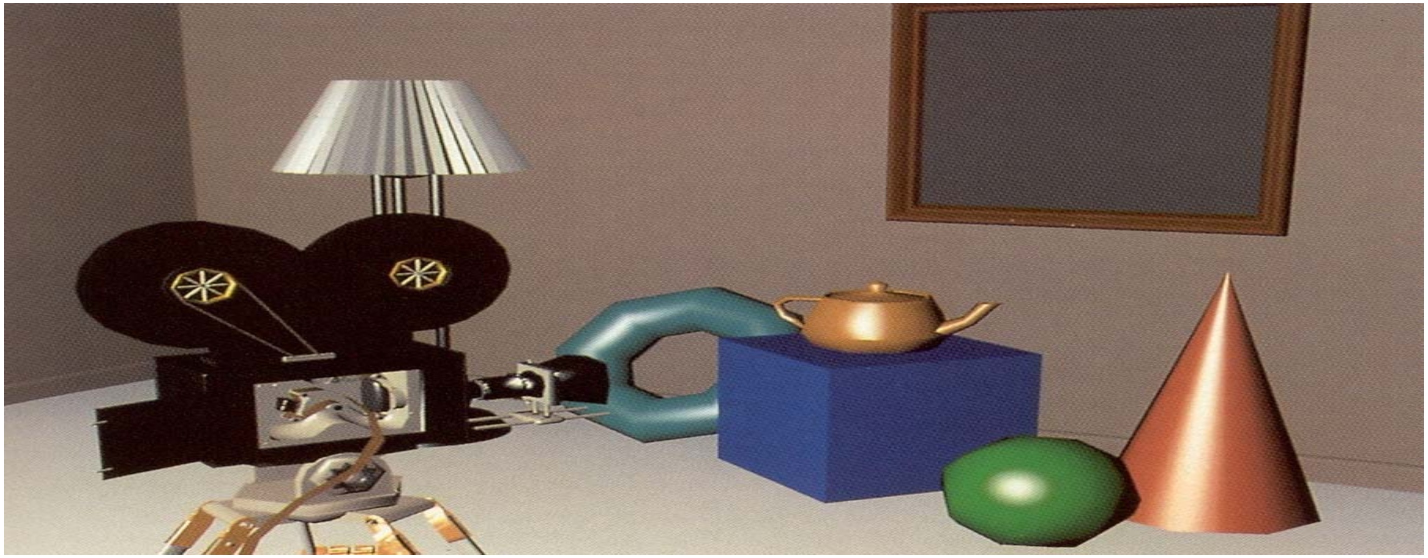
- To get an even smoother result with fewer artifacts, we can perform **Phong *interpolation***.
- Here's how it works:
  1. Compute normals at the vertices.
  2. Interpolate normals and normalize.
  3. Shade using the interpolated normals.





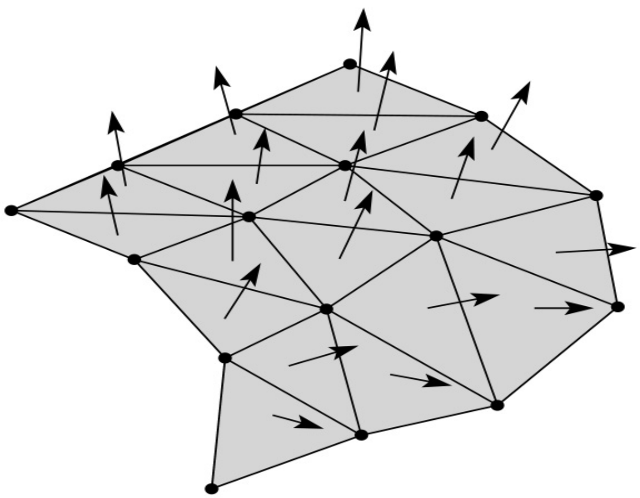
Gouraud vs. Phong interpolation





How to compute vertex normals

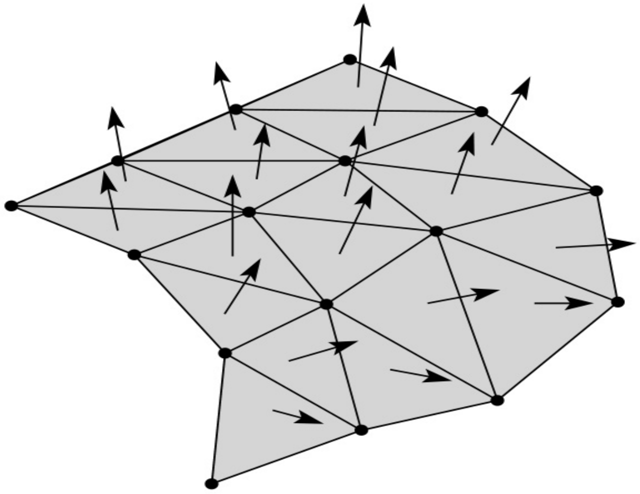
A weighted average of normals of neighboring triangles



$$\mathbf{n}_{vertex} = \frac{\sum_{triangle} area_{triangle} \mathbf{n}_{triangle}}{\sum_{triangle} area_{triangle}}$$

How to compute vertex normals

A weighted average of normals of neighboring triangles



$$\mathbf{n}_{vertex} = \frac{\sum_{triangle} area_{triangle} \mathbf{n}_{triangle}}{\left\| \sum_{triangle} area_{triangle} \mathbf{n}_{triangle} \right\|}$$