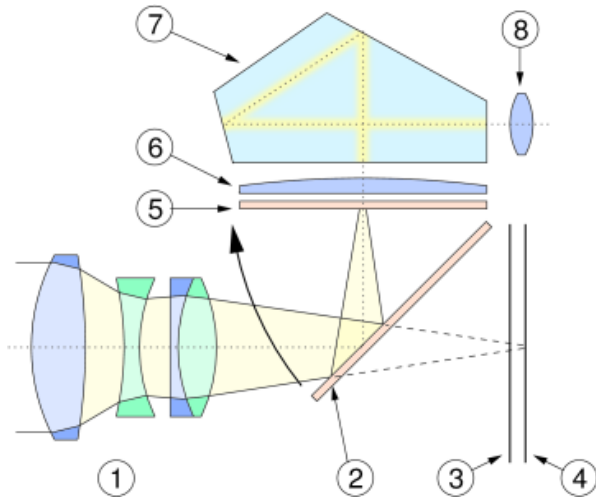


Last 4 lectures



Camera Structure



HDR

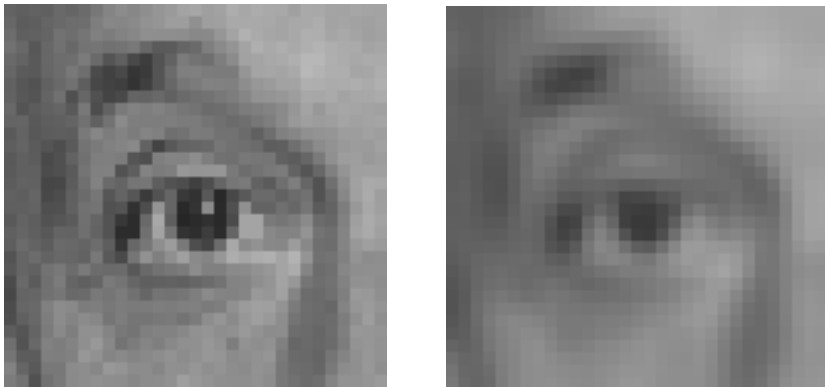


Image Filtering



Image Transform

Today

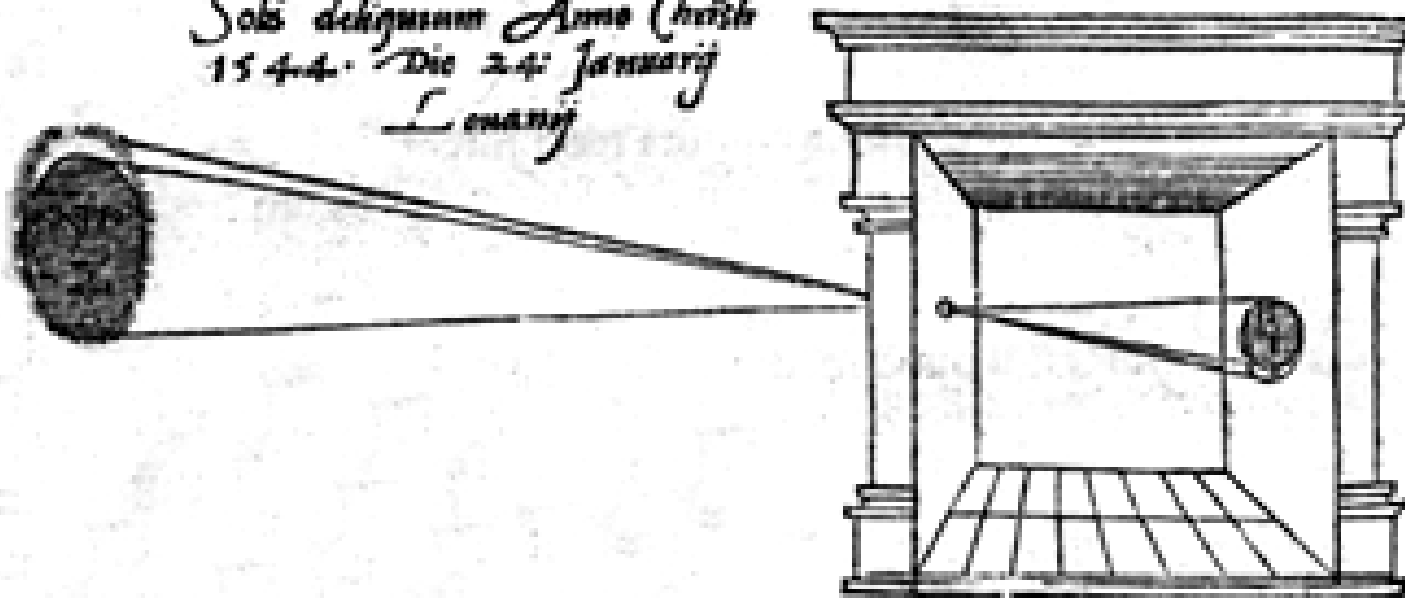
Camera Projection

Camera Calibration

Pinhole camera

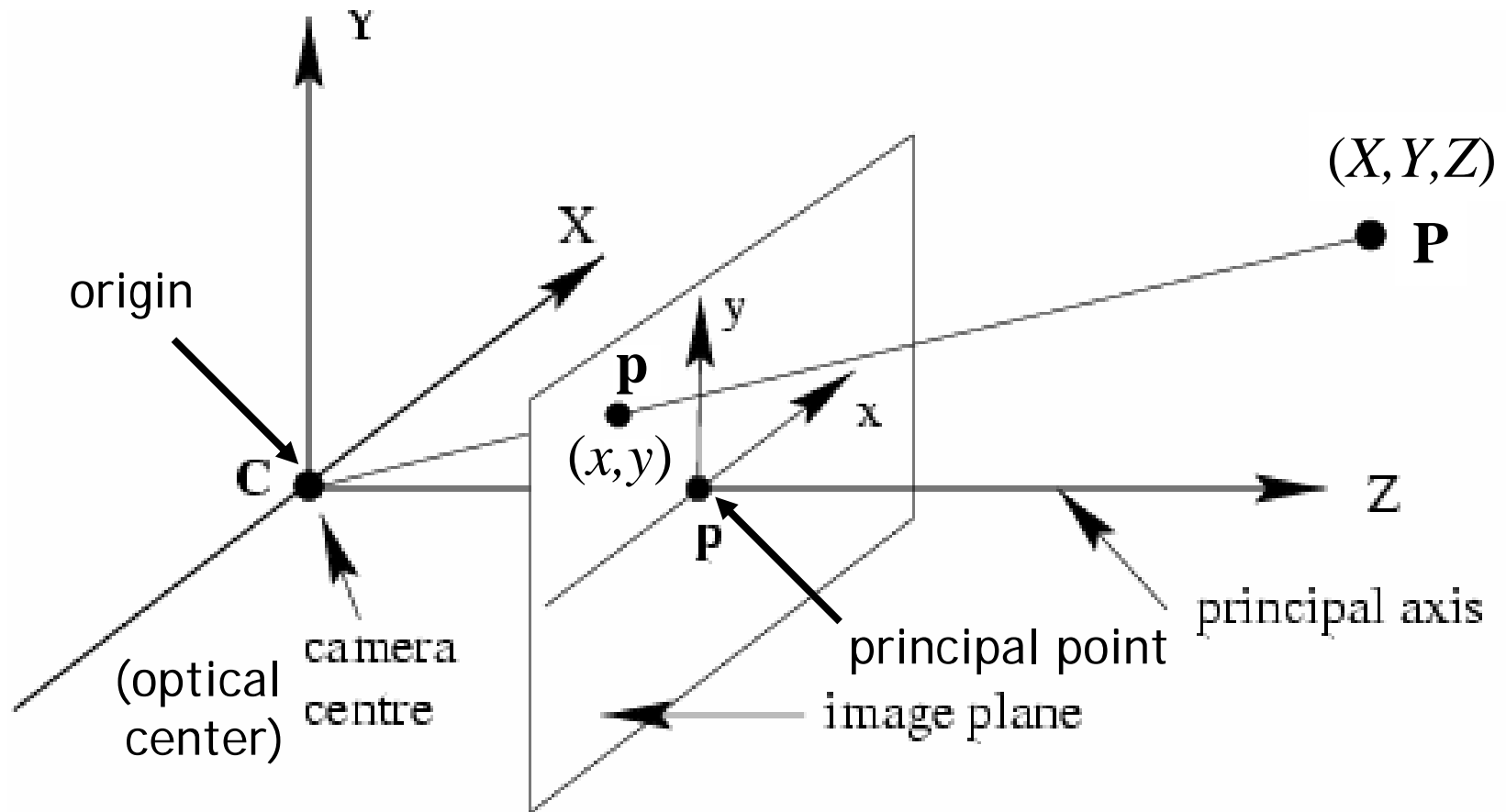
illum in tabula per radios Solis, quàm in cælo contin-
git: hoc est, si in cælo superior pars deliquiū patiatur, in
radiis apparebit inferior deficere, vt ratio exigit optica.

*Soli deliquium Anno Christi
1544. Die 24. Januarij
Louanij*



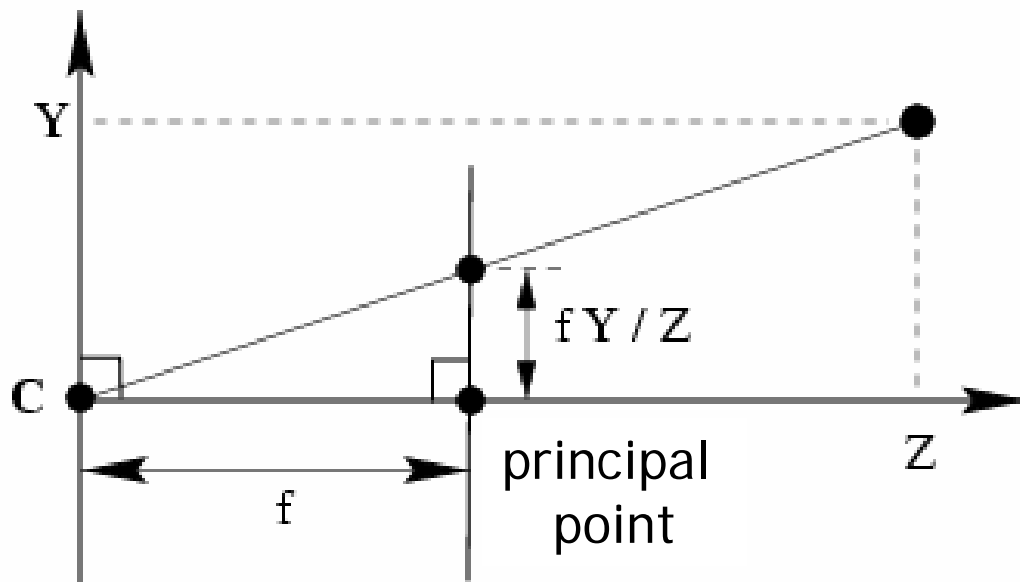
Sic nos exactè Anno .1544. Louanii eclipsim Solis
obseruauimus, inuenimusq; deficere paulò plus q̄ dex-

Pinhole camera model



- The coordinate system
 - We will use the pin-hole model as an approximation
 - Put the optical center (**C**enter **O**f **P**rojection) at the origin
 - Put the image plane (**P**rojection **P**lane) *in front* of the COP (Why?)

Pinhole camera model

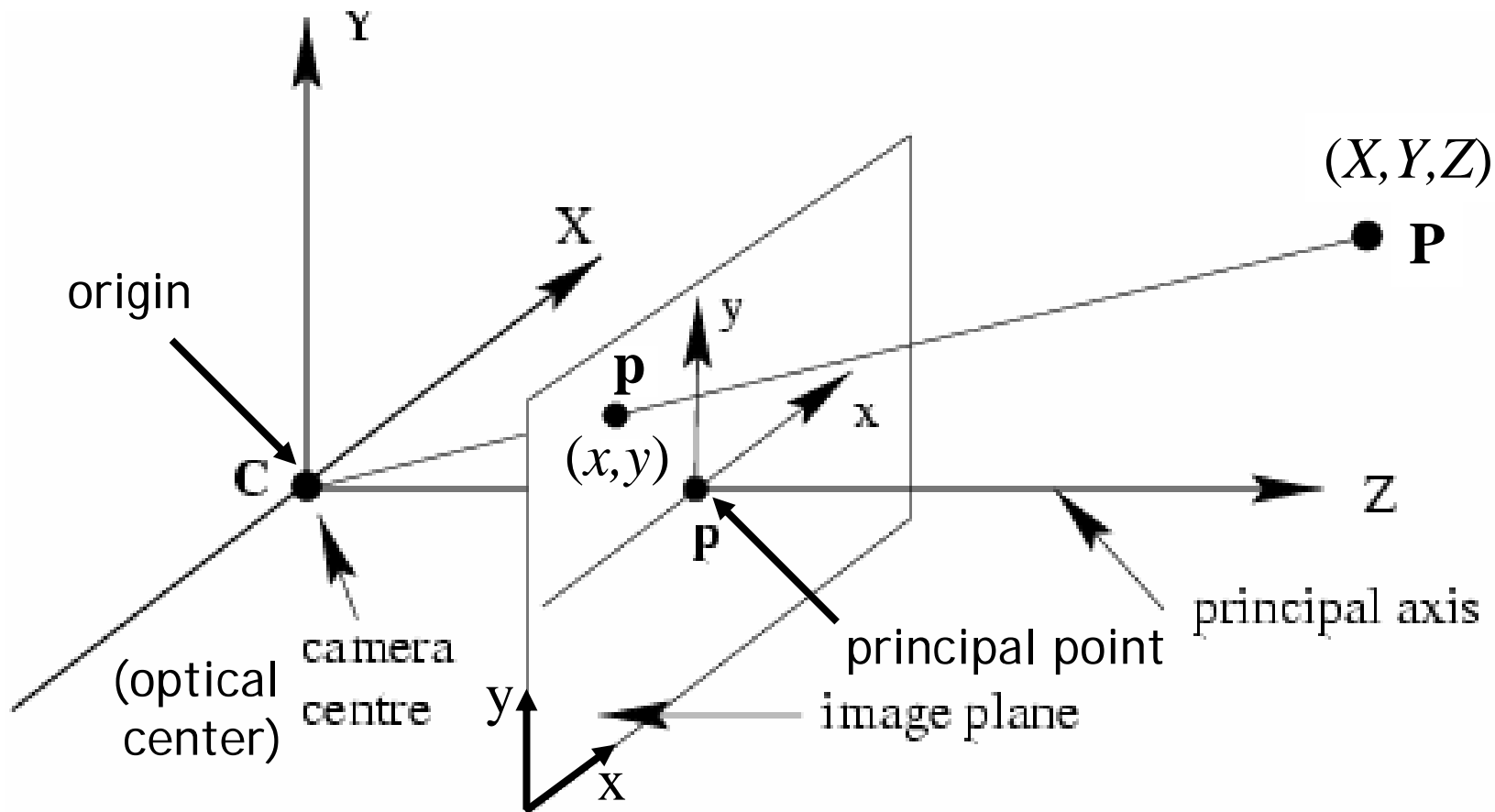


$$x = \frac{fX}{Z}$$

$$y = \frac{fY}{Z}$$

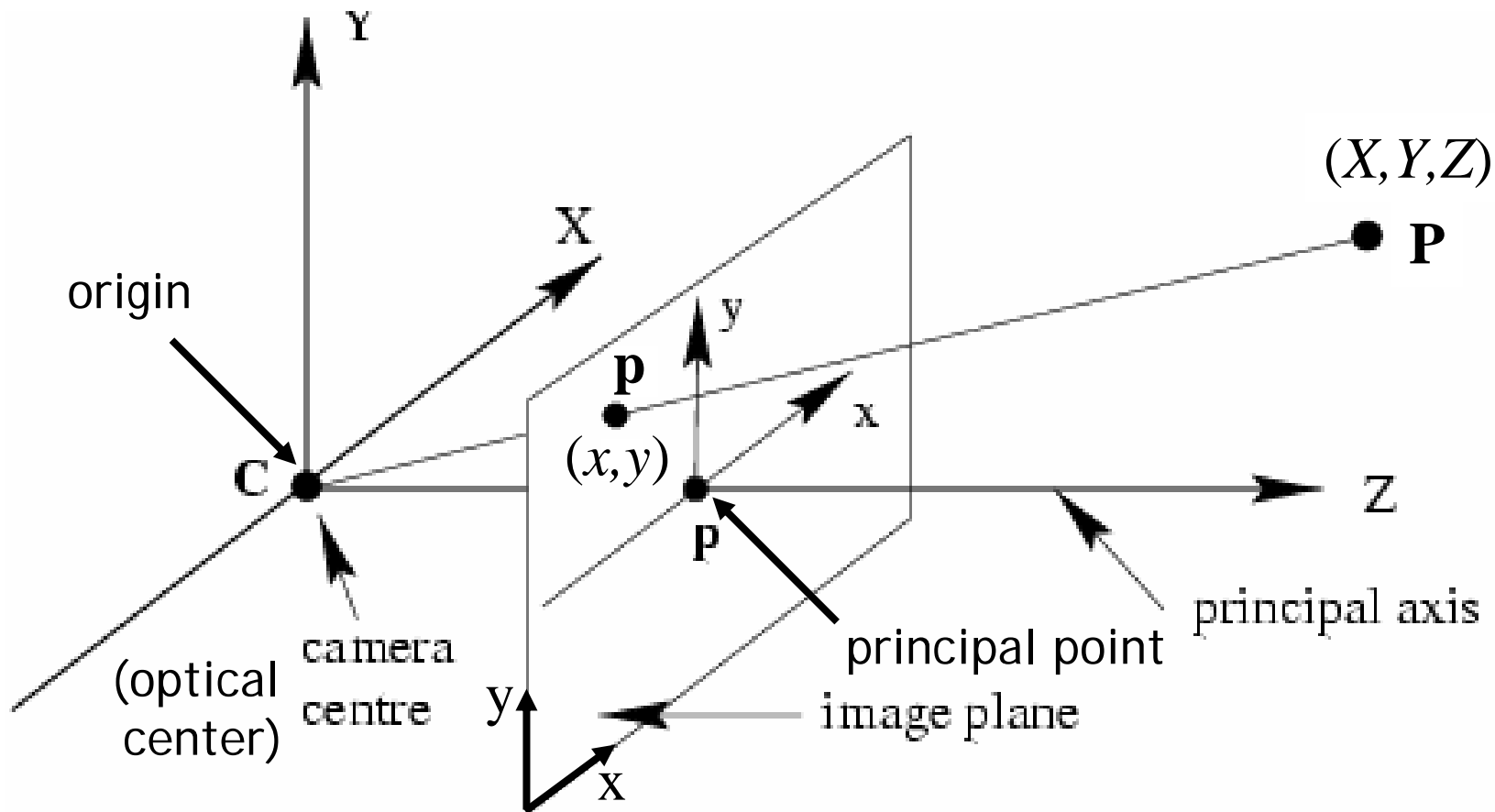
$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \sim \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

Pinhole camera model



$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \sim \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

Pinhole camera model



$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \sim \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

Intrinsic matrix

Is this form of \mathbf{K} good enough?

$$\mathbf{K} = \begin{bmatrix} f & 0 & x_c \\ 0 & f & y_c \\ 0 & 0 & 1 \end{bmatrix}$$

- non-square pixels (digital video)

$$\mathbf{K} = \begin{bmatrix} f_x & 0 & x_c \\ 0 & f_y & y_c \\ 0 & 0 & 1 \end{bmatrix}$$

Intrinsic matrix

Is this form of \mathbf{K} good enough?

$$\mathbf{K} = \begin{bmatrix} f & 0 & x_c \\ 0 & f & y_c \\ 0 & 0 & 1 \end{bmatrix}$$

- non-square pixels (digital video)
- skew

$$\mathbf{K} = \begin{bmatrix} f_x & s & x_c \\ 0 & f_y & y_c \\ 0 & 0 & 1 \end{bmatrix}$$

Intrinsic matrix

Is this form of \mathbf{K} good enough?

$$\mathbf{K} = \begin{bmatrix} f & 0 & x_c \\ 0 & f & y_c \\ 0 & 0 & 1 \end{bmatrix}$$

- non-square pixels (digital video)
- skew
- radial distortion

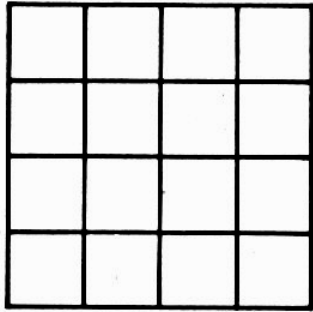
$$\mathbf{K} = \begin{bmatrix} f_x & s & x_c \\ 0 & f_y & y_c \\ 0 & 0 & 1 \end{bmatrix}$$

Distortion

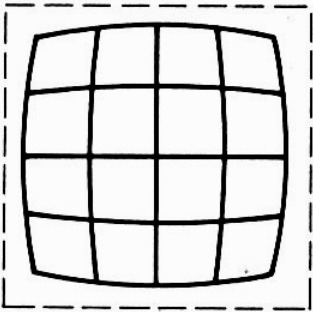


- Radial distortion of the image
 - Caused by imperfect lenses
 - Deviations are most noticeable for rays that pass through the edge of the lens

Barrel Distortion



No distortion

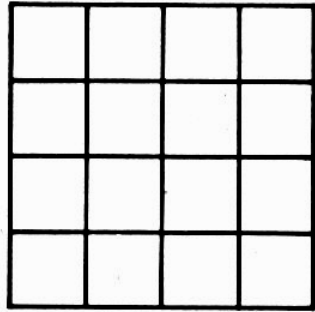


Barrel

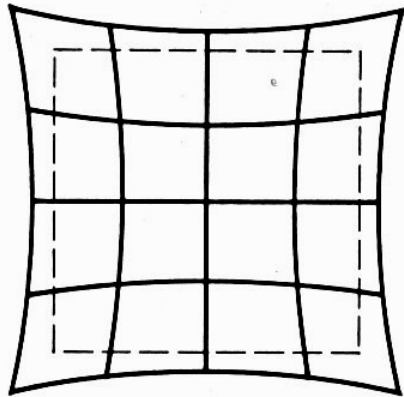


Wide Angle Lens

Pin Cushion Distortion



No distortion



Pin cushion



Telephoto lens

Modeling distortion

Distortion-Free:

$$x = \frac{fX}{Z}$$

$$y = \frac{fY}{Z}$$

With Distortion:

1. Project (X, Y, Z)
to “normalized”
image coordinates

$$x_n = \frac{X}{Z}$$

$$y_n = \frac{Y}{Z}$$

2. Apply radial distortion

$$r^2 = x_n^2 + y_n^2$$

$$x_d = x_n \left(1 + \kappa_1 r^2 + \kappa_2 r^4 \right)$$

$$y_d = y_n \left(1 + \kappa_1 r^2 + \kappa_2 r^4 \right)$$

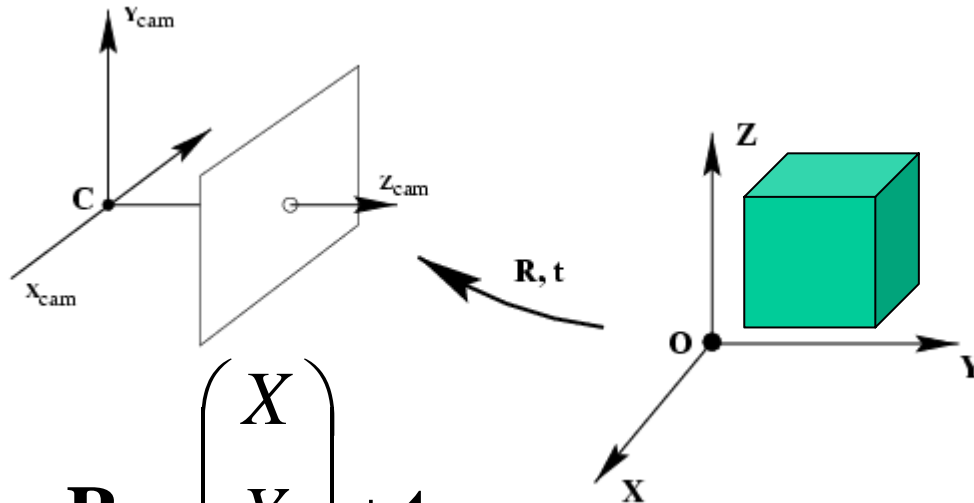
3. Apply focal length
translate image center

$$x = fx_d + x_c$$

$$y = fy_d + y_c$$

- To model lens distortion
 - Use above projection operation instead of standard projection matrix multiplication

Camera rotation and translation



$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \mathbf{R}_{3 \times 3} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \mathbf{t}$$

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \sim \begin{bmatrix} f & 0 & x_c \\ 0 & f & y_c \\ 0 & 0 & 1 \end{bmatrix} [\mathbf{R} | \mathbf{t}] \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\mathbf{x} \sim \mathbf{K} [\mathbf{R} | \mathbf{t}] \mathbf{X}$$

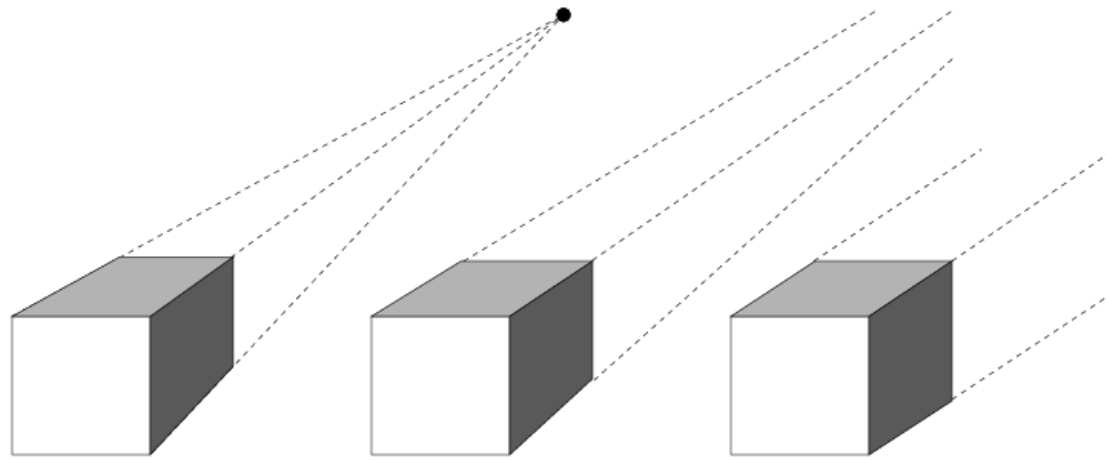
extrinsic matrix

Two kinds of parameters

- *internal* or *intrinsic* parameters: focal length, optical center, skew
- *external* or *extrinsic* (pose): rotation and translation:

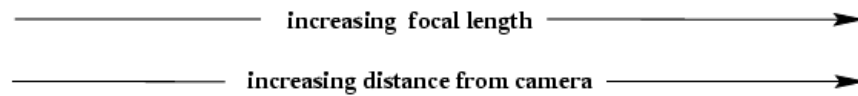
$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \sim \begin{bmatrix} f & 0 & x_c \\ 0 & f & y_c \\ 0 & 0 & 1 \end{bmatrix} [\mathbf{R}|\mathbf{t}] \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Other projection models



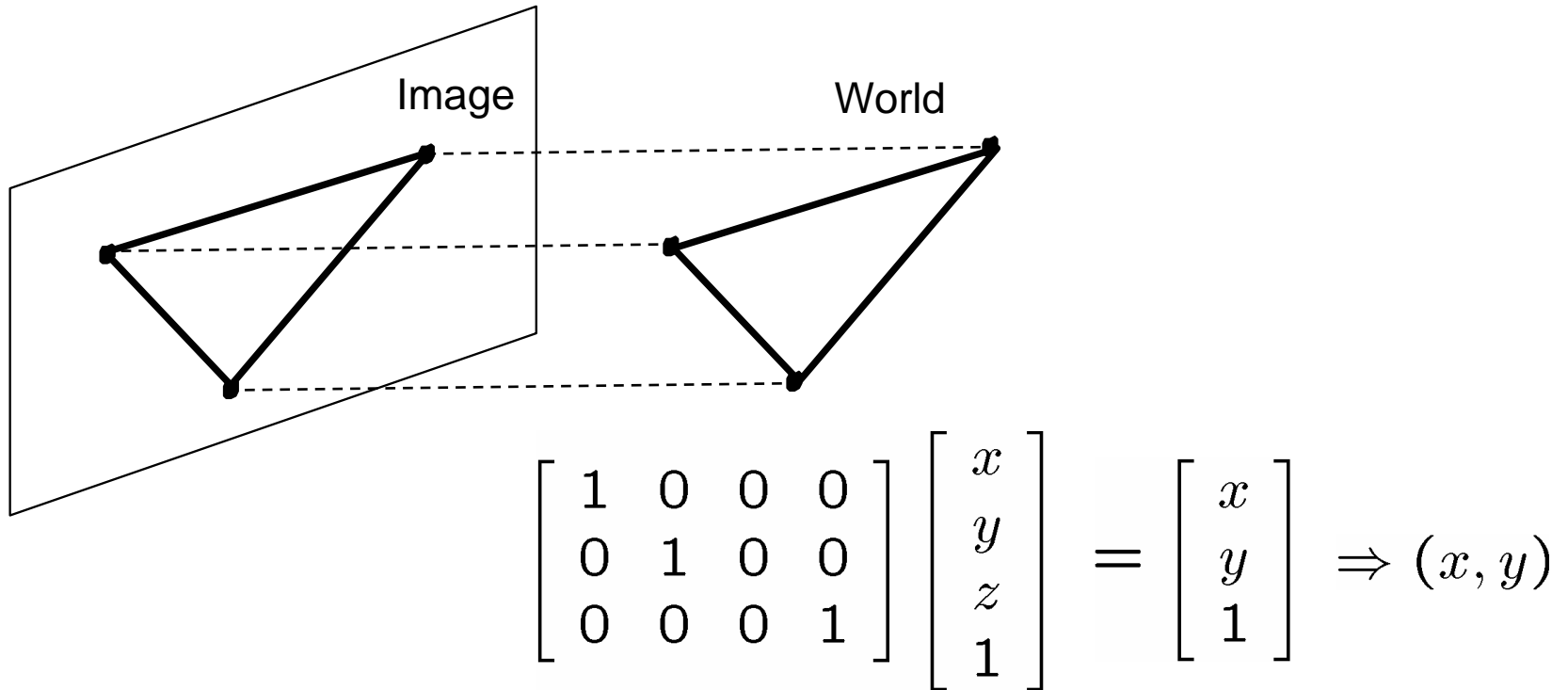
perspective

weak perspective



Orthographic projection

- Special case of perspective projection
 - Distance from the COP to the PP is infinite



- Also called “parallel projection”: $(x, y, z) \rightarrow (x, y)$

Other types of projections

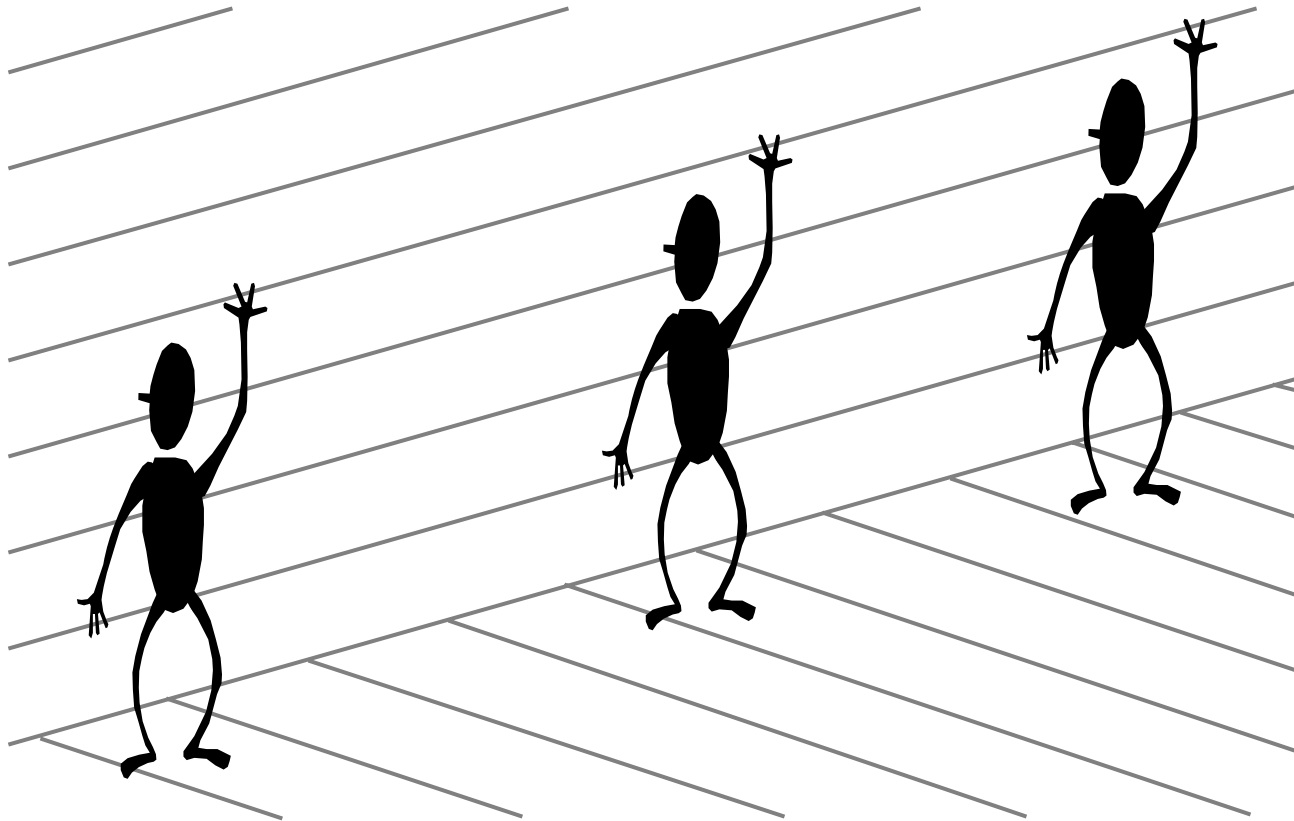
- Scaled orthographic
 - Also called “weak perspective”

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/d \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1/d \end{bmatrix} \Rightarrow (dx, dy)$$

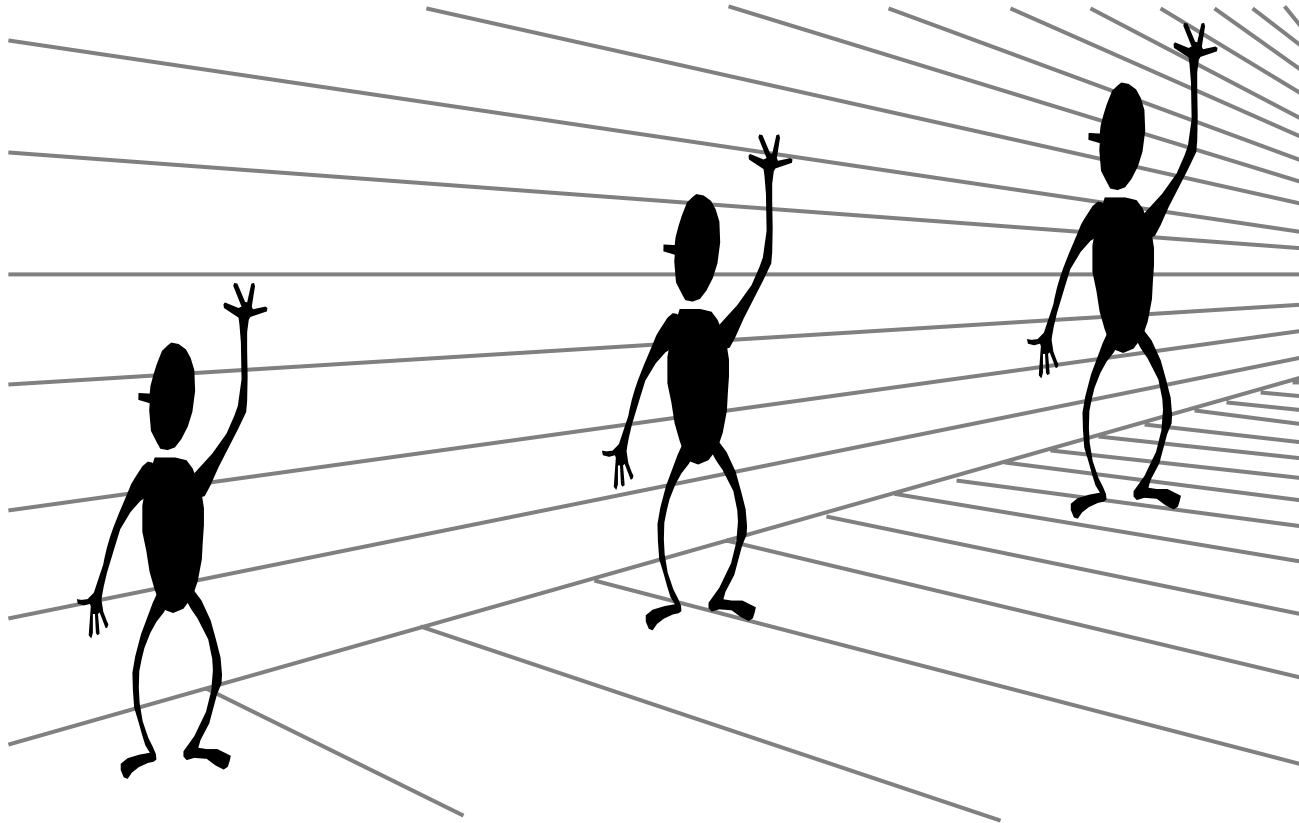
- Affine projection
 - Also called “paraperspective”

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

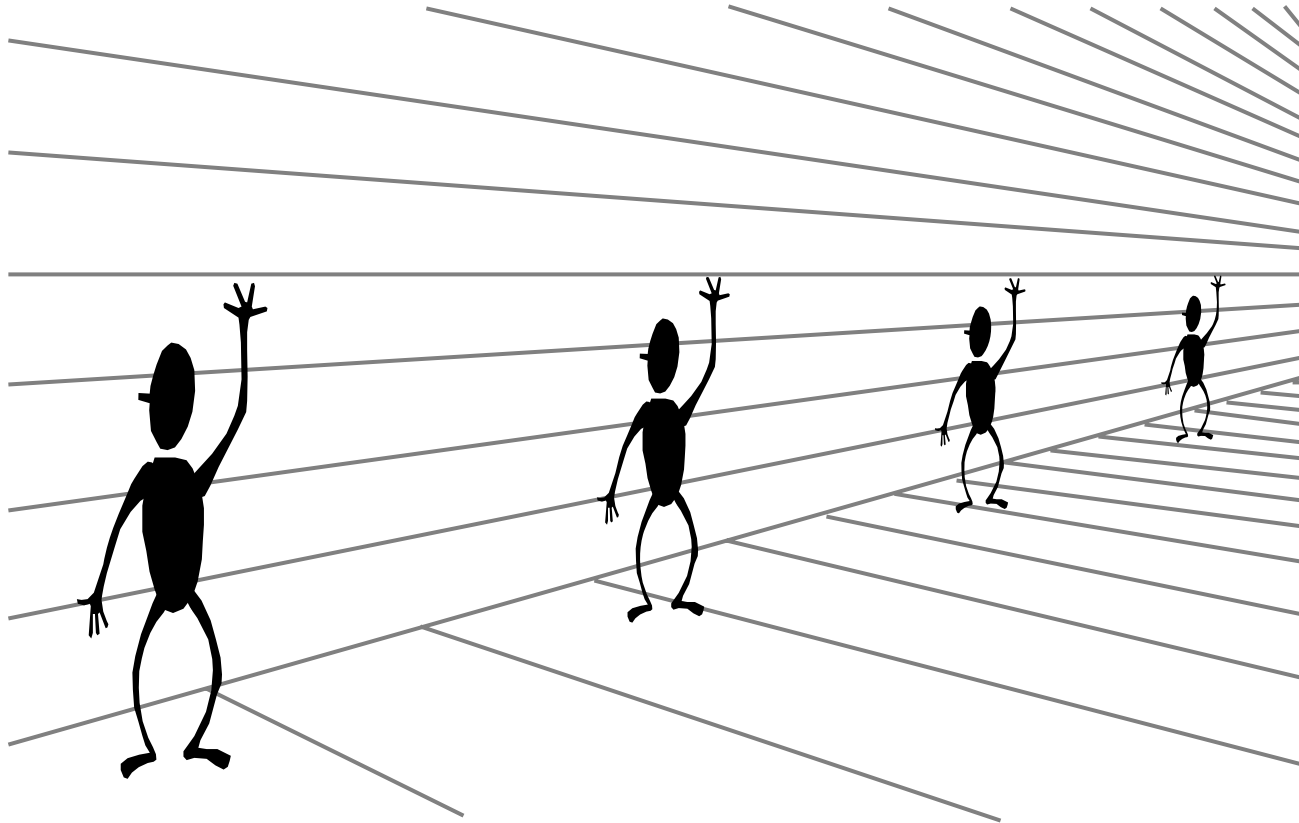
Fun with perspective



Perspective cues



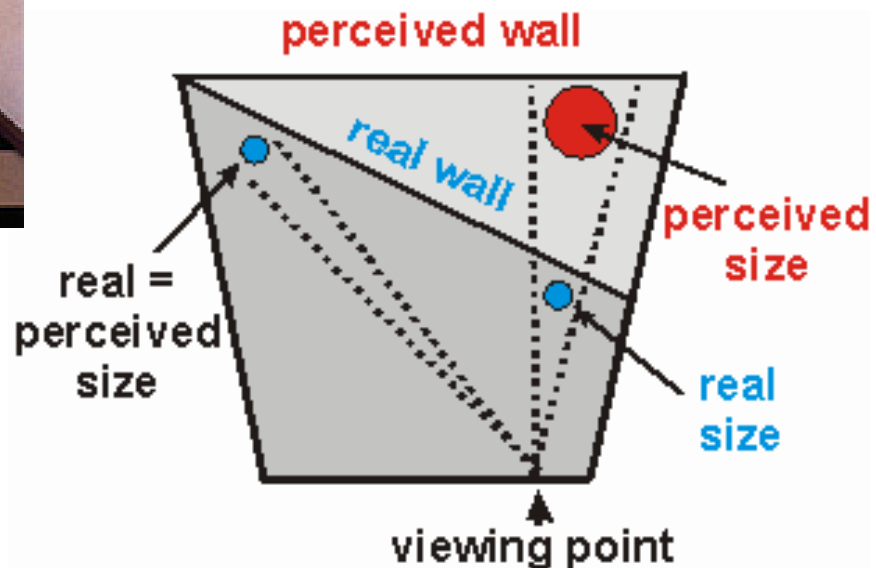
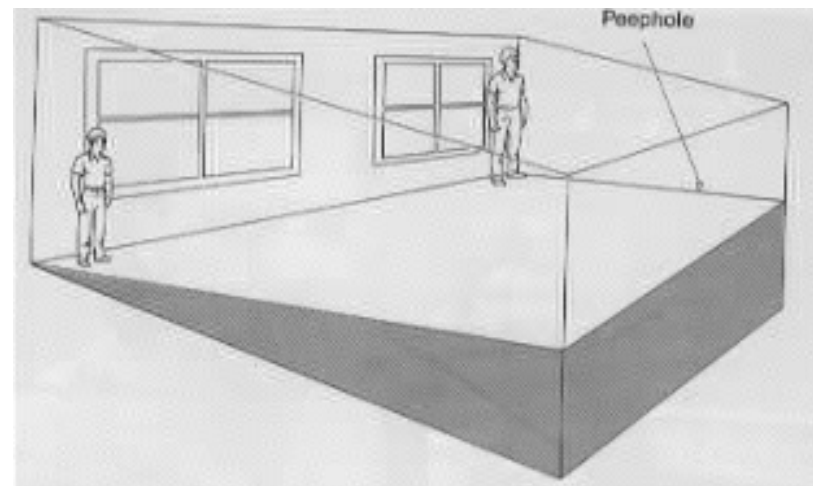
Perspective cues



Fun with perspective



Ames room



Forced perspective in LOTR



Elijah Wood: 5' 6" (1.68 m)

Ian McKellen 5' 11" (1.80 m)

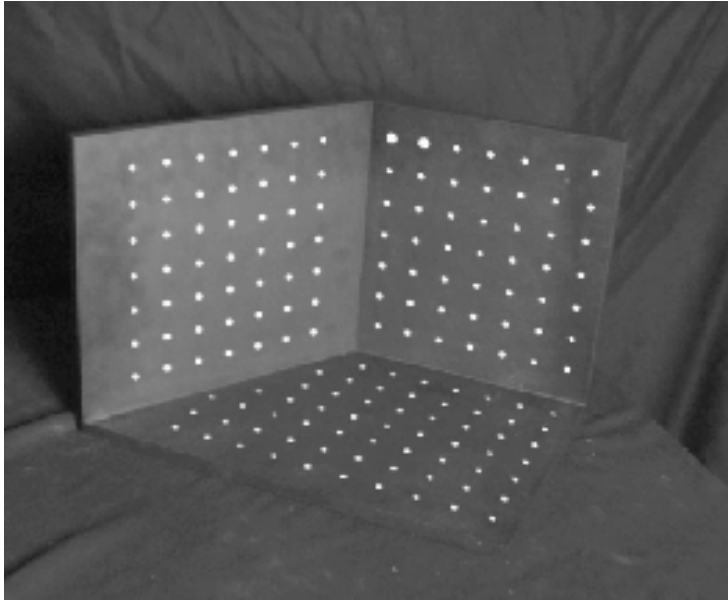
Camera calibration

Camera calibration

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \sim \begin{bmatrix} f & 0 & x_c \\ 0 & f & y_c \\ 0 & 0 & 1 \end{bmatrix} [\mathbf{R}|\mathbf{t}] \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

- Estimate both intrinsic and extrinsic parameters
- Mainly, two categories:
 1. Using objects with known geometry as reference
 2. Self calibration (structure from motion)

Camera calibration approaches



- Directly estimate 11 unknowns in the \mathbf{M} matrix using known 3D points (X_i, Y_i, Z_i) and measured feature positions (u_i, v_i)

$$\mathbf{x} \sim \mathbf{K}[\mathbf{R}|\mathbf{t}]\mathbf{X} = \mathbf{M}\mathbf{X}$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \sim \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Linear regression

$$u_i = \frac{m_{00}X_i + m_{01}Y_i + m_{02}Z_i + m_{03}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1}$$

$$v_i = \frac{m_{10}X_i + m_{11}Y_i + m_{12}Z_i + m_{13}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1}$$

Linear regression

$$u_i(m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1) = m_{00}X_i + m_{01}Y_i + m_{02}Z_i + m_{03}$$

$$v_i(m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1) = m_{10}X_i + m_{11}Y_i + m_{12}Z_i + m_{13}$$

$$\begin{bmatrix} X_i & Y_i & Z_i & 1 & 0 & 0 & 0 & 0 & -u_iX_i & -u_iY_i & -u_iZ_i \\ 0 & 0 & 0 & 0 & X_i & Y_i & Z_i & 1 & -v_iX_i & -v_iY_i & -v_iZ_i \end{bmatrix} \begin{bmatrix} m_{00} \\ m_{01} \\ m_{02} \\ m_{03} \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{13} \\ m_{20} \\ m_{21} \\ m_{22} \end{bmatrix} = \begin{bmatrix} u_i \\ v_i \end{bmatrix}$$

Linear regression

$$\begin{bmatrix}
 & X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1 X_1 & -u_1 Y_1 & -u_1 Z_1 \\
 & 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1 X_1 & -v_1 Y_1 & -v_1 Z_1 \\
 & & & & & & & \vdots & & & & \\
 X_N & Y_N & Z_N & 1 & 0 & 0 & 0 & 0 & 0 & -u_N X_N & -u_N Y_N & -u_N Z_N \\
 0 & 0 & 0 & 0 & X_N & Y_N & Z_N & 1 & 1 & -v_N X_N & -v_N Y_N & -v_N Z_N
 \end{bmatrix}
 \begin{bmatrix}
 m_{00} \\
 m_{01} \\
 m_{02} \\
 m_{03} \\
 m_{10} \\
 m_{11} \\
 m_{12} \\
 m_{13} \\
 m_{20} \\
 m_{21} \\
 m_{22}
 \end{bmatrix}
 =
 \begin{bmatrix}
 u_i \\
 v_i
 \end{bmatrix}$$

Solve for Projection Matrix M using least-square techniques

Normal equation (Geometric Interpretation)

Given an overdetermined system

$$\mathbf{Ax} = \mathbf{b}$$

the normal equation is that which minimizes the sum of the square differences between left and right sides

$$\min \|\mathbf{Ax} - \mathbf{b}\|^2$$

$$\Leftrightarrow \mathbf{A}^T (\mathbf{Ax} - \mathbf{b}) = 0$$

$$\Leftrightarrow x = \left(\mathbf{A}^T \mathbf{A} \right)^{-1} \mathbf{A}^T \mathbf{b}$$

Normal equation (Differential Interpretation)

$$E(\mathbf{x}) = \|\mathbf{Ax} - \mathbf{b}\|^2$$

$$\begin{bmatrix} a_{11} & \dots & a_{1m} \\ \vdots & & \vdots \\ \vdots & & \vdots \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nm} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ \vdots \\ b_n \end{bmatrix}$$

$n \times m$, n equations, m variables

Normal equation

$$\begin{aligned} E(\mathbf{x}) &= \|\mathbf{Ax} - \mathbf{b}\|^2 \\ &= (\mathbf{Ax} - \mathbf{b})^T (\mathbf{Ax} - \mathbf{b}) \\ &= (\mathbf{x}^T \mathbf{A}^T - \mathbf{b}^T) (\mathbf{Ax} - \mathbf{b}) \\ &= \mathbf{x}^T \mathbf{A}^T \mathbf{Ax} - \mathbf{b}^T \mathbf{Ax} - \mathbf{x}^T \mathbf{A}^T \mathbf{b} + \mathbf{b}^T \mathbf{b} \\ &= \mathbf{x}^T \mathbf{A}^T \mathbf{Ax} - (\mathbf{A}^T \mathbf{b})^T \mathbf{x} - (\mathbf{A}^T \mathbf{b})^T \mathbf{x} + \mathbf{b}^T \mathbf{b} \end{aligned}$$



[Carl Friedrich Gauss](#)

$$\frac{\partial E}{\partial \mathbf{x}} = 2\mathbf{A}^T \mathbf{Ax} - 2\mathbf{A}^T \mathbf{b}$$

Who invented Least Square?

Nonlinear optimization

- A probabilistic view of least square
- Feature measurement equations

$$u_i = f(\mathbf{M}, \mathbf{x}_i) + n_i = \hat{u}_i + n_i, \quad n_i \sim N(0, \sigma)$$

$$v_i = g(\mathbf{M}, \mathbf{x}_i) + m_i = \hat{v}_i + m_i, \quad m_i \sim N(0, \sigma)$$

- Likelihood of \mathbf{M} given $\{(u_j, v_j)\}$

$$\begin{aligned} L &= \prod_i p(u_i | \hat{u}_i) p(v_i | \hat{v}_i) \\ &= \prod_i e^{-(u_i - \hat{u}_i)^2 / \sigma^2} e^{-(v_i - \hat{v}_i)^2 / \sigma^2} \end{aligned}$$

Optimal estimation

- Log likelihood of \mathbf{M} given $\{(u_i, v_i)\}$

$$C = -\log L = \sum_i (u_i - \hat{u}_i)^2 / \sigma_i^2 + (v_i - \hat{v}_i)^2 / \sigma_i^2$$

$$= \sum_{i=1}^N \frac{1}{\sigma_i^2} \left(u_i - \frac{m_{00}X_i + m_{01}Y_i + m_{02}Z_i + m_{03}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1} \right)^2 + \frac{1}{\sigma_i^2} \left(v_i - \frac{m_{10}X_i + m_{11}Y_i + m_{12}Z_i + m_{13}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1} \right)^2$$

- It is a least square problem (but not necessarily linear least square)
- How do we minimize C ?

Nonlinear least square methods

Least square fitting

Least Squares Problem

Find \mathbf{x}^* , a local minimizer for

$$F(\mathbf{x}) = \frac{1}{2} \sum_{i=1}^m (f_i(\mathbf{x}))^2 ,$$

where $f_i : \mathbb{R}^n \mapsto \mathbb{R}$, $i = 1, \dots, m$ are given functions, and $m \geq n$.

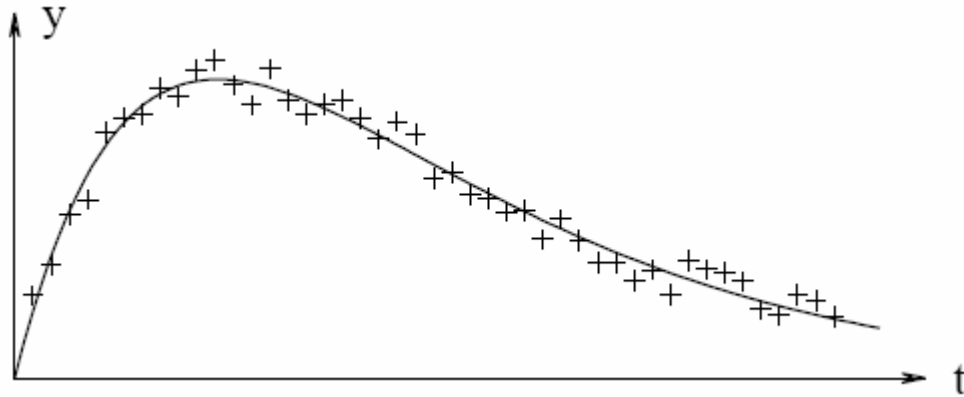
number of data points



number of parameters



Nonlinear least square fitting



$$\text{model } M(\mathbf{x}, t) = x_3 e^{x_1 t} + x_4 e^{x_2 t}$$

$$\text{parameters } \mathbf{x} = [x_1, x_2, x_3, x_4]^\top$$

$$\begin{aligned} \text{residuals } f_i(\mathbf{x}) &= y_i - M(\mathbf{x}, t_i) \\ &= y_i - x_3 e^{x_1 t_i} - x_4 e^{x_2 t_i} \end{aligned}$$

Function minimization

Least square is related to function minimization.

Global Minimizer

Given $F : \mathbb{R}^n \mapsto \mathbb{R}$. Find

$$\mathbf{x}^+ = \operatorname{argmin}_{\mathbf{x}} \{F(\mathbf{x})\} .$$

It is very hard to solve in general. Here, we only consider a simpler problem of finding local minimum.

Local Minimizer

Given $F : \mathbb{R}^n \mapsto \mathbb{R}$. Find \mathbf{x}^* so that

$$F(\mathbf{x}^*) \leq F(\mathbf{x}) \quad \text{for} \quad \|\mathbf{x} - \mathbf{x}^*\| < \delta .$$

Function minimization

We assume that the cost function F is differentiable and so smooth that the following *Taylor expansion* is valid,²⁾

$$F(\mathbf{x}+\mathbf{h}) = F(\mathbf{x}) + \mathbf{h}^\top \mathbf{g} + \frac{1}{2}\mathbf{h}^\top \mathbf{H} \mathbf{h} + O(\|\mathbf{h}\|^3),$$

where \mathbf{g} is the *gradient*,

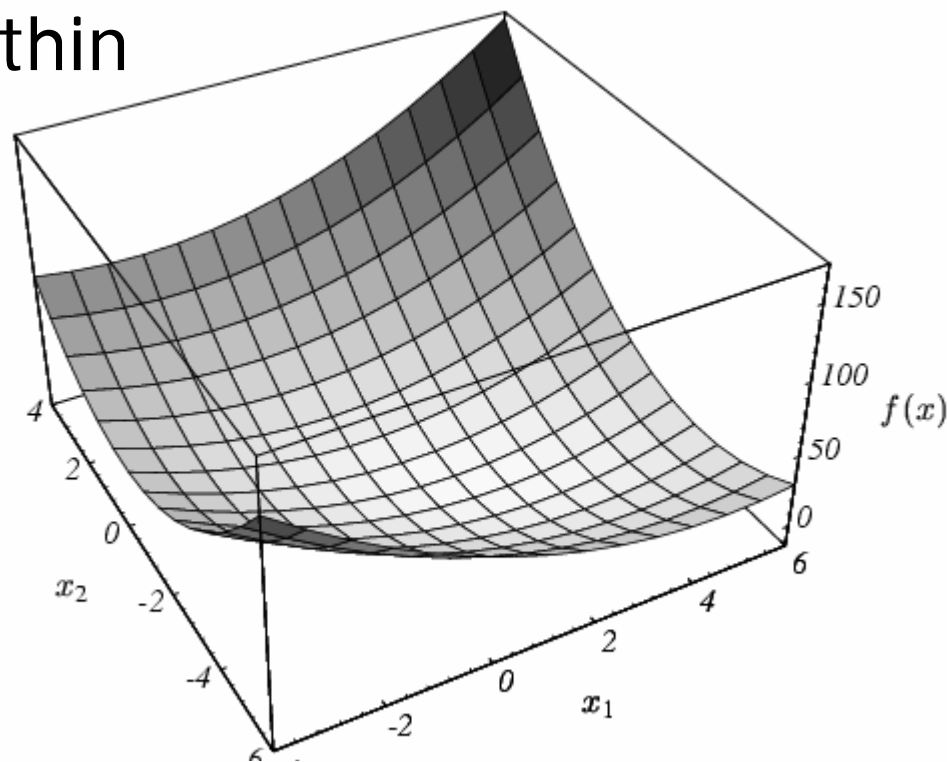
$$\mathbf{g} \equiv \mathbf{F}'(\mathbf{x}) = \begin{bmatrix} \frac{\partial F}{\partial x_1}(\mathbf{x}) \\ \vdots \\ \frac{\partial F}{\partial x_n}(\mathbf{x}) \end{bmatrix},$$

and \mathbf{H} is the *Hessian*,

$$\mathbf{H} \equiv \mathbf{F}''(\mathbf{x}) = \left[\frac{\partial^2 F}{\partial x_i \partial x_j}(\mathbf{x}) \right].$$

Quadratic functions

Approximate the function with
a quadratic function within
a small neighborhood



$$f(x) = \frac{1}{2}x^T A x - b^T x + c$$

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ -8 \end{bmatrix}, \quad c = 0.$$

Function minimization

Theorem 1.5. Necessary condition for a local minimizer.

If \mathbf{x}^* is a local minimizer, then

$$\mathbf{g}^* \equiv \mathbf{F}'(\mathbf{x}^*) = \mathbf{0}.$$

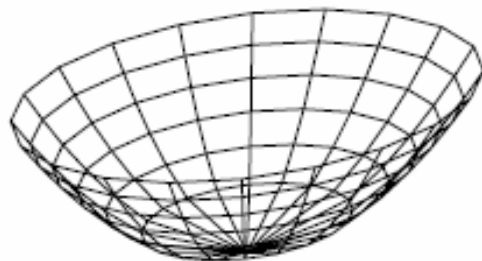
Definition 1.6. Stationary point. If

$$\mathbf{g}_s \equiv \mathbf{F}'(\mathbf{x}_s) = \mathbf{0},$$

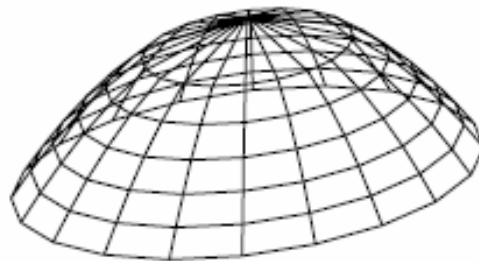
then \mathbf{x}_s is said to be a *stationary point* for F .

$$F(\mathbf{x}_s + \mathbf{h}) = F(\mathbf{x}_s) + \frac{1}{2} \mathbf{h}^\top \mathbf{H}_s \mathbf{h} + O(\|\mathbf{h}\|^3)$$

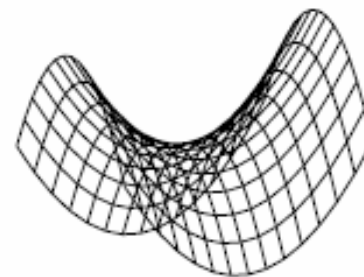
\mathbf{H}_s is *positive definite*.



a) *minimum*



b) *maximum*



c) *saddle point*

Computing gradient and Hessian

$$F(\mathbf{x} + \mathbf{h}) = \frac{1}{2} \sum_{i=1}^m (f_i(\mathbf{x} + \mathbf{h}))^2$$

Computing gradient and Hessian

$$F(\mathbf{x} + \mathbf{h}) = \frac{1}{2} \sum_{i=1}^m (f_i(\mathbf{x} + \mathbf{h}))^2$$
$$= \frac{1}{2} \sum_{i=1}^m \left(f_i(\mathbf{x}) + \frac{\partial f_i}{\partial \mathbf{x}}{}^T \mathbf{h} + \frac{1}{2} \mathbf{h}^T \frac{\partial^2 f_i}{\partial \mathbf{x}^2} \mathbf{h} \right)^2$$

Computing gradient and Hessian

$$\begin{aligned} F(\mathbf{x} + \mathbf{h}) &= \frac{1}{2} \sum_{i=1}^m (f_i(\mathbf{x} + \mathbf{h}))^2 \\ &= \frac{1}{2} \sum_{i=1}^m \left(f_i(\mathbf{x}) + \frac{\partial f_i}{\partial \mathbf{x}}{}^T \mathbf{h} + \frac{1}{2} \mathbf{h}^T \frac{\partial^2 f_i}{\partial \mathbf{x}^2} \mathbf{h} \right)^2 \\ &= \frac{1}{2} \sum_{i=1}^m \left((f_i(\mathbf{x}))^2 + 2 \left(f_i(\mathbf{x}) \frac{\partial f_i}{\partial \mathbf{x}} \right)^T \mathbf{h} + \mathbf{h}^T \left(f_i(\mathbf{x}) \frac{\partial^2 f_i}{\partial \mathbf{x}^2} + \frac{\partial f_i}{\partial \mathbf{x}} \frac{\partial f_i}{\partial \mathbf{x}}{}^T \right) \mathbf{h} \right) \end{aligned}$$

Computing gradient and Hessian

$$\begin{aligned} F(\mathbf{x} + \mathbf{h}) &= \frac{1}{2} \sum_{i=1}^m (f_i(\mathbf{x} + \mathbf{h}))^2 \\ &= \frac{1}{2} \sum_{i=1}^m \left(f_i(\mathbf{x}) + \frac{\partial f_i}{\partial \mathbf{x}}^T \mathbf{h} + \frac{1}{2} \mathbf{h}^T \frac{\partial^2 f_i}{\partial \mathbf{x}^2} \mathbf{h} \right)^2 \\ &= \frac{1}{2} \sum_{i=1}^m \left((f_i(\mathbf{x}))^2 + 2 \left(f_i(\mathbf{x}) \frac{\partial f_i}{\partial \mathbf{x}} \right)^T \mathbf{h} + \mathbf{h}^T \left(f_i(\mathbf{x}) \frac{\partial^2 f_i}{\partial \mathbf{x}^2} + \frac{\partial f_i}{\partial \mathbf{x}} \frac{\partial f_i}{\partial \mathbf{x}}^T \right) \mathbf{h} \right) \\ &= \frac{1}{2} \sum_{i=1}^m (f_i(\mathbf{x}))^2 + \left(\sum_{i=1}^m f_i(\mathbf{x}) \frac{\partial f_i}{\partial \mathbf{x}} \right)^T \mathbf{h} + \frac{1}{2} \mathbf{h}^T \left(\sum_{i=1}^m f_i(\mathbf{x}) \frac{\partial^2 f_i}{\partial \mathbf{x}^2} + \frac{\partial f_i}{\partial \mathbf{x}} \frac{\partial f_i}{\partial \mathbf{x}}^T \right) \mathbf{h} \end{aligned}$$

Searching for update \mathbf{h}

$$F(\mathbf{x} + \mathbf{h}) \approx \frac{1}{2} \sum_{i=1}^m (f_i(\mathbf{x}))^2 + \overset{\text{Gradient}}{\mathbf{g}^T \mathbf{h}} + \frac{1}{2} \overset{\text{Hessian}}{\mathbf{h}^T \mathbf{H} \mathbf{h}}$$

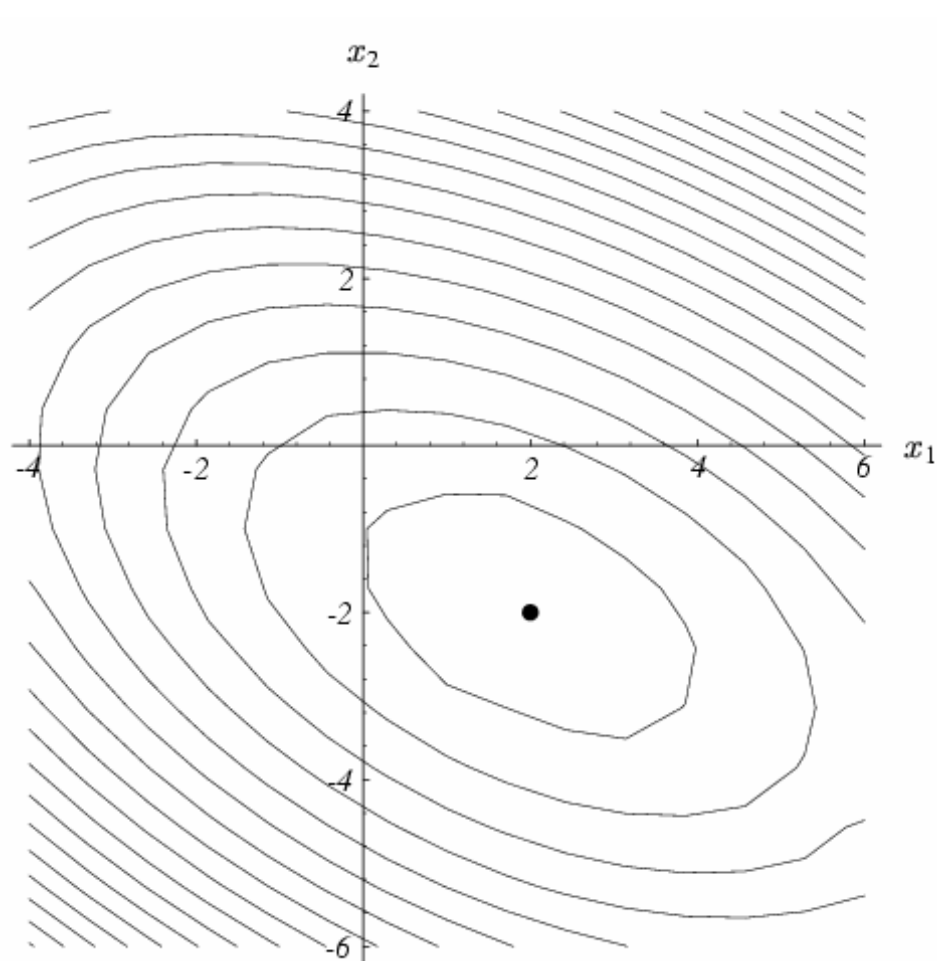
Idea 1: Steepest Descent

Let $\mathbf{h} = -\alpha \mathbf{g}$

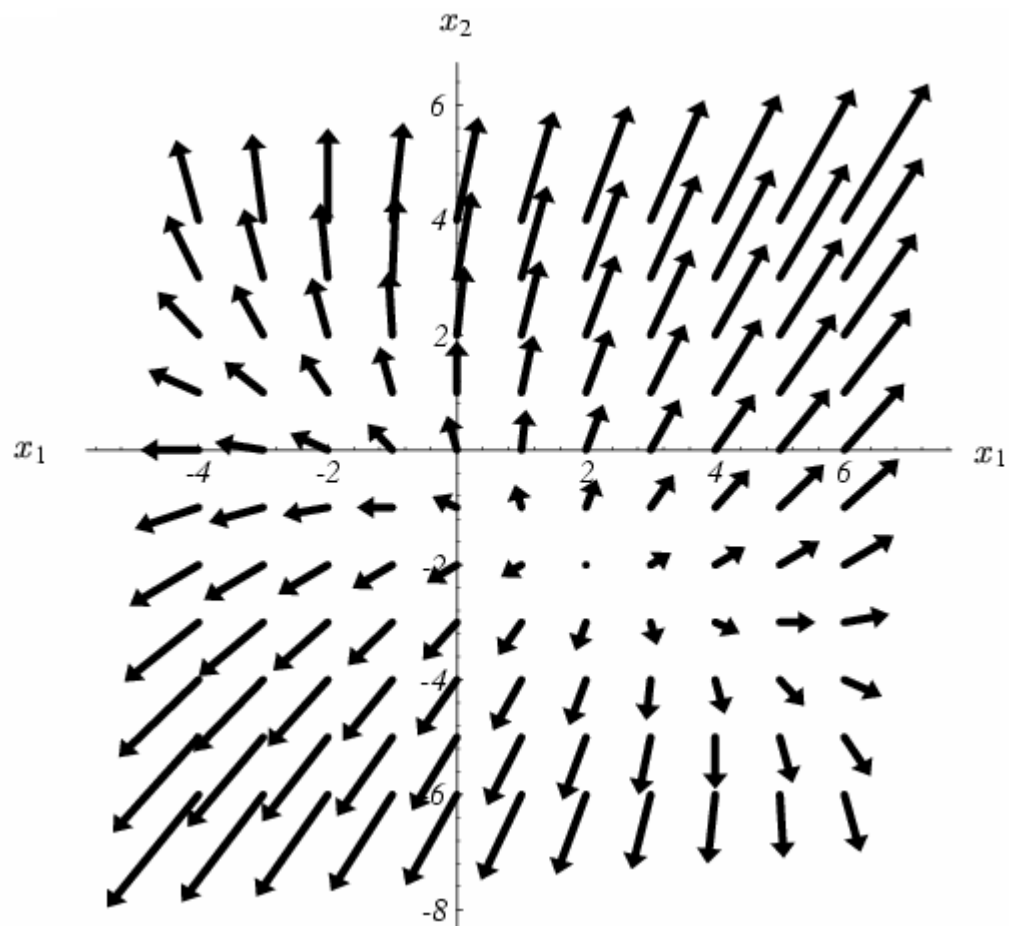
$$F(\mathbf{x} + \mathbf{h}) \approx \frac{1}{2} \sum_{i=1}^m (f_i(\mathbf{x}))^2 - \alpha \mathbf{g}^T \mathbf{g} + \frac{\alpha^2}{2} \mathbf{g}^T \mathbf{H} \mathbf{g}$$

$$\alpha = \frac{\mathbf{g}^T \mathbf{g}}{\mathbf{g}^T \mathbf{H} \mathbf{g}}$$

Steepest descent method

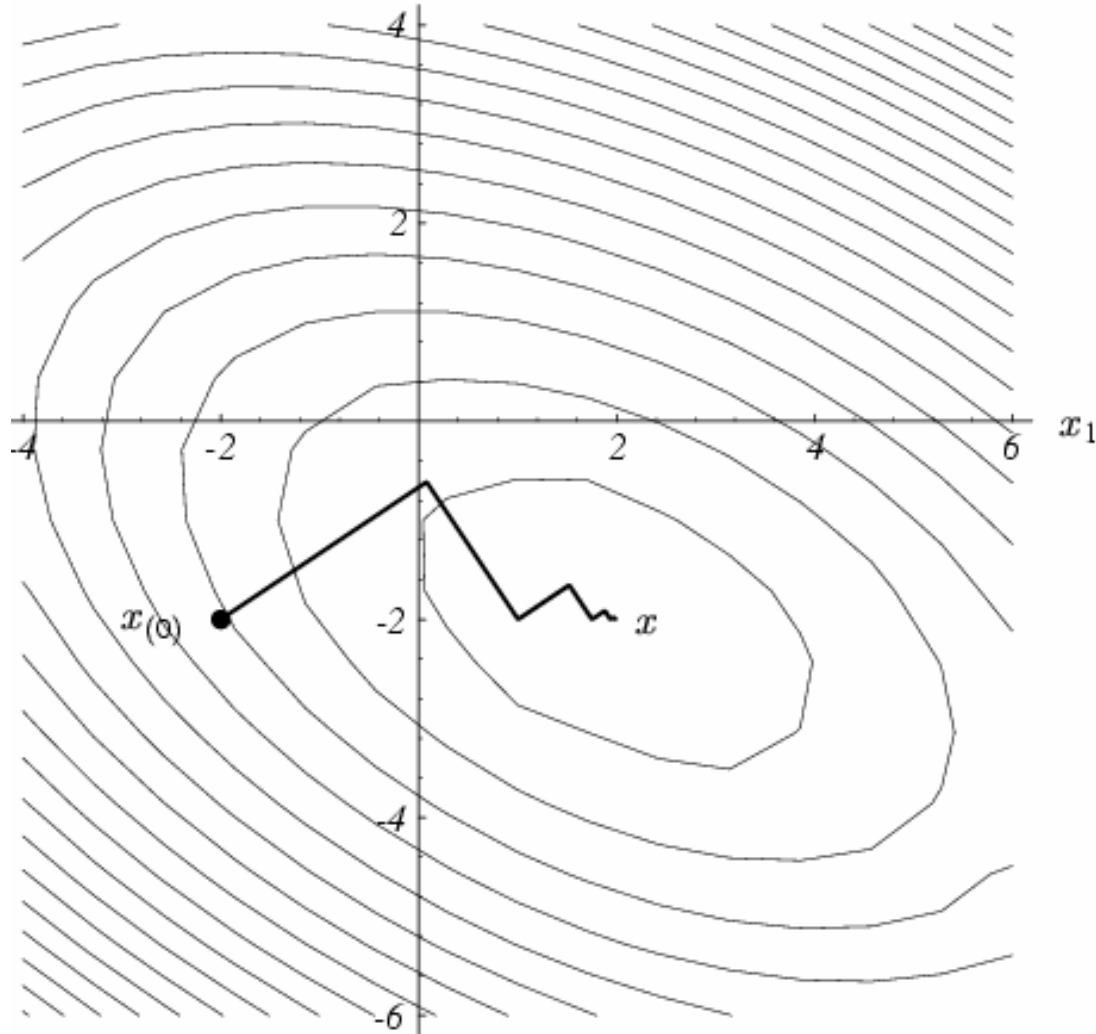


isocontour



gradient

Steepest descent method



It has good performance in the initial stage of the iterative process. Converge very slow with a linear rate.

Searching for update \mathbf{h}

$$F(\mathbf{x} + \mathbf{h}) \approx \frac{1}{2} \sum_{i=1}^m (f_i(\mathbf{x}))^2 + \overset{\text{Gradient}}{\mathbf{g}^T \mathbf{h}} + \frac{1}{2} \overset{\text{Hessian}}{\mathbf{h}^T \mathbf{H} \mathbf{h}}$$

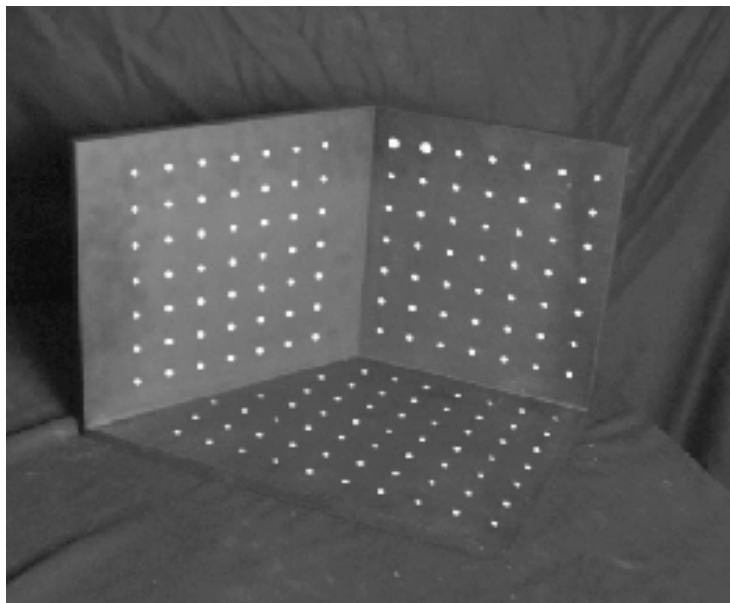
Idea 2: minimizing the quadric directly

$$\text{Let } \frac{\partial}{\partial \mathbf{h}} F(\mathbf{x} + \mathbf{h}) \approx \mathbf{g} + \mathbf{H} \mathbf{h} = 0$$

$$\mathbf{h} = \mathbf{H}^{-1} \mathbf{g}$$

Converge faster but needs to solve the linear system

Recap: Calibration



- Directly estimate 11 unknowns in the \mathbf{M} matrix using known 3D points (X_i, Y_i, Z_i) and measured feature positions (u_i, v_i)

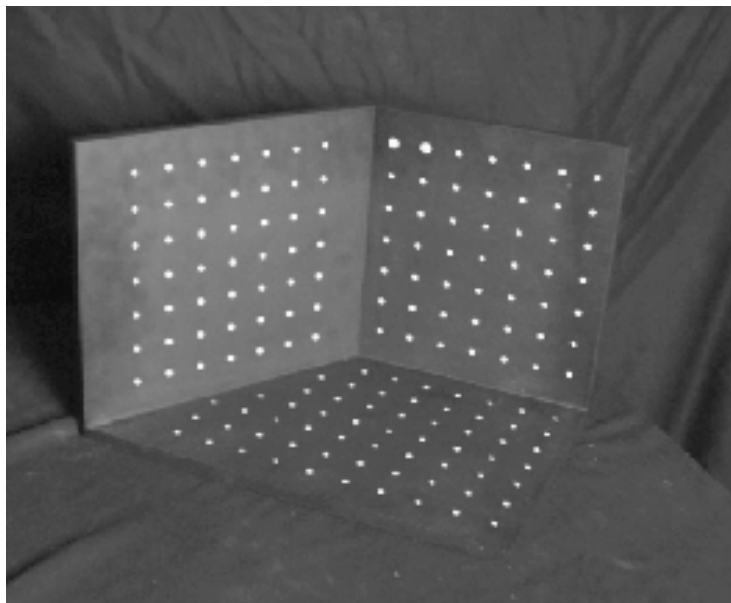
$$\mathbf{x} \sim \mathbf{K}[\mathbf{R}|\mathbf{t}]\mathbf{X} = \mathbf{M}\mathbf{X}$$

Camera Model:

$$u_i = \frac{m_{00}X_i + m_{01}Y_i + m_{02}Z_i + m_{03}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1}$$

$$v_i = \frac{m_{10}X_i + m_{11}Y_i + m_{12}Z_i + m_{13}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1}$$

Recap: Calibration



- Directly estimate 11 unknowns in the \mathbf{M} matrix using known 3D points (X_i, Y_i, Z_i) and measured feature positions (u_i, v_i)

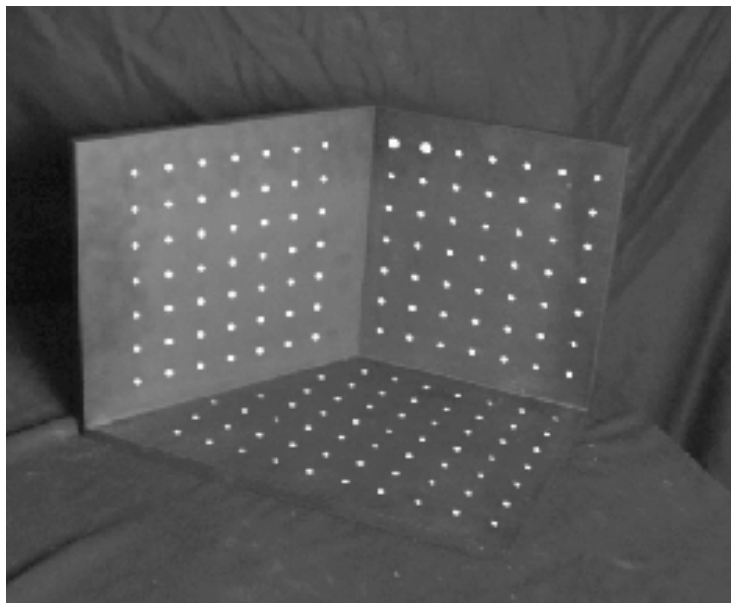
$$\mathbf{x} \sim \mathbf{K} \begin{bmatrix} \mathbf{R} | \mathbf{t} \end{bmatrix} \mathbf{X} = \mathbf{M} \mathbf{X}$$

Linear Approach:

$$u_i(m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1) = m_{00}X_i + m_{01}Y_i + m_{02}Z_i + m_{03}$$

$$v_i(m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1) = m_{10}X_i + m_{11}Y_i + m_{12}Z_i + m_{13}$$

Recap: Calibration



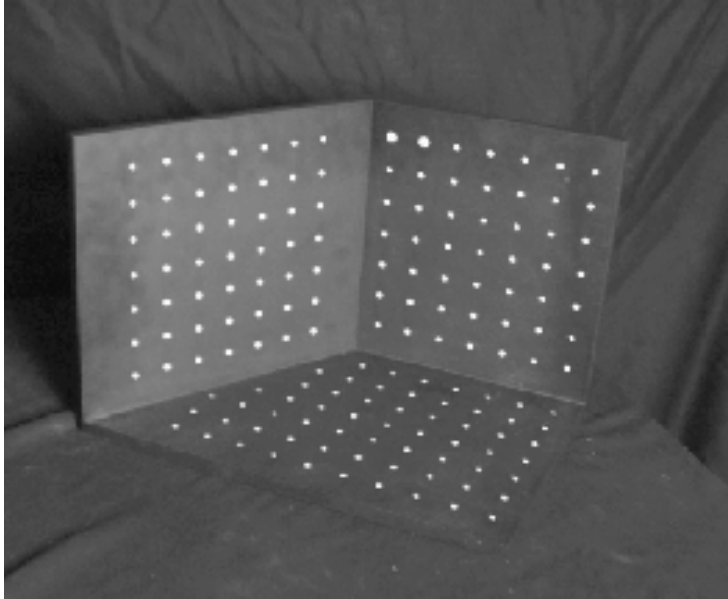
- Directly estimate 11 unknowns in the \mathbf{M} matrix using known 3D points (X_i, Y_i, Z_i) and measured feature positions (u_i, v_i)

$$\mathbf{x} \sim \mathbf{K}[\mathbf{R}|\mathbf{t}]\mathbf{X} = \mathbf{M}\mathbf{X}$$

NonLinear Approach:

$$\sum_{i=1}^N \left(u_i - \frac{m_{00}X_i + m_{01}Y_i + m_{02}Z_i + m_{03}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1} \right)^2 + \left(v_i - \frac{m_{10}X_i + m_{11}Y_i + m_{12}Z_i + m_{13}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1} \right)^2$$

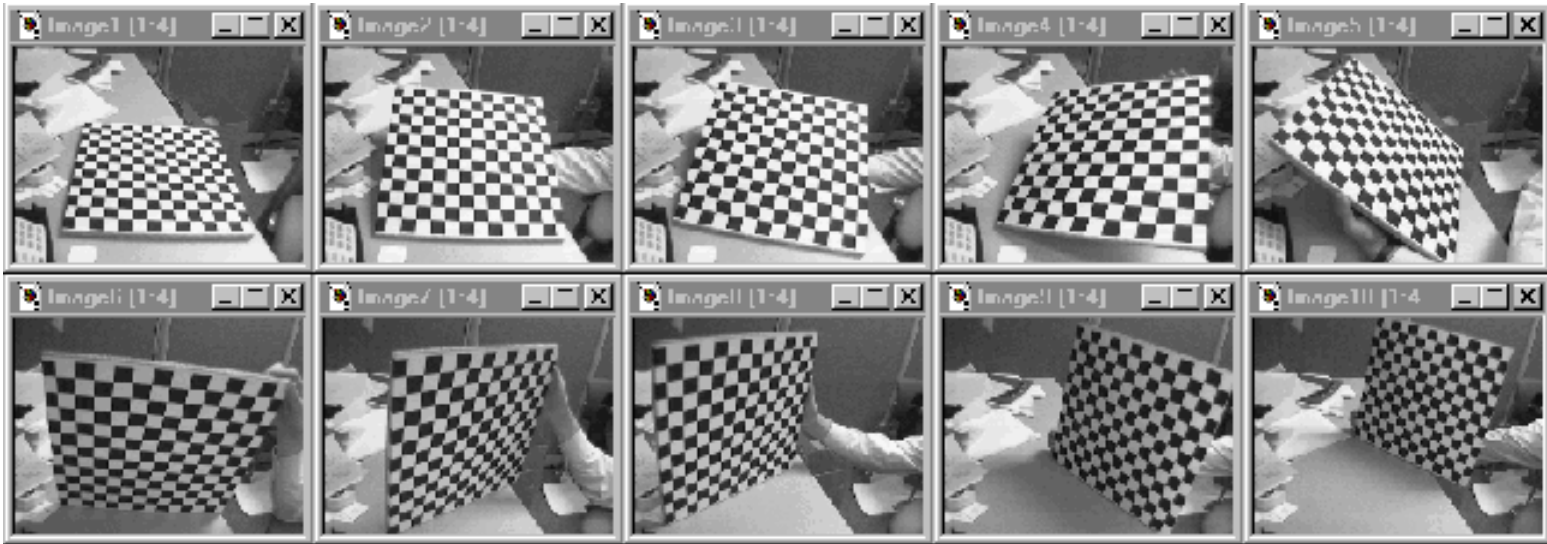
Practical Issue



is hard to make and the 3D feature positions are difficult to measure!

A popular calibration tool

Multi-plane calibration

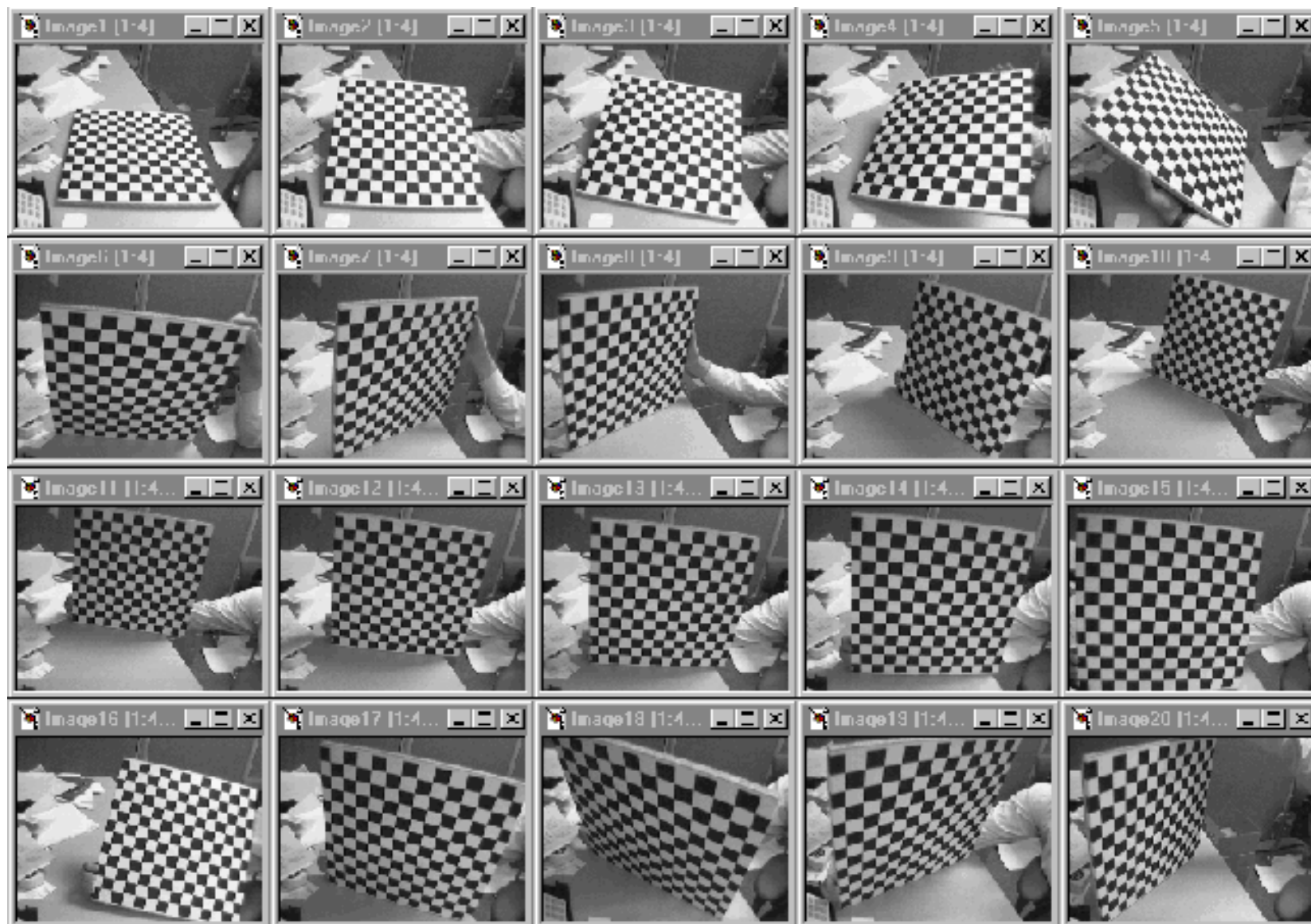


Images courtesy Jean-Yves Bouguet, Intel Corp.

Advantage

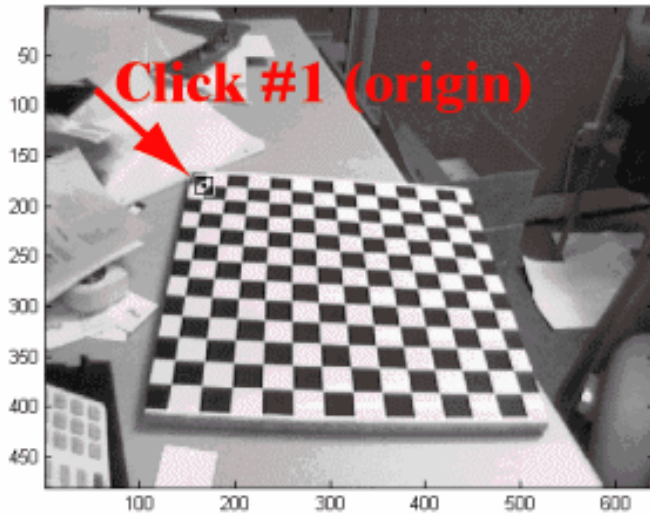
- Only requires a plane
- Don't have to know positions/orientations
- Good code available online!
 - Intel's OpenCV library: <http://www.intel.com/research/mrl/research/opencv/>
 - Matlab version by Jean-Yves Bouguet: http://www.vision.caltech.edu/bouguetj/calib_doc/index.html
 - Zhengyou Zhang's web site: <http://research.microsoft.com/~zhang/Calib/>

Step 1: data acquisition

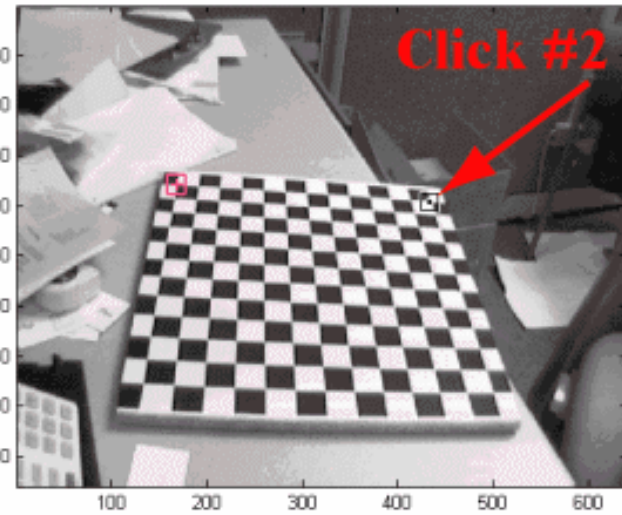


Step 2: specify corner order

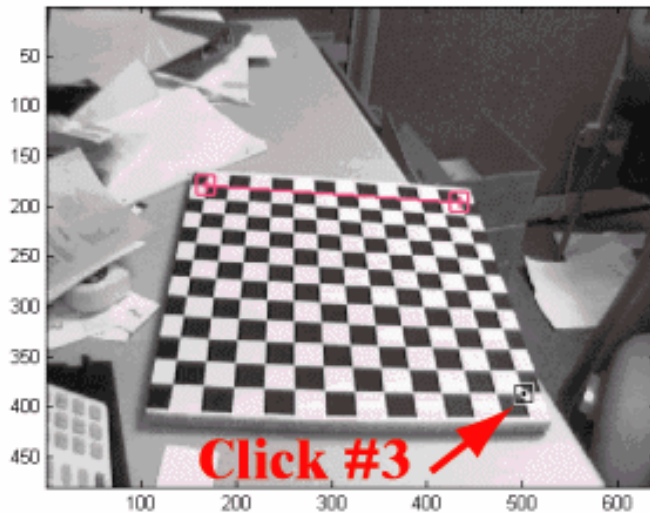
Click on the four extreme corners of the rectangular pattern (first corner = origin)... Image 1



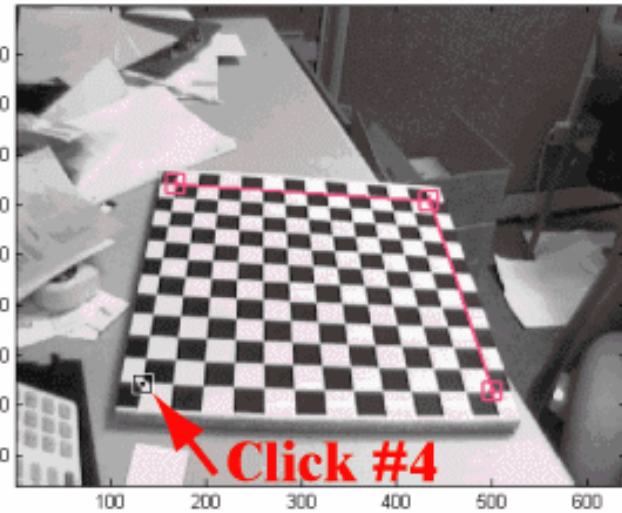
Click on the four extreme corners of the rectangular pattern (first corner = origin)... Image 1



Click on the four extreme corners of the rectangular pattern (first corner = origin)... Image 1

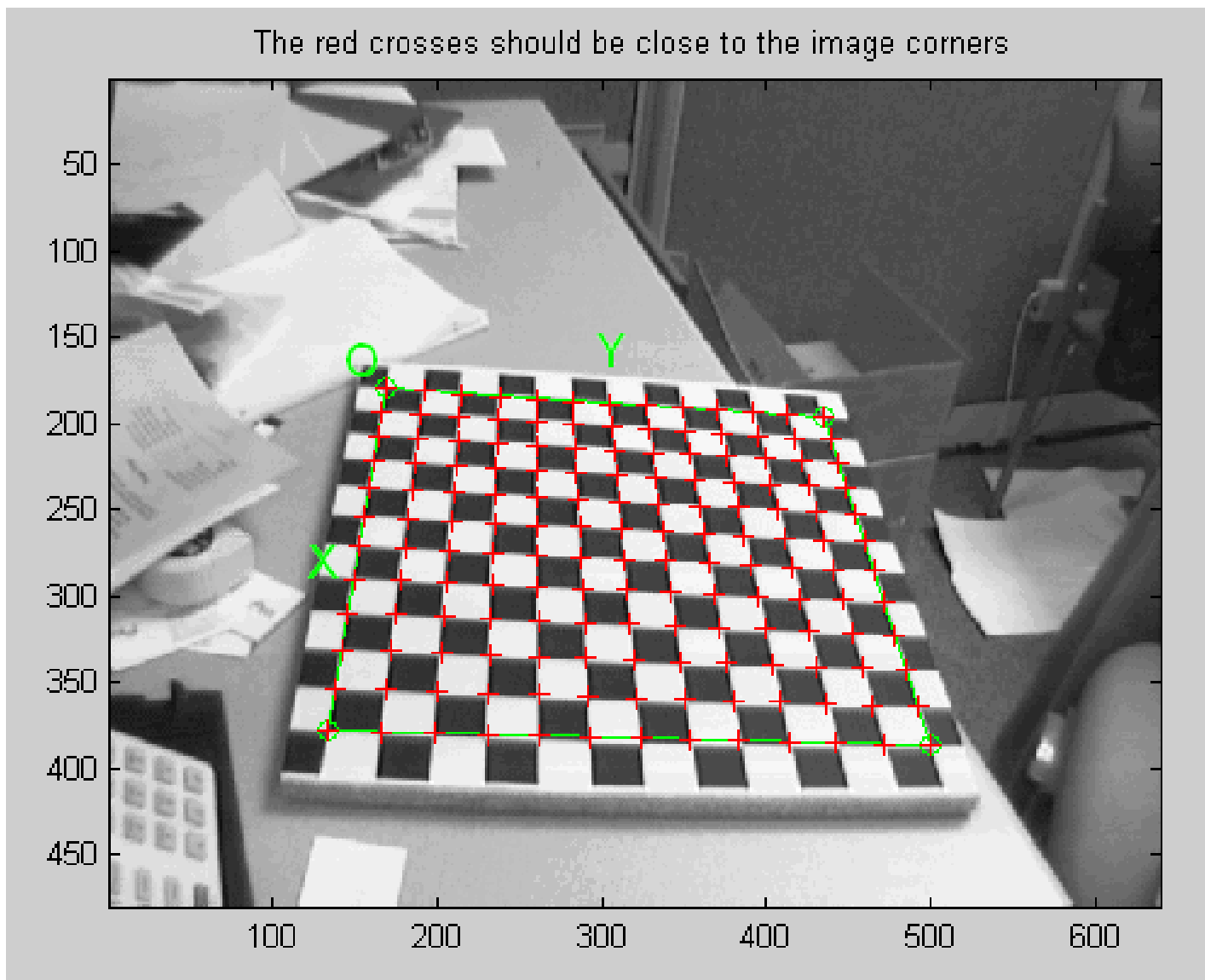


Click on the four extreme corners of the rectangular pattern (first corner = origin)... Image 1

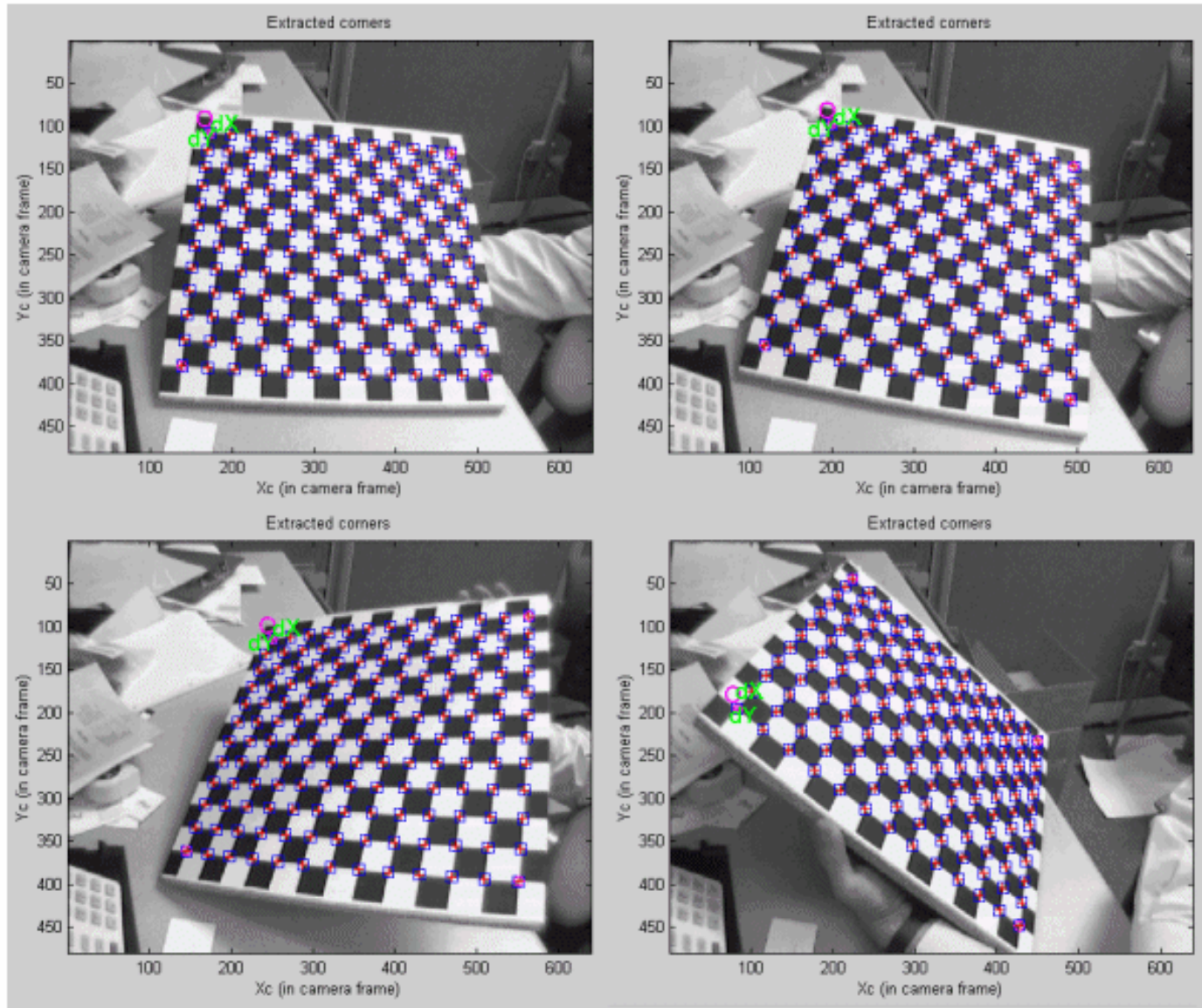


Step 3: corner extraction

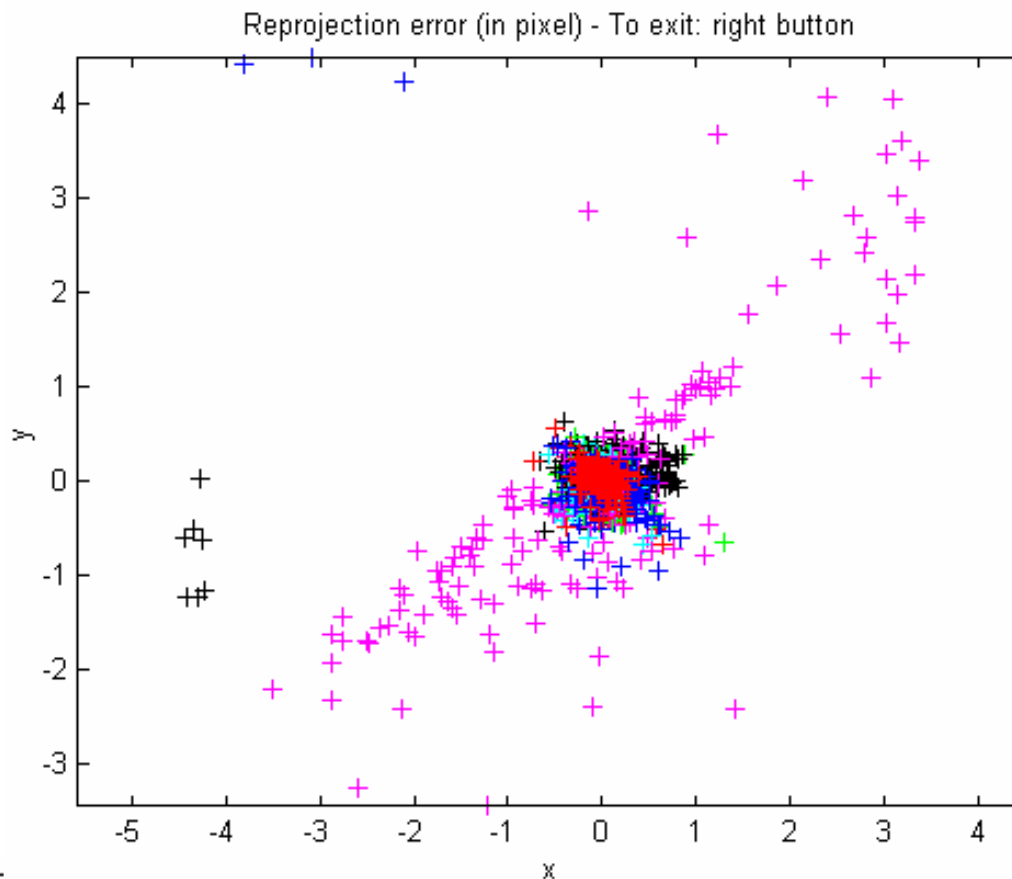
The red crosses should be close to the image corners



Step 3: corner extraction



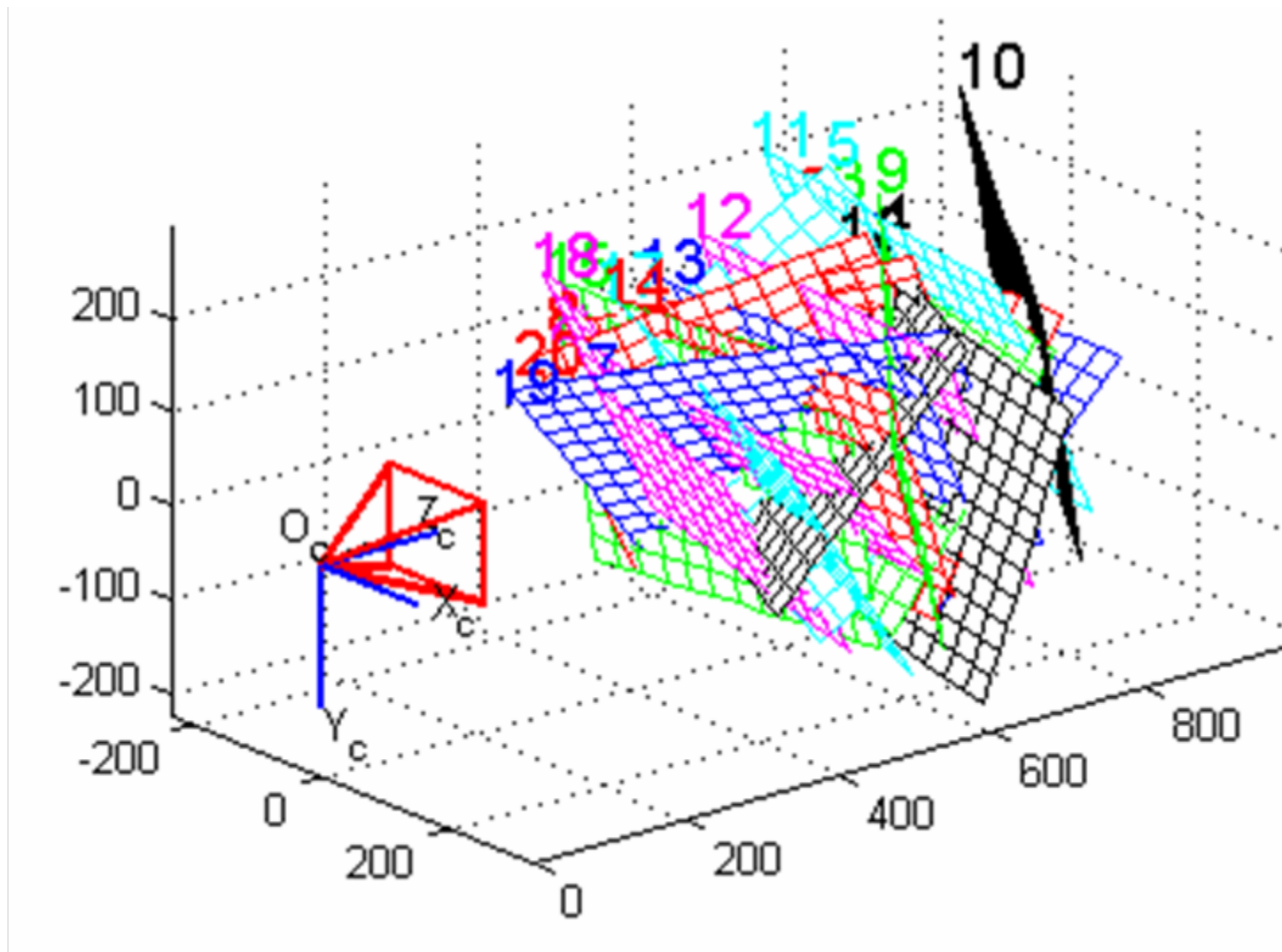
Step 4: minimize projection error



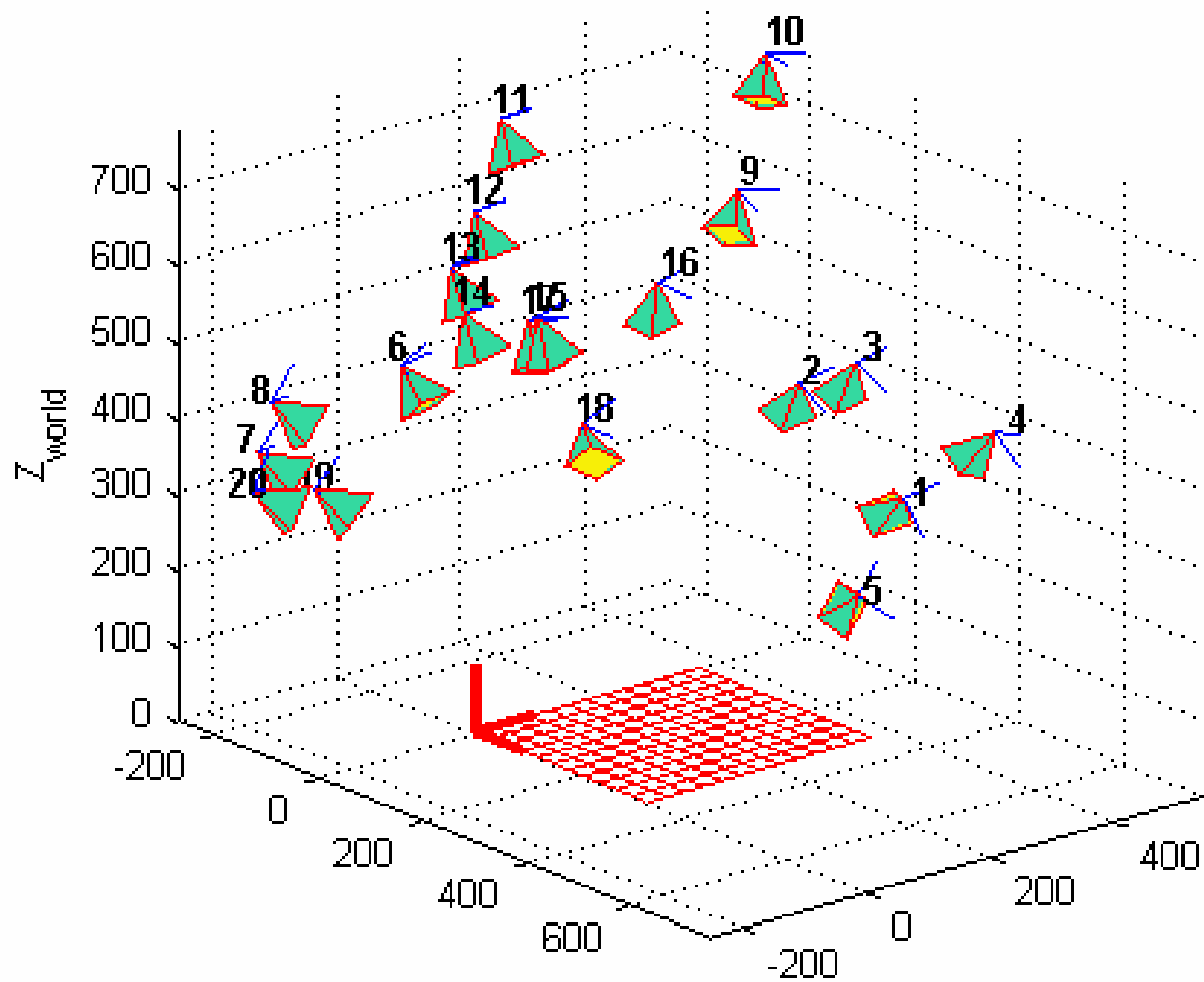
Calibration res

```
Focal Length:      fc = [ 657.46290  657.94673 ] ± [ 0.31819  0.34046 ]
Principal point:   cc = [ 303.13665  242.56935 ] ± [ 0.64682  0.59218 ]
Skew:             alpha_c = [ 0.00000 ] ± [ 0.00000 ] => angle of pixel axes =
Distortion:       kc = [ -0.25403  0.12143  -0.00021  0.00002  0.00000 ]
Pixel error:      err = [ 0.11689  0.11500 ]
```

Step 4: camera calibration



Step 4: camera calibration



Step 5: refinement

