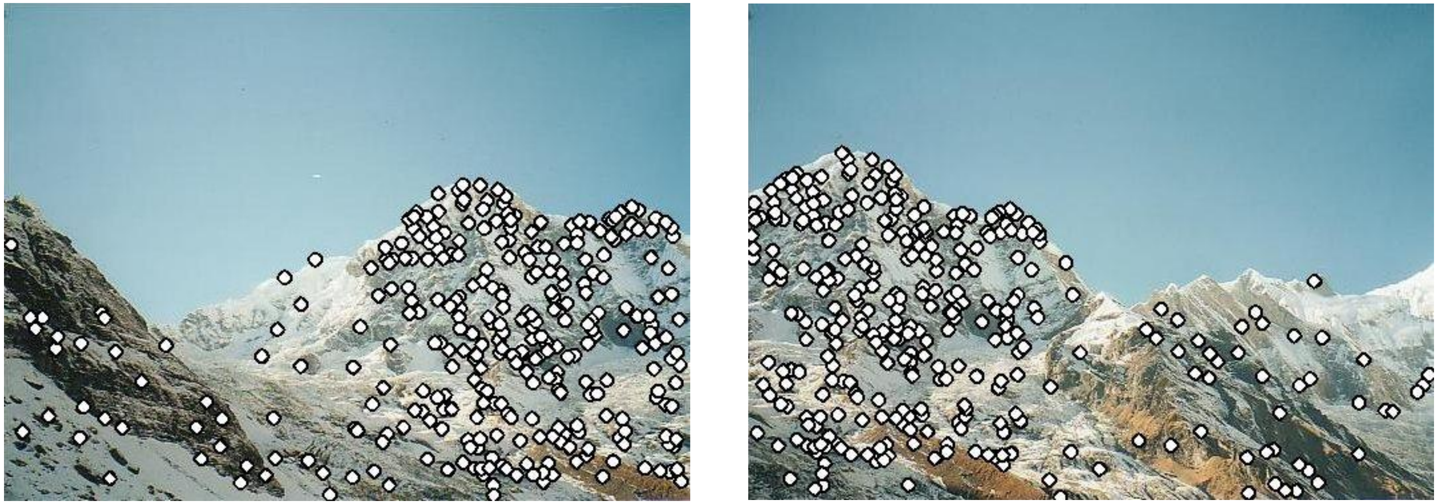
Last Two Lectures

Panoramic Image Stitching



Feature Detection and Matching

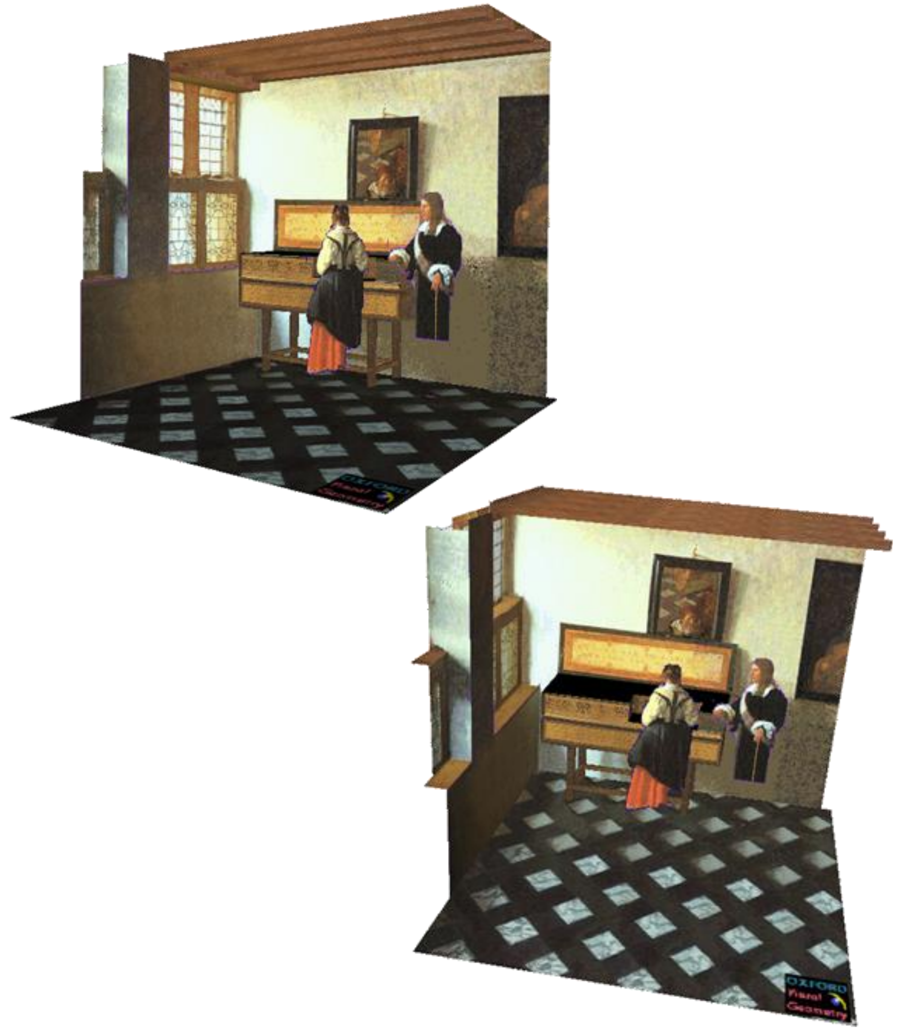
Today

More on Mosaic

Projective Geometry

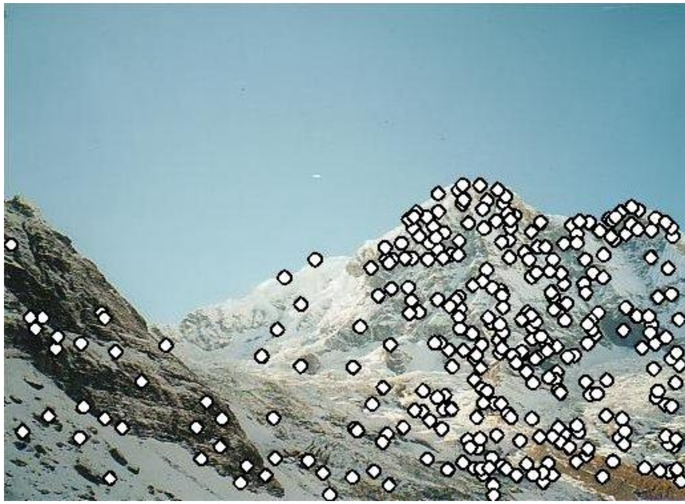
Single View Modeling

Vermeer's *Music Lesson*



Reconstructions by Criminisi et al.

Image Alignment

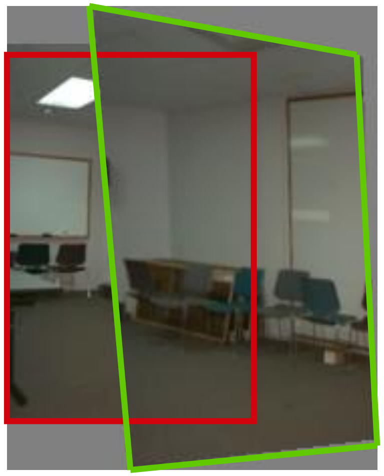
Feature Detection and Matching

Cylinder:
Translation
2 DoF

Plane:
Homography
8 DoF

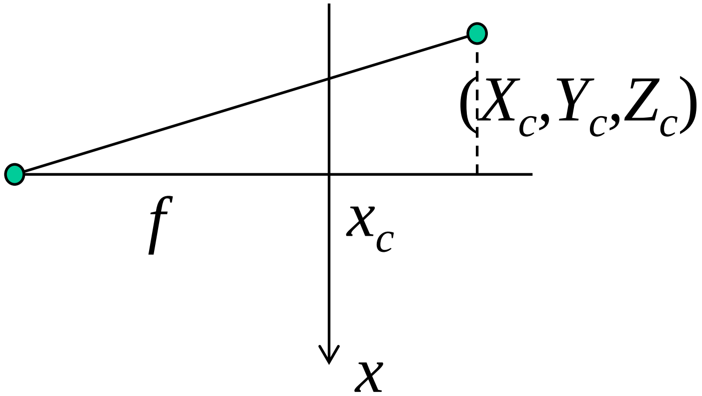
Plane perspective mosaics

- 8-parameter generalization of affine motion
  - works for pure rotation or planar surfaces
- Limitations:
  - local minima
  - slow convergence



Revisit Homography

$$\begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} \sim \begin{bmatrix} f & 0 & x_c \\ 0 & f & y_c \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$



$$\begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} \sim \begin{bmatrix} f & 0 & x_c \\ 0 & f & y_c \\ 0 & 0 & 1 \end{bmatrix} \mathbf{R} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$\mathbf{R}(\mathbf{K}^{-1}\mathbf{x}_1) \sim \mathbf{K}^{-1}\mathbf{x}_2$$

Absolute orientation

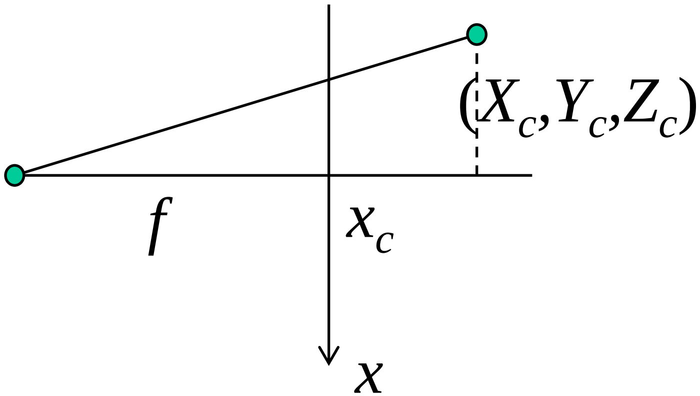
[Arun *et al.*, PAMI 1987] [Horn *et al.*, JOSA A 1988]
Procrustes Algorithm [Golub & VanLoan]

- Given two sets of matching points, compute R such that $p_i' = \boldsymbol{R}\, p_i$

- $\boldsymbol{A} = \Sigma_{\mathbf{i}}\, p_i\, p_i'^T = \boldsymbol{U}\, \boldsymbol{S}\, \boldsymbol{V}^T$
- $\boldsymbol{R} = \boldsymbol{V}\, \boldsymbol{U}^T$

What if we don't know f?

$$\begin{pmatrix} x_1 - x_c \\ y_1 - y_c \\ 1 \end{pmatrix} \sim \begin{bmatrix} f_1 & 0 & 0 \\ 0 & f_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$



$$\begin{pmatrix} x_2 - x_c \\ y_2 - y_c \\ 1 \end{pmatrix} \sim \begin{bmatrix} f_2 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{R} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$\mathbf{R} \sim \mathbf{K}_2^{-1}\mathbf{H}\mathbf{K}_1$$

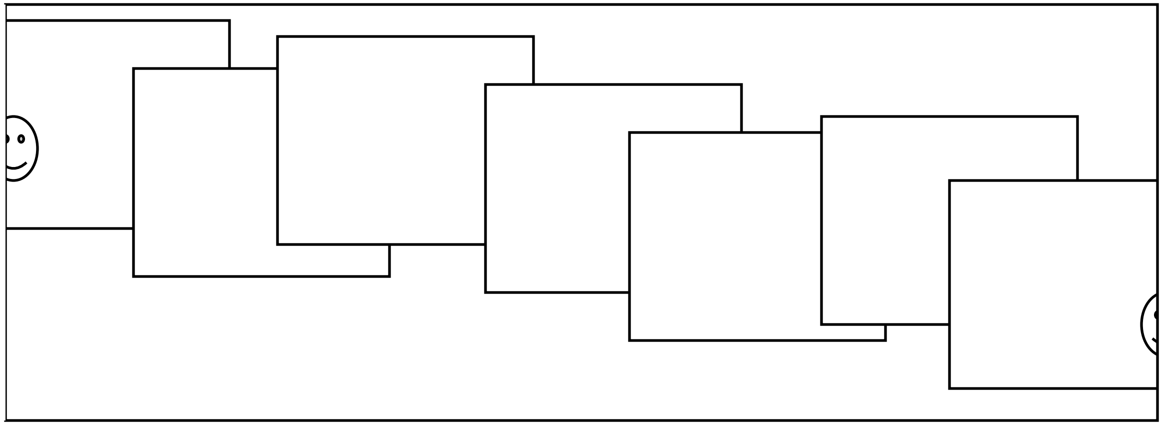
$$= \begin{bmatrix} a & b & c/f_1 \\ d & e & g/f_1 \\ h * f_2 & i * f_2 & j * \dfrac{f_2}{f_1} \end{bmatrix}$$

$$\mathbf{R}(\mathbf{K}_1^{-1}\mathbf{x}_1) \sim \mathbf{K}_2^{-1}\mathbf{x}_2$$

$$(\underbrace{\mathbf{K}_2\mathbf{R}\mathbf{K}_1^{-1}}_{\mathbf{H}})\mathbf{x}_1 \sim \mathbf{x}_2$$

$$f_1 = ?,\ f_2 = ?$$

The drifting problem



- Error accumulation
  - small errors accumulate over time

Bundle Adjustment

Associate each image i with $\mathbf{K}_i \quad \mathbf{R}_i$

Each image i has features $\mathbf{p}_{ij}$

Trying to minimize total matching residuals

$$E(\text{all } f_i \text{ and } \mathbf{R}_i) = \sum_{(i,m)} \sum_j \left\| \mathbf{p}_{ij} \sim \mathbf{K}_i \mathbf{R}_i \mathbf{R}_m^{-1} \mathbf{K}_m^{-1} \mathbf{p}_{mj} \right\|^2$$

Rotations

- How do we represent rotation matrices?

1. Axis / angle $(\boldsymbol{n},\theta)$

   $\boldsymbol{R} = \boldsymbol{I} + \sin\theta\,[\boldsymbol{n}]_\times + (1-\cos\theta)\,[\boldsymbol{n}]_\times^2$

   (Rodriguez Formula), with

   $[\boldsymbol{n}]_\times$ be the cross product matrix.

$$\mathbf{a} \times \mathbf{b} = [\mathbf{a}]_\times \mathbf{b} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Incremental rotation update

1. Small angle approximation

   $$\Delta \boldsymbol{R} = \boldsymbol{I} + \sin\theta\, [\boldsymbol{n}]_\times + (1 - \cos\theta)\, [\boldsymbol{n}]_\times^2$$

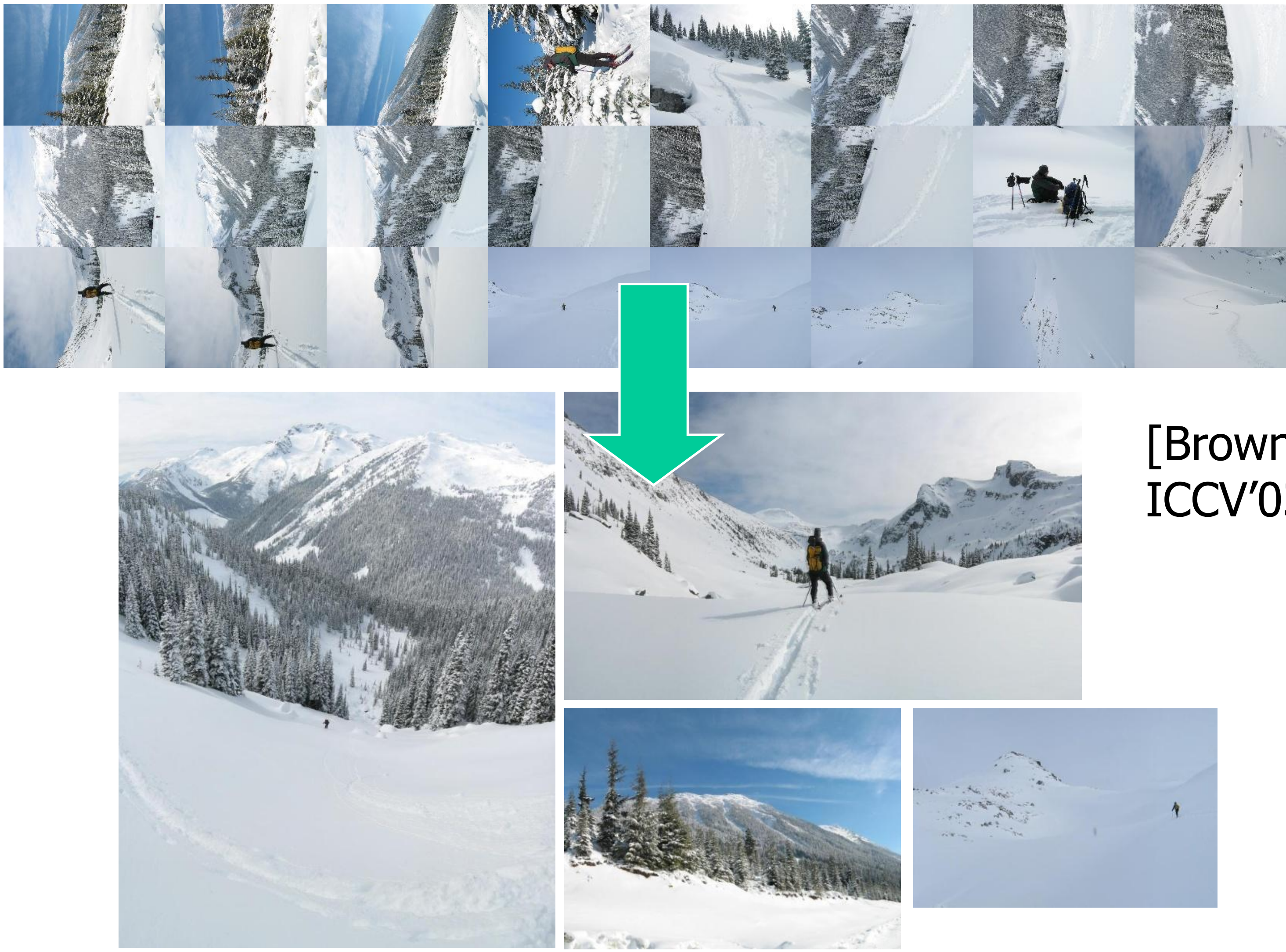
   $$\approx \boldsymbol{I} + \theta\, [\boldsymbol{n}]_\times = \boldsymbol{I} + [\boldsymbol{\omega}]_\times$$

   linear in $\boldsymbol{\omega} = \theta \boldsymbol{n}$

2. Update original $\boldsymbol{R}$ matrix

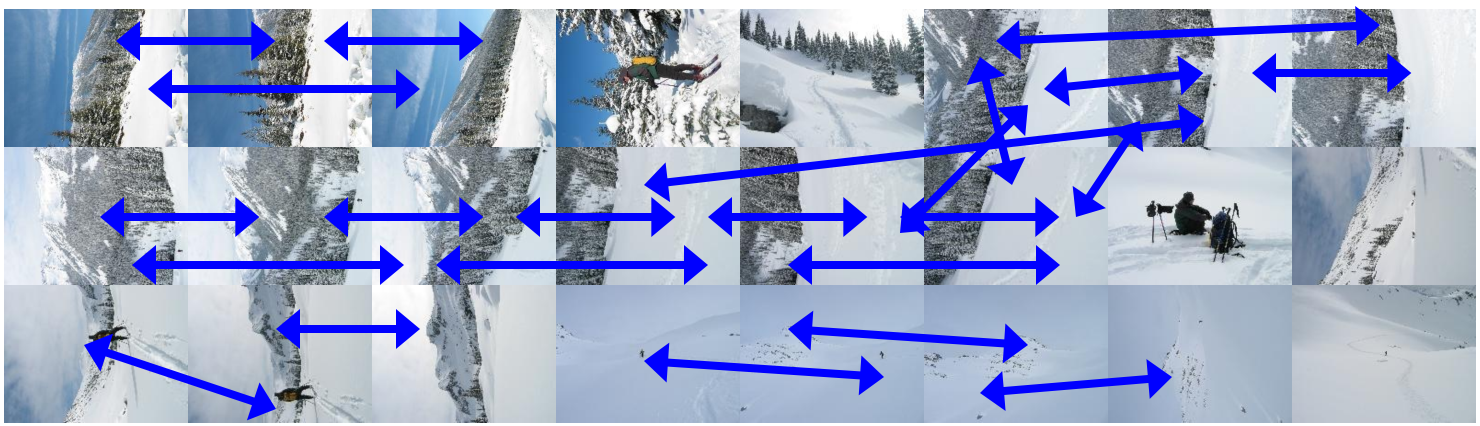
   $$\boldsymbol{R} \leftarrow \boldsymbol{R}\, \Delta \boldsymbol{R}$$

Recognizing Panoramas

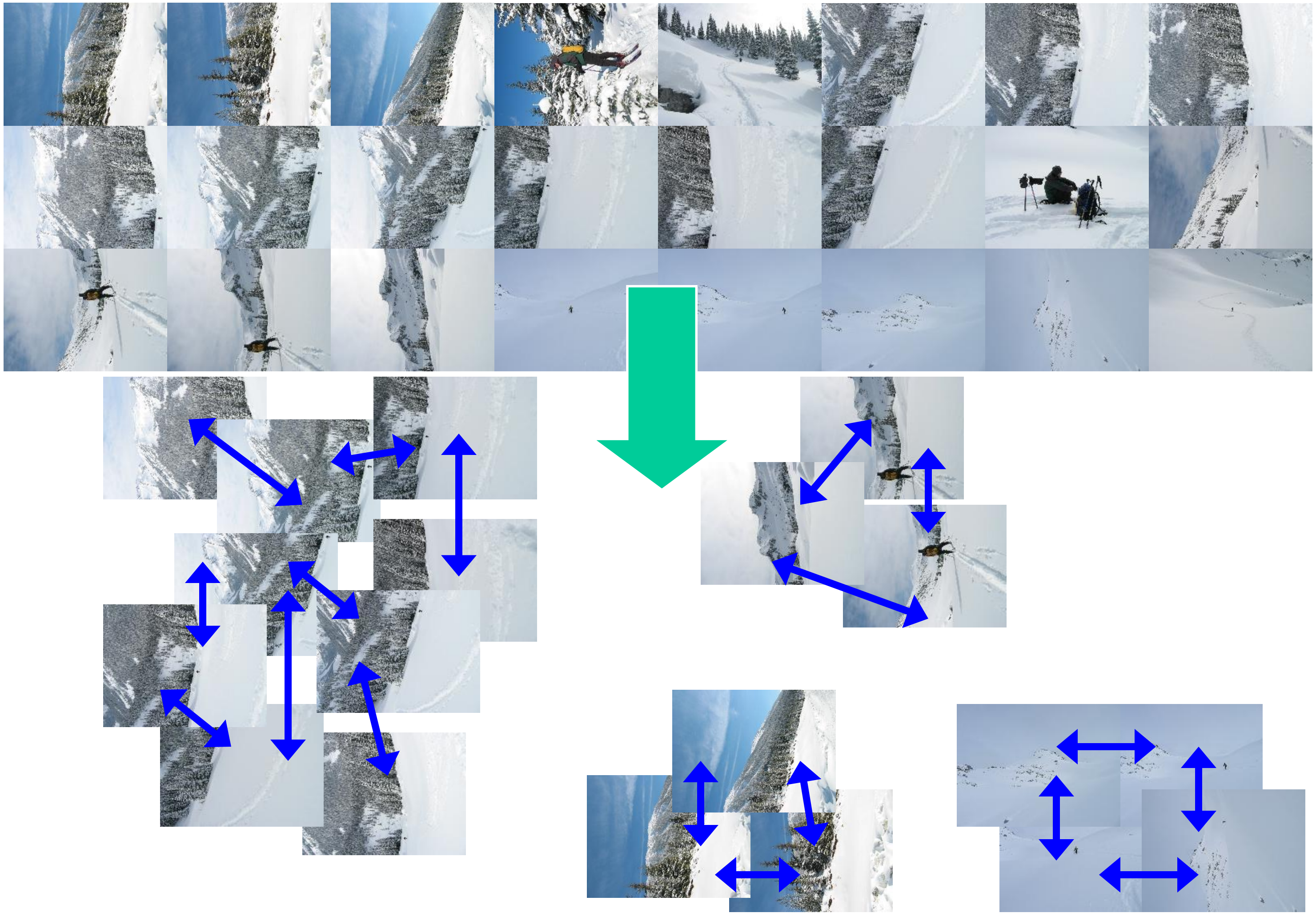


[Brown & Lowe, ICCV'03]

Finding the panoramas



Finding the panoramas



Algorithm overview

Algorithm: Panoramic Recognition

Input: n unordered images

Algorithm overview

Algorithm: Panoramic Recognition

Input: n unordered images

I. Extract SIFT features from all n images

Algorithm overview

Algorithm: Panoramic Recognition

Input: n unordered images

I. Extract SIFT features from all n images

II. Find k nearest-neighbours for each feature using a k-d tree

Algorithm overview

Algorithm: Panoramic Recognition

Input: n unordered images

I. Extract SIFT features from all n images

II. Find k nearest-neighbours for each feature using a k-d tree

III. For each image:
   (i) Select m candidate matching images (with the maximum number of feature matches to this image)

Algorithm overview

Algorithm: Panoramic Recognition

Input: n unordered images

I. Extract SIFT features from all n images

II. Find k nearest-neighbours for each feature using a k-d tree

III. For each image:

  (i) Select m candidate matching images (with the maximum number of feature matches to this image)

  (ii) Find geometrically conisistent feature matches using RANSAC to solve for the homography between pairs of images

Algorithm overview

Algorithm: Panoramic Recognition

Input: n unordered images

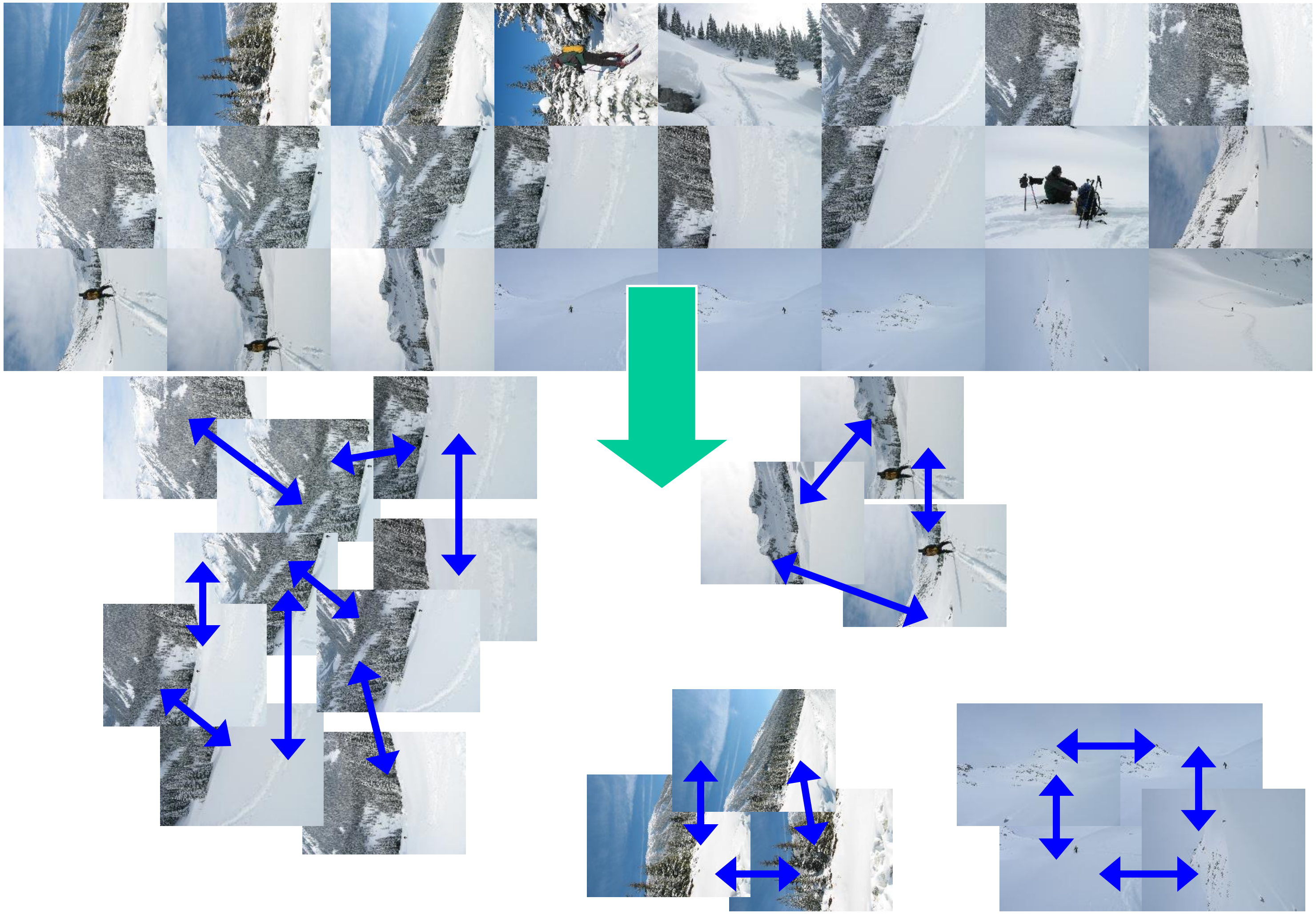
I. Extract SIFT features from all n images

II. Find k nearest-neighbours for each feature using a k-d tree

III. For each image:
   (i) Select m candidate matching images (with the maximum number of feature matches to this image)
   (ii) Find geometrically conisistent feature matches using RANSAC to solve for the homography between pairs of images
   (iii) Verify image matches using probabilistic model

IV. Find connected components of image matches

Finding the panoramas



Finding the panoramas

Algorithm overview

Algorithm: Panoramic Recognition

Input: n unordered images

I. Extract SIFT features from all n images

II. Find k nearest-neighbours for each feature using a k-d tree

III. For each image:
   (i) Select m candidate matching images (with the maximum number of feature matches to this image)
   (ii) Find geometrically conisistent feature matches using RANSAC to solve for the homography between pairs of images
   (iii) Verify image matches using probabilistic model

IV. Find connected components of image matches

Algorithm overview

Algorithm: Panoramic Recognition

Input: n unordered images

I. Extract SIFT features from all n images

II. Find k nearest-neighbours for each feature using a k-d tree

III. For each image:
   (i) Select m candidate matching images (with the maximum number of feature matches to this image)

   (ii) Find geometrically conisistent feature matches using RANSAC to solve for the homography between pairs of images

   (iii) Verify image matches using probabilistic model

IV. Find connected components of image matches

V. For each connected component:
   (i) Perform bundle adjustment to solve for the rotation $\theta_1, \theta_2, \theta_3$ and focal length f of all cameras

Algorithm overview

Algorithm: Panoramic Recognition

Input: n unordered images

I. Extract SIFT features from all n images

II. Find k nearest-neighbours for each feature using a k-d tree

III. For each image:
   (i) Select m candidate matching images (with the maximum number of feature matches to this image)
   (ii) Find geometrically conisistent feature matches using RANSAC to solve for the homography between pairs of images
   (iii) Verify image matches using probabilistic model

IV. Find connected components of image matches

V. For each connected component:
   (i) Perform bundle adjustment to solve for the rotation $\theta_1, \theta_2, \theta_3$ and focal length f of all cameras
   (ii) Render panorama using multi-band blending

Output: Panoramic image(s)

Get you own copy!

[Brown & Lowe, ICCV 2003]
[Brown, Szeliski, Winder, CVPR’05]

How well does this work?

Test on 100s of examples…

How well does this work?

Test on 100s of examples…

…still too many failures (5-10%)
for <u>consumer</u> application

Matching Mistakes: False Positive

Matching Mistakes: False Positive

Matching Mistakes: False Negative

- Moving objects: large areas of disagreement

Matching Mistakes

- Accidental alignment
  - repeated / similar regions
- Failed alignments
  - moving objects / parallax
  - low overlap
  - “feature-less” regions (more variety?)
- No 100% reliable algorithm?

How can we fix these?

- Tune the feature detector
- Tune the feature matcher (cost metric)
- Tune the RANSAC stage (motion model)
- Tune the verification stage
- Use “higher-level” knowledge
  - e.g., typical camera motions

- → Sounds like a big “learning” problem
  - Need a large training/test data set (panoramas)

on to 3D…

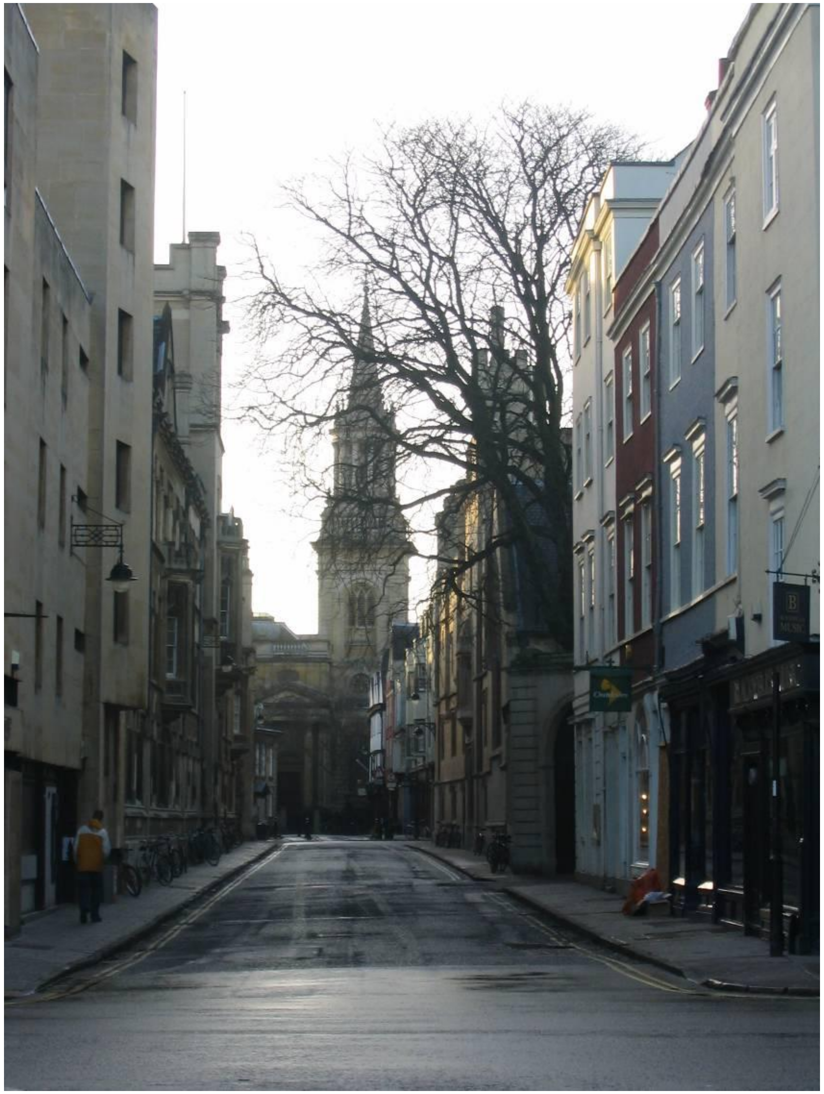
Enough of images!

We want more from the image

We want real 3D scene

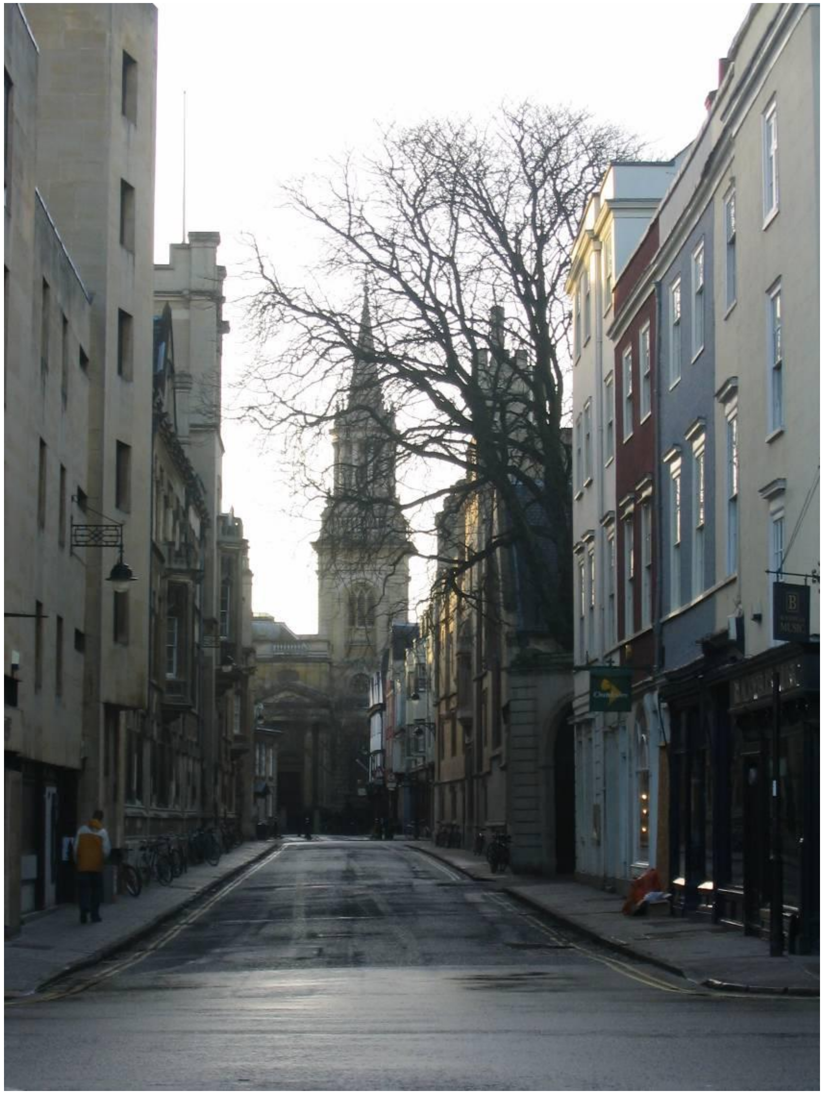
walk-throughs:

- Camera rotation
- Camera translation



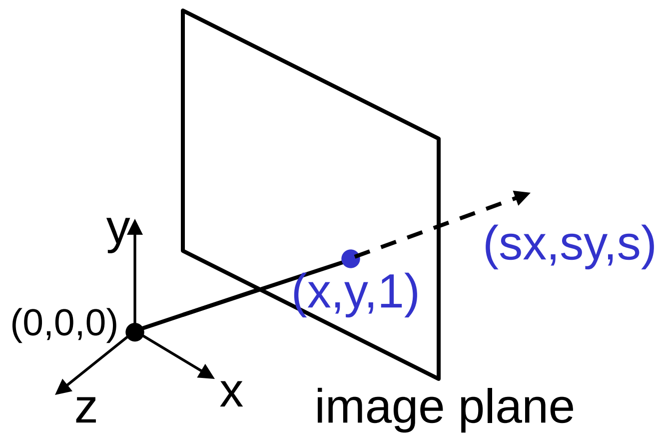
So, what can we do here?

- Model the scene as a set of planes!



The projective plane

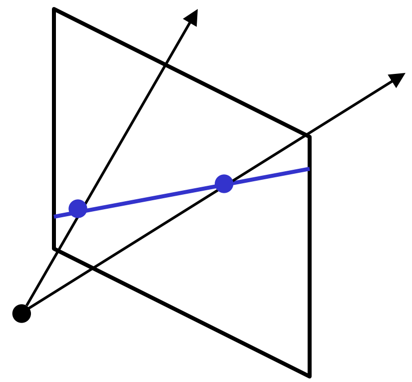
- Why do we need homogeneous coordinates?
  - represent points at infinity, homographies, perspective projection, multi-view relationships
- What is the geometric intuition?
  - a point in the image is a *ray* in projective space



- Each *point* (x,y) on the plane is represented by a *ray* (sx,sy,s)
  - all points on the ray are equivalent: $(x, y, 1) \cong (sx, sy, s)$

Projective lines

- What does a line in the image correspond to in projective space?



- A line is a *plane* of rays through origin
  - all rays (x,y,z) satisfying: ax + by + cz = 0

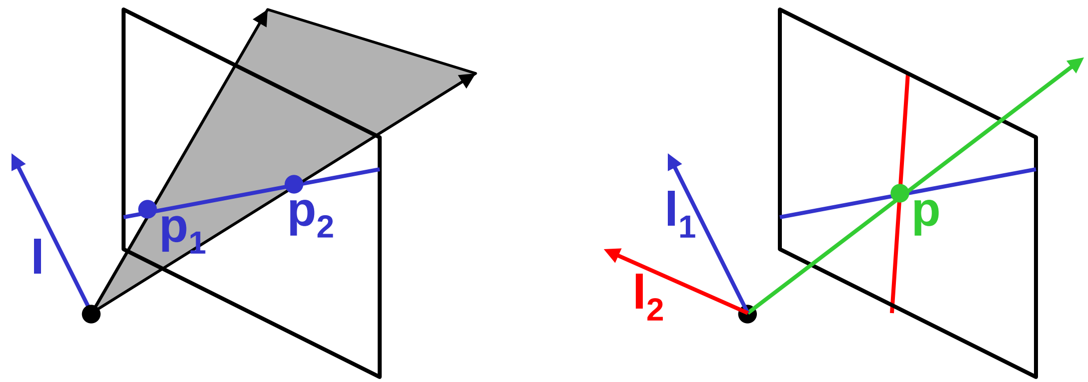
$$\text{in vector notation :} \quad 0 = \underbrace{\begin{bmatrix} a & b & c \end{bmatrix}}_{\mathbf{l}} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\mathbf{p}}$$

- A line is also represented as a homogeneous 3-vector **l**

Point and line duality

– A line **l** is a homogeneous 3-vector

– It is $\perp$ to every point (ray) **p** on the line: **l p**=0



What is the line **l** spanned by rays $\mathbf{p_1}$ and $\mathbf{p_2}$?

- **l** is $\perp$ to $\mathbf{p_1}$ and $\mathbf{p_2}$ $\Rightarrow$ $\mathbf{l} = \mathbf{p_1} \times \mathbf{p_2}$
- **l** is the plane normal

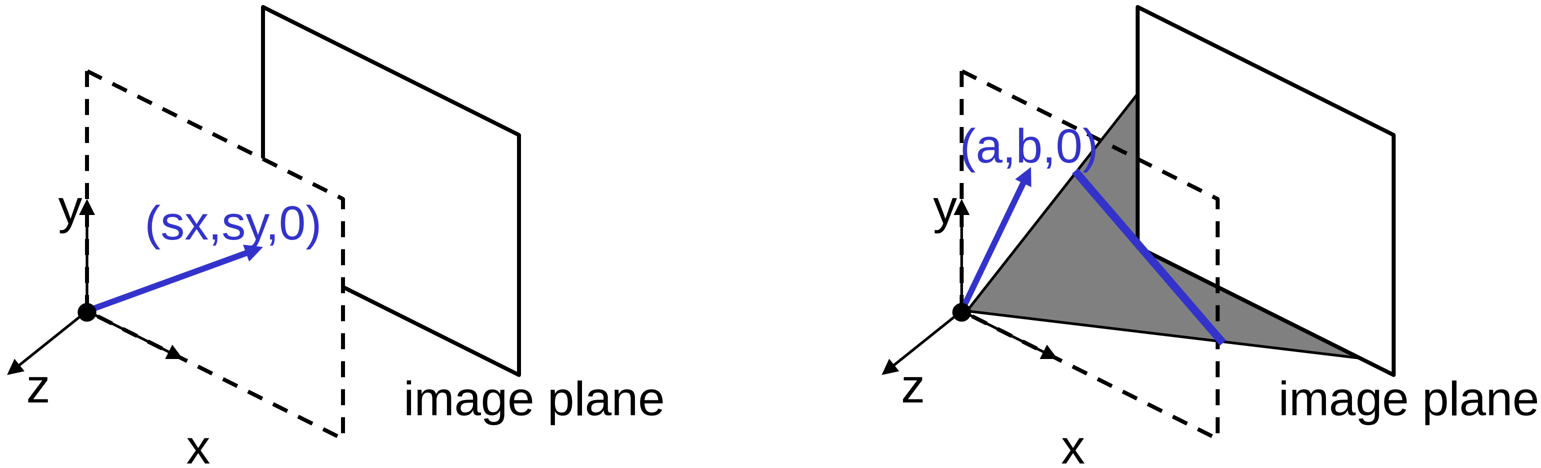
What is the intersection of two lines $\mathbf{l_1}$ and $\mathbf{l_2}$?

- **p** is $\perp$ to $\mathbf{l_1}$ and $\mathbf{l_2}$ $\Rightarrow$ $\mathbf{p} = \mathbf{l_1} \times \mathbf{l_2}$

Points and lines are *dual* in projective space

- given any formula, can switch the meanings of points and lines to get another formula

Ideal points and lines



- Ideal point ("point at infinity")
  - p ≅ (x, y, 0) – parallel to image plane
  - It has infinite image coordinates

Ideal line

- l ≅ (a, b, 0) – parallel to image plane
- Corresponds to a line in the image (finite coordinates)

Homographies of points and lines

- Computed by 3x3 matrix multiplication
  - To transform a point: $\mathbf{p'} = \mathbf{Hp}$
  - To transform a line: $\mathbf{lp}=0 \rightarrow \mathbf{l'p'}=0$
    - $0 = \mathbf{lp} = \mathbf{lH^{-1}Hp} = \mathbf{lH^{-1}p'} \Rightarrow \mathbf{l' = lH^{-1}}$
    - lines are transformed by postmultiplication of $\mathbf{H^{-1}}$

3D projective geometry

- These concepts generalize naturally to 3D
  - Homogeneous coordinates
    - Projective 3D points have four coords: **P** = (X,Y,Z,W)
  - Duality
    - A plane **N** is also represented by a 4-vector
    - Points and planes are dual in 4D: **N P**=0
  - Projective transformations
    - Represented by 4x4 matrices **T**: **P' = TP, N' = N** $\mathbf{T^{-1}}$

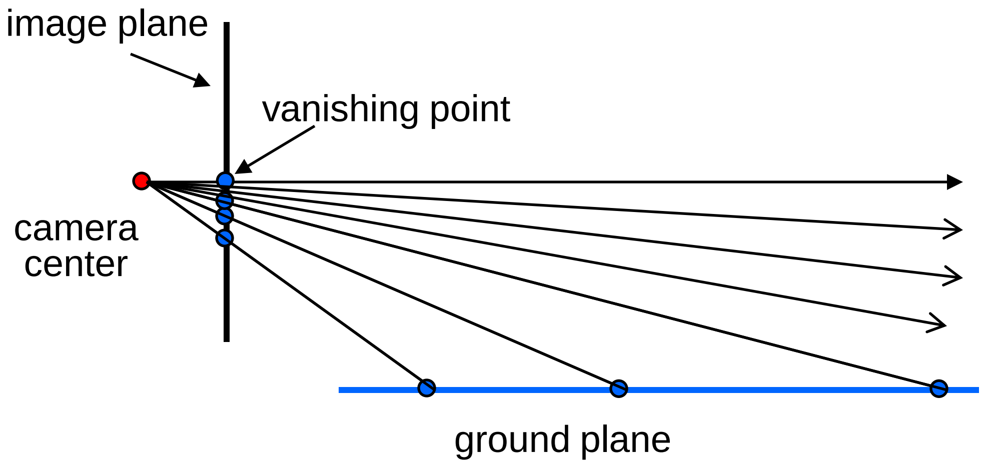
3D to 2D: "perspective" projection

- Matrix Projection: $\mathbf{p} = \begin{bmatrix} wx \\ wy \\ w \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{\Pi P}$

What is *not* preserved under perspective projection?

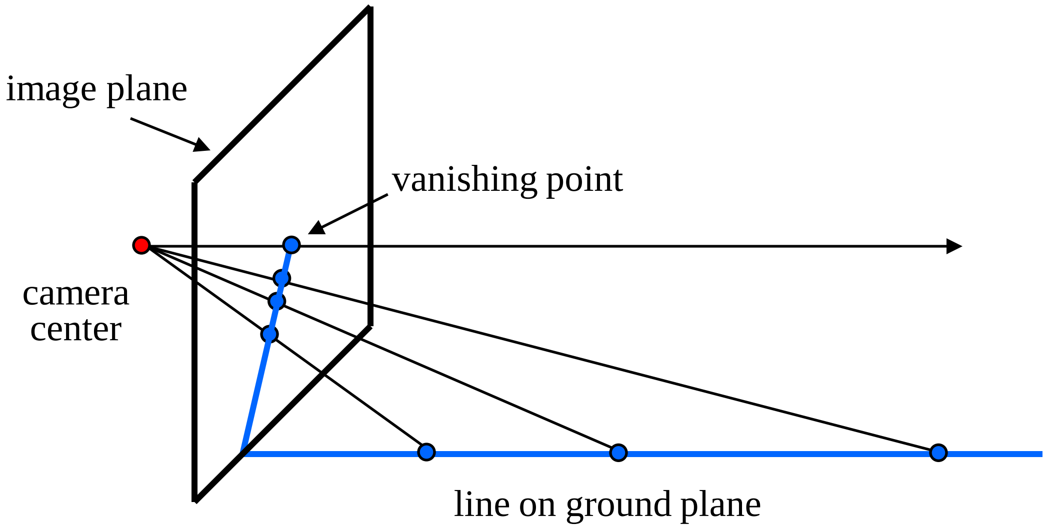
What IS preserved?

Vanishing points

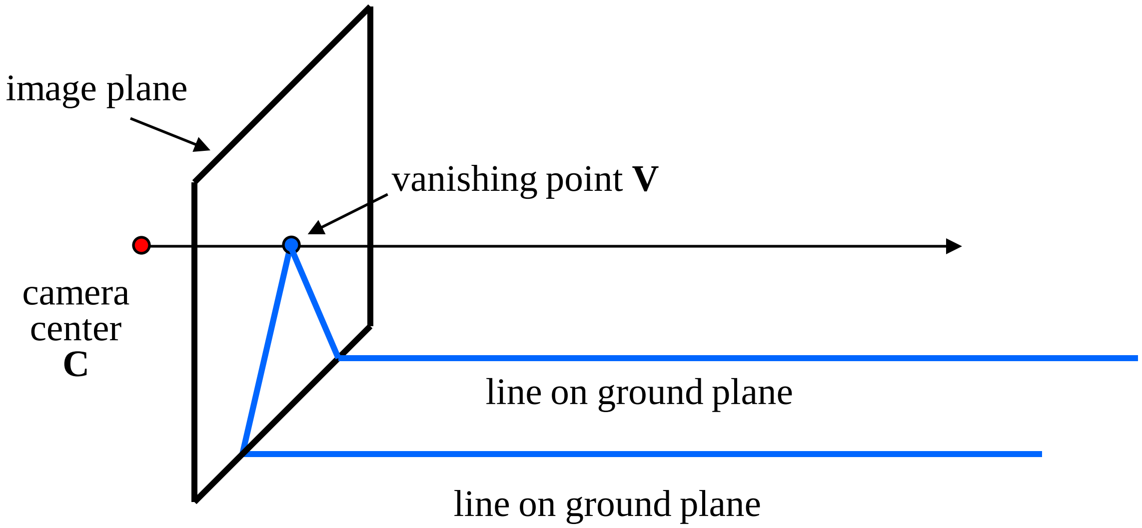


- Vanishing point
  – projection of a point at infinity

Vanishing points (2D)

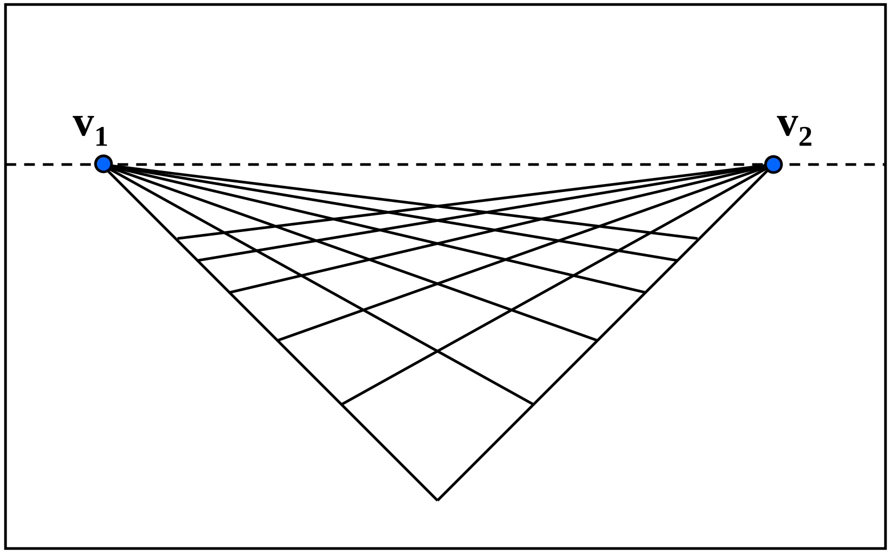


Vanishing points



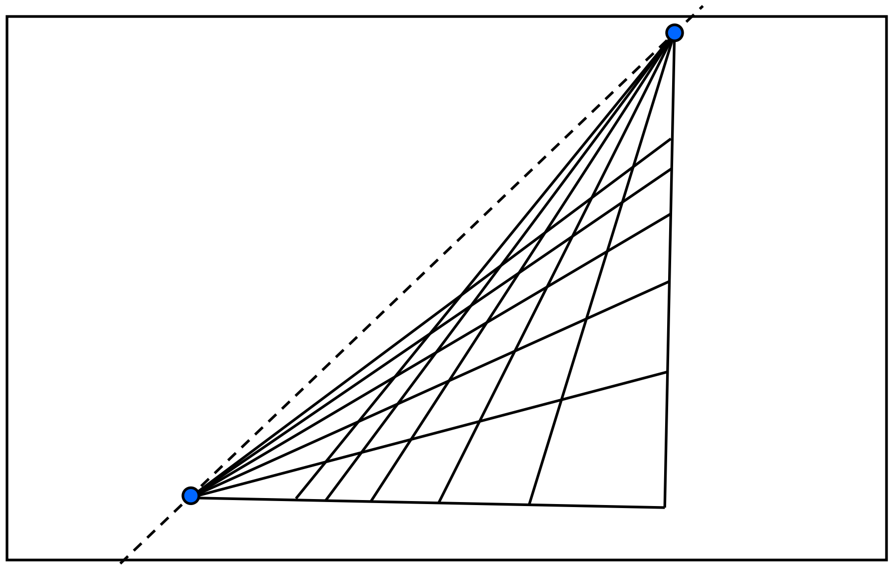
- Properties
  - Any two parallel lines have the same vanishing point **v**
  - The ray from **C** through **v** is parallel to the lines
  - An image may have more than one vanishing point
    - in fact every pixel is a potential vanishing point

Vanishing lines



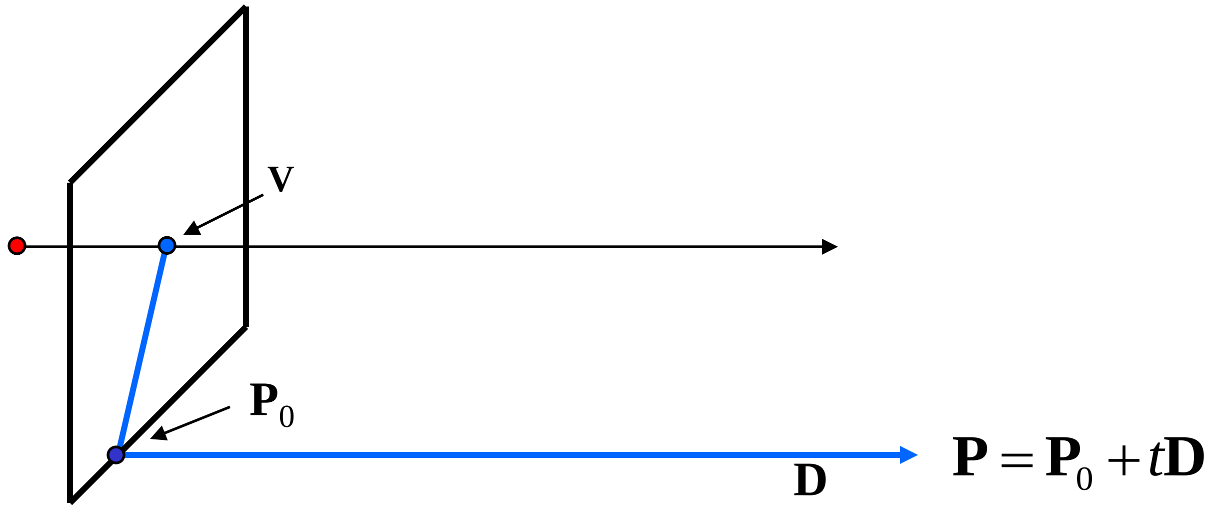
- Multiple Vanishing Points
  - Any set of parallel lines on the plane define a vanishing point
  - The union of all of these vanishing points is the *horizon line*
    - also called *vanishing line*
  - Note that different planes define different vanishing lines

Vanishing lines



- Multiple Vanishing Points
  - Any set of parallel lines on the plane define a vanishing point
  - The union of all of these vanishing points is the *horizon line*
    - also called *vanishing line*
  - Note that different planes define different vanishing lines

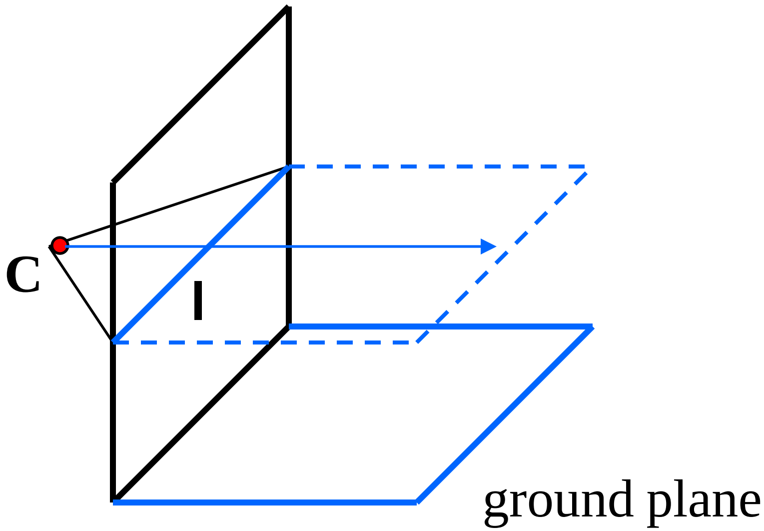
Computing vanishing points



$$\mathbf{P}_t = \begin{bmatrix} P_X + tD_X \\ P_Y + tD_Y \\ P_Z + tD_Z \\ 1 \end{bmatrix} \cong \begin{bmatrix} P_X / t + D_X \\ P_Y / t + D_Y \\ P_Z / t + D_Z \\ 1/t \end{bmatrix} \qquad t \to \infty \qquad \mathbf{P}_\infty \cong \begin{bmatrix} D_X \\ D_Y \\ D_Z \\ 0 \end{bmatrix}$$

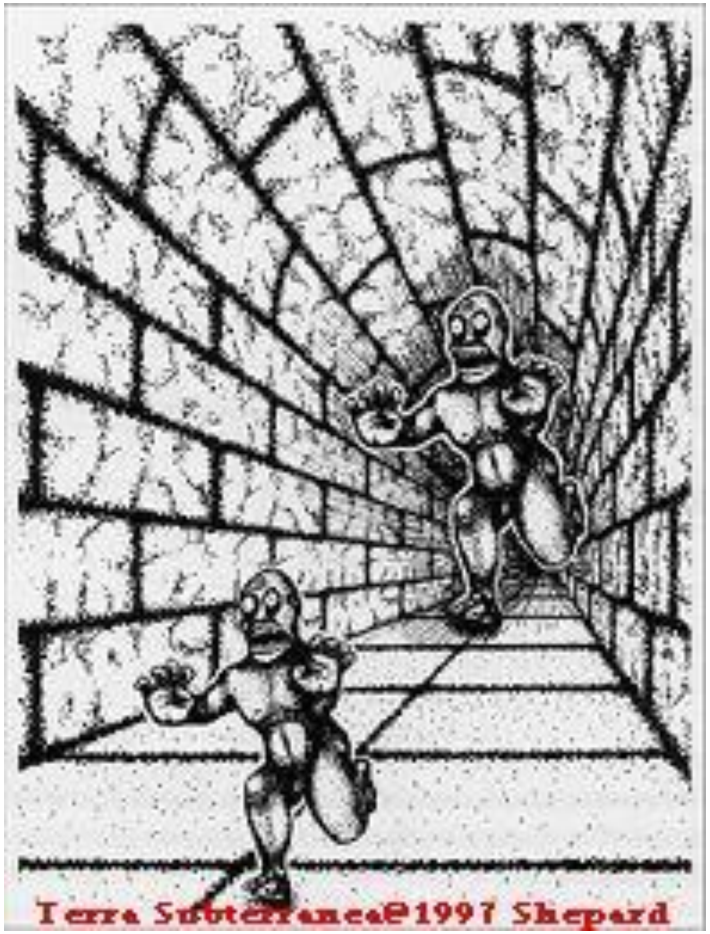
- Properties $\mathbf{v} = \mathbf{\Pi}\mathbf{P}_\infty$
  - $\mathbf{P}_\infty$ is a point at *infinity*, **v** is its projection
  - They depend only on line *direction*
  - Parallel lines $\mathbf{P}_0$ + t**D**, $\mathbf{P}_1$ + t**D** intersect at $\mathbf{P}_\infty$

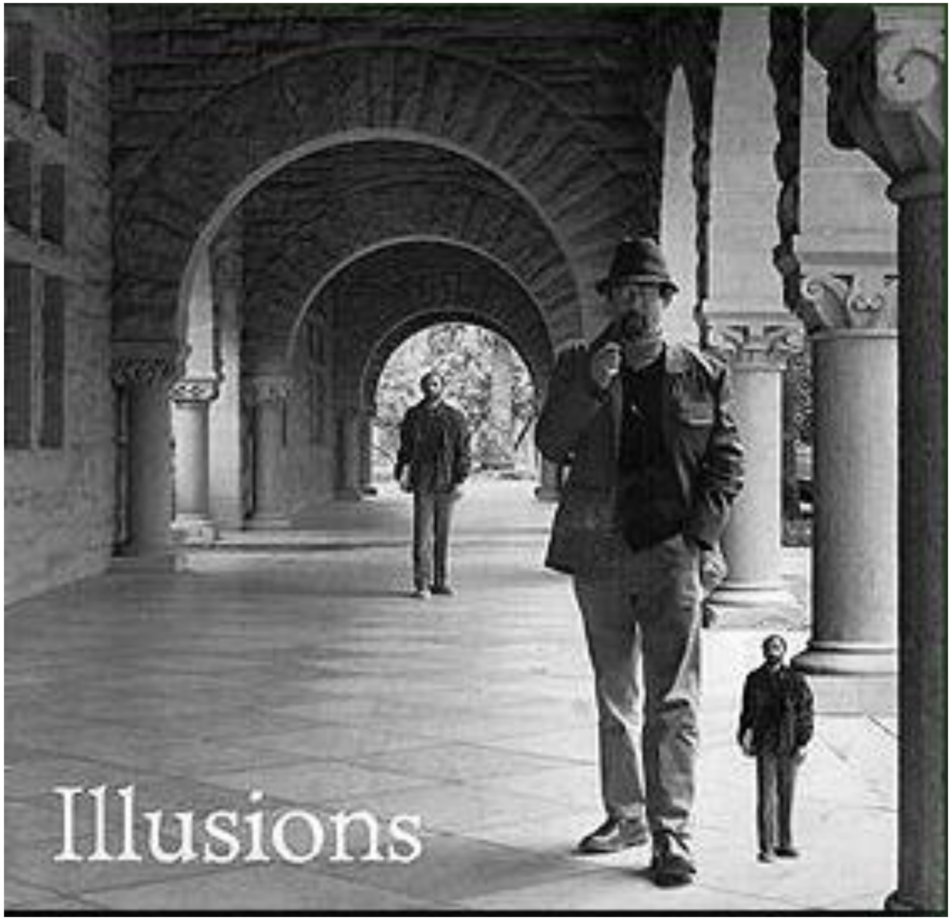
Computing vanishing lines



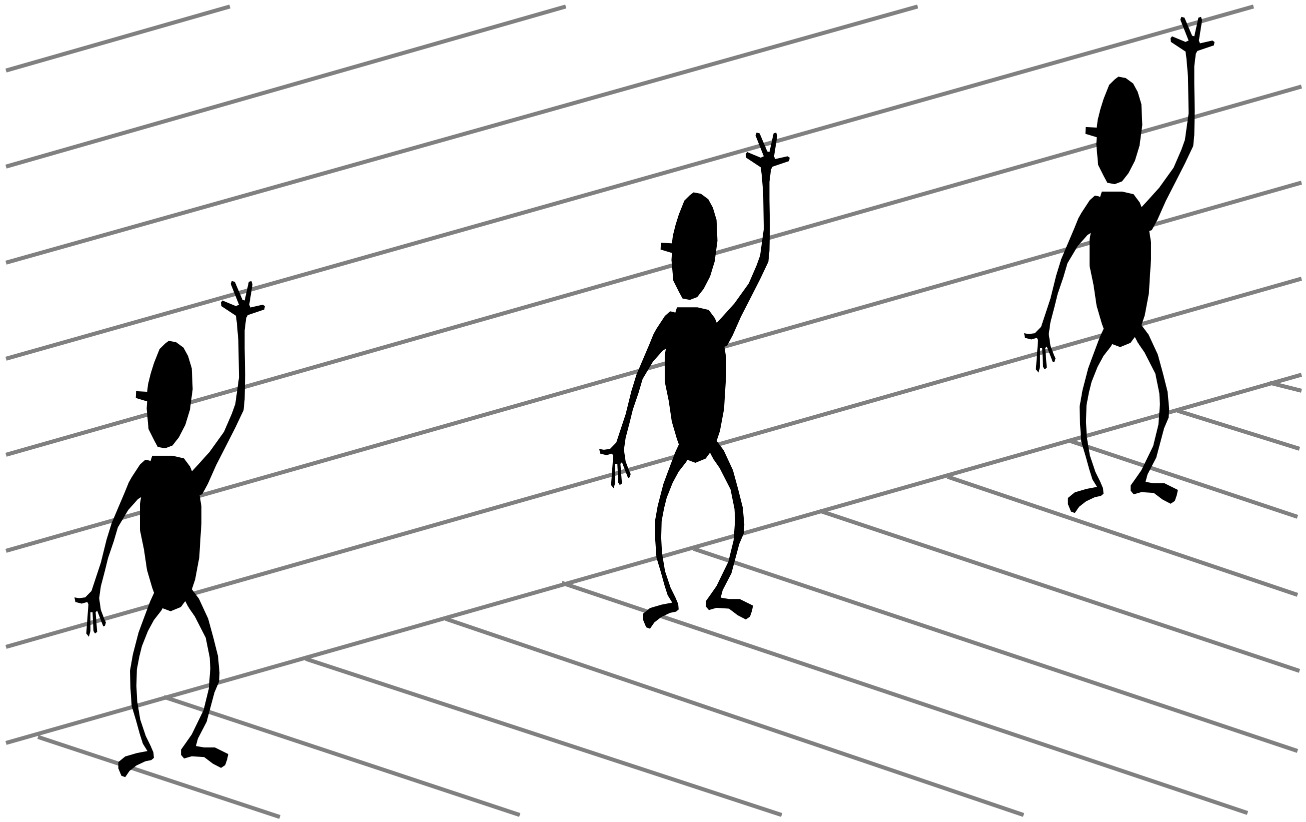
- Properties
  - **l** is intersection of horizontal plane through **C** with image plane
  - Compute **l** from two sets of parallel lines on ground plane
  - All points at same height as **C** project to **l**
    - points higher than C project above l
  - Provides way of comparing height of objects in the scene

Fun with vanishing points

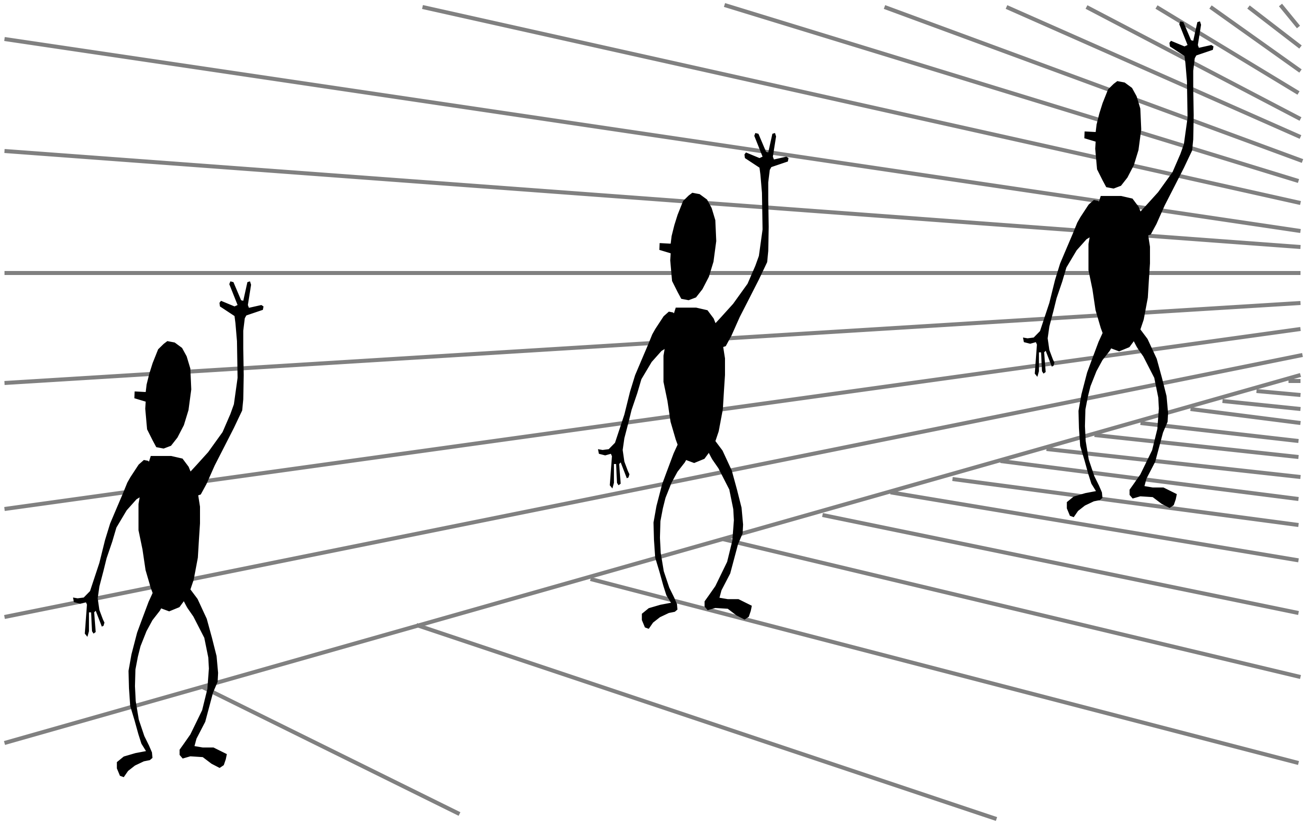




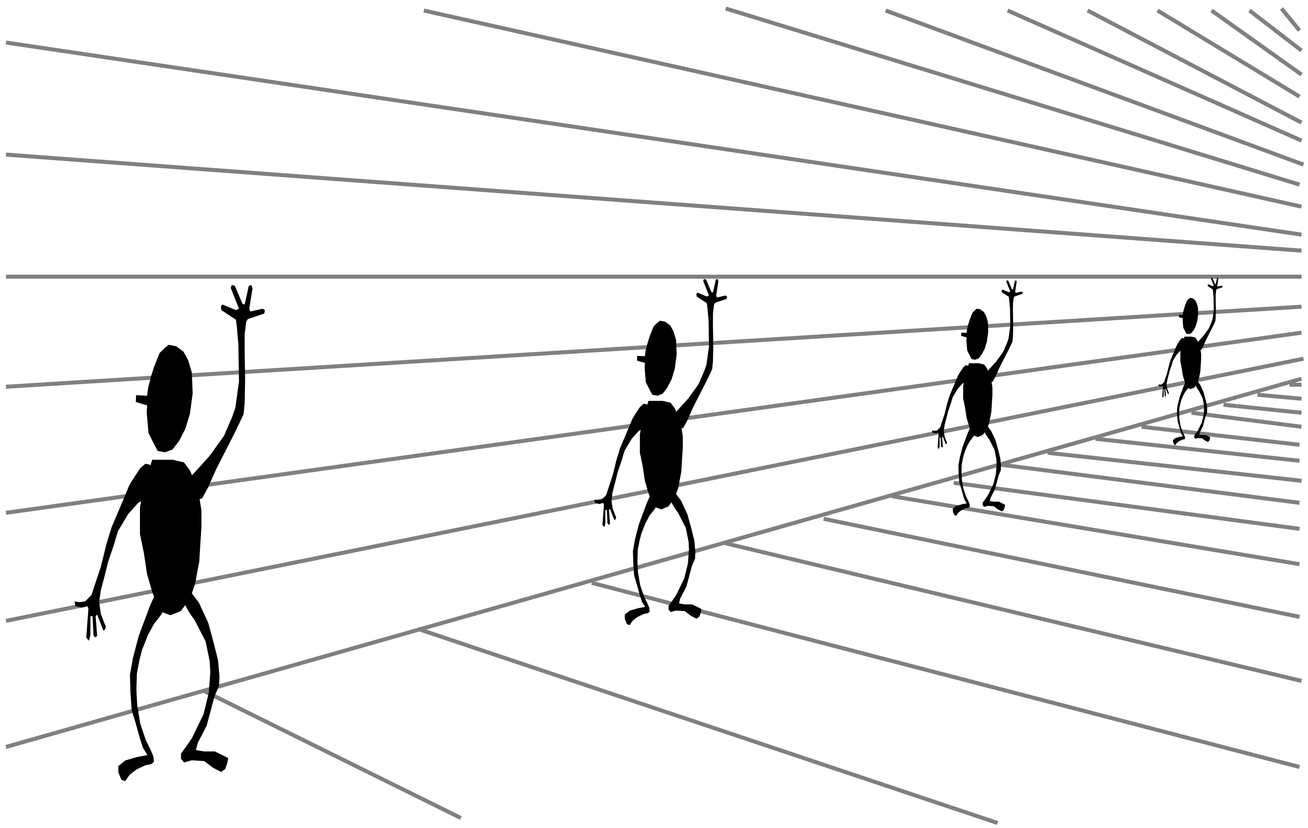
Perspective cues



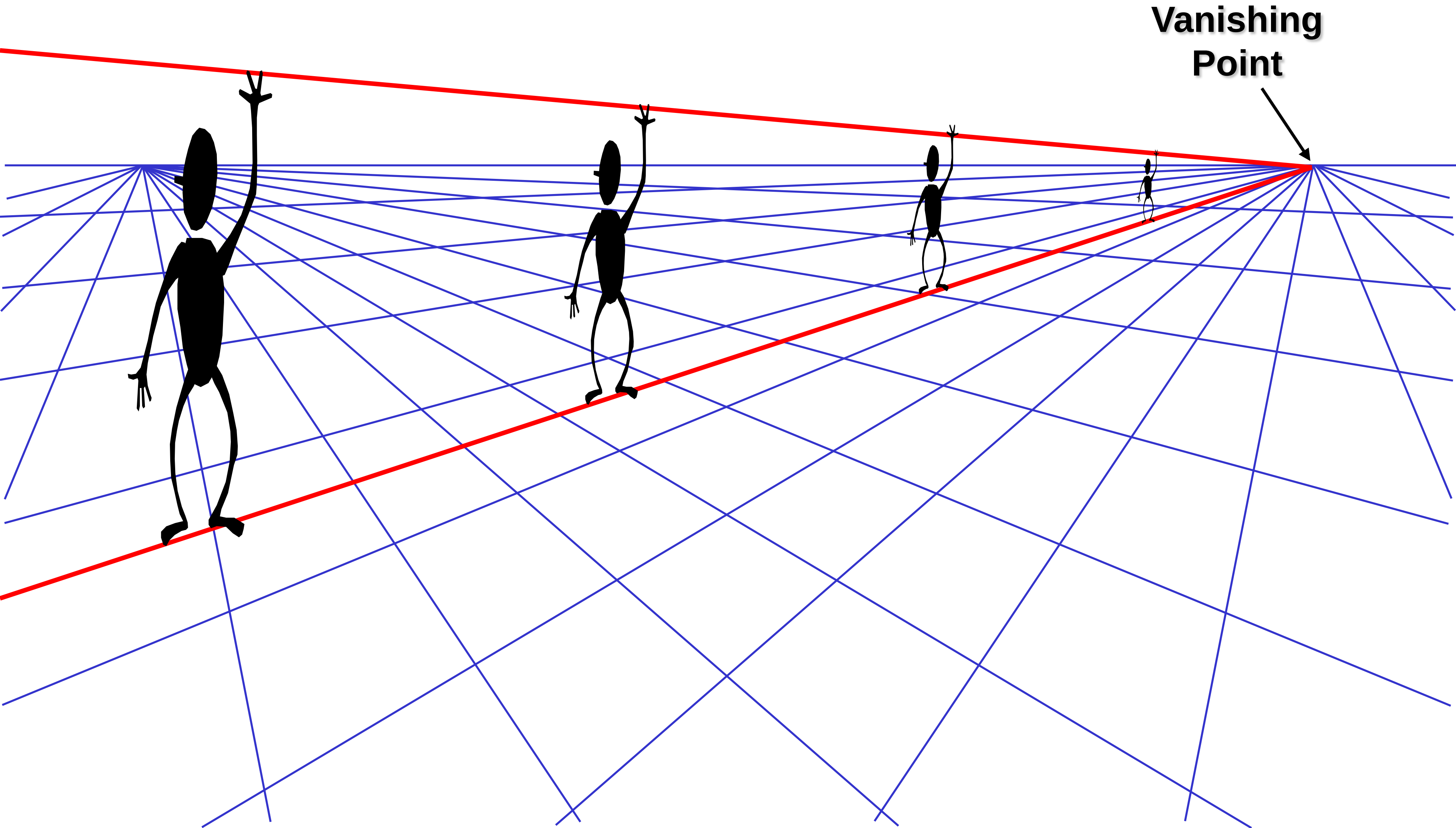
Perspective cues



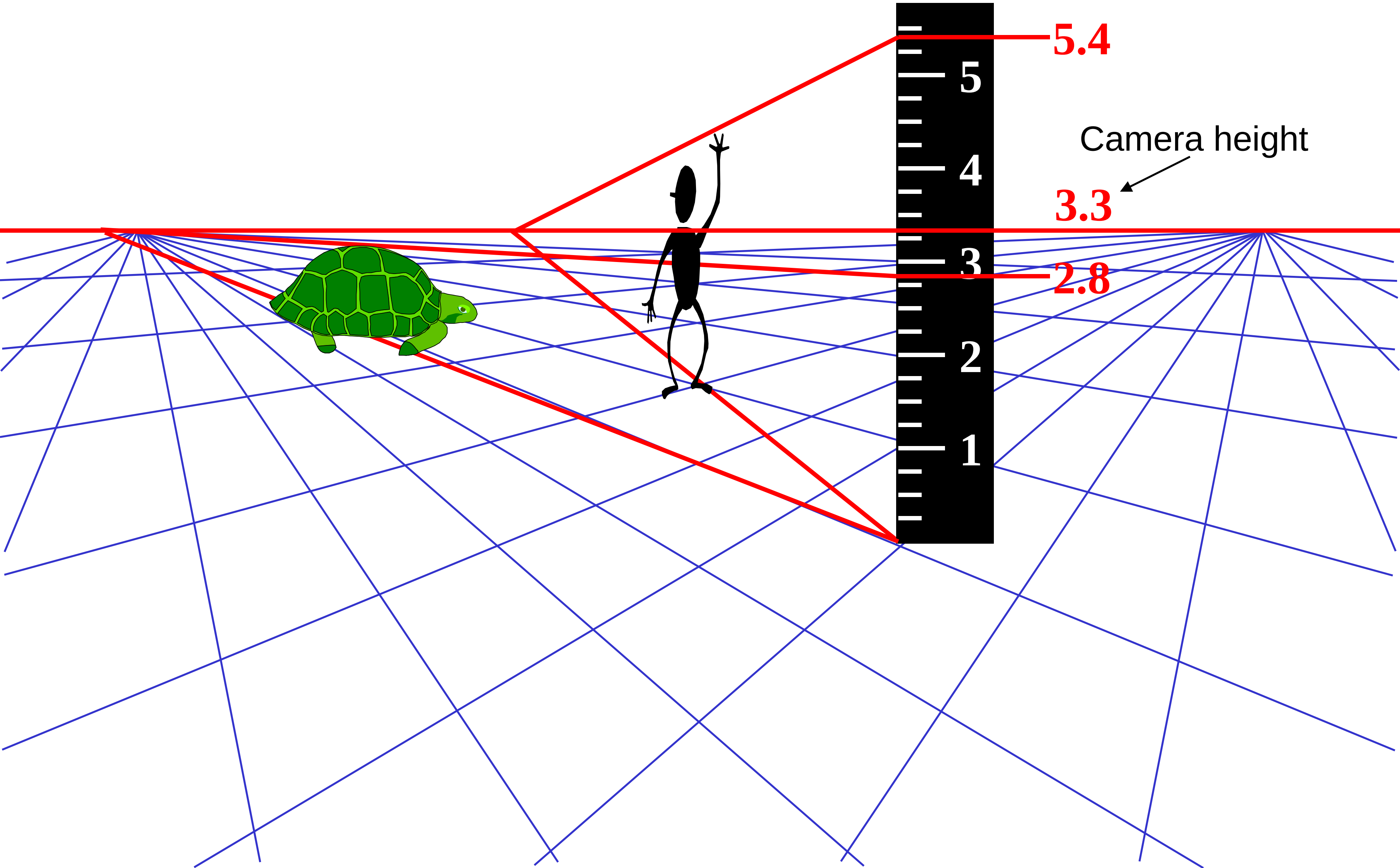
Perspective cues



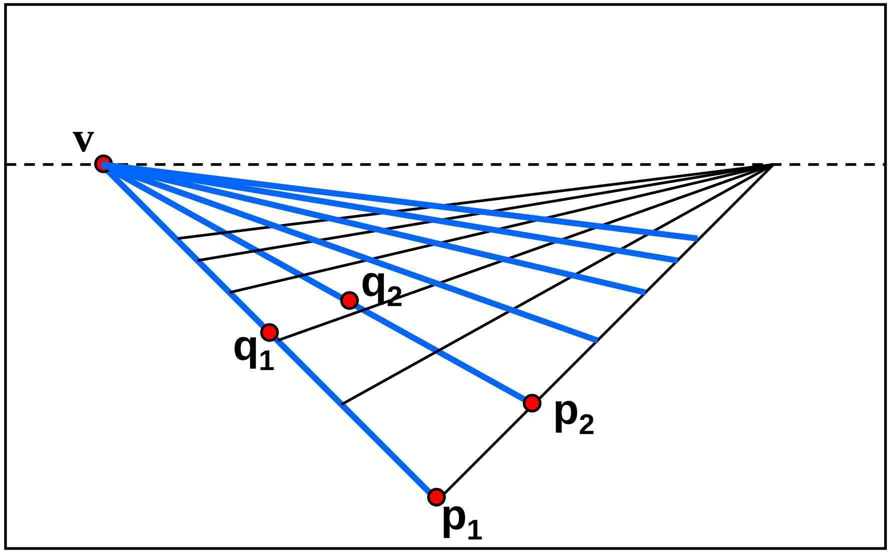
Comparing heights



Measuring height



Computing vanishing points (from lines)



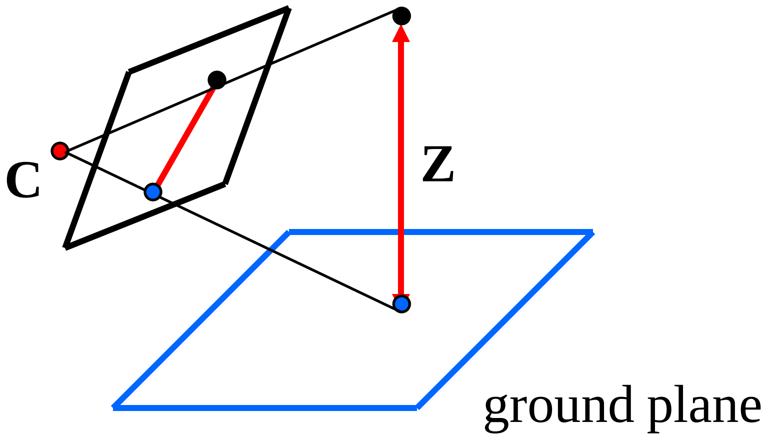
- Intersect p_1q_1 with p_2q_2

$$v = (p_1 \times q_1) \times (p_2 \times q_2)$$

Least squares version

- Better to use more than two lines and compute the "closest" point of intersection
- See notes by Bob Collins for one good way of doing this:
  - http://www-2.cs.cmu.edu/~ph/869/www/notes/vanishing.txt

Measuring height without a ruler



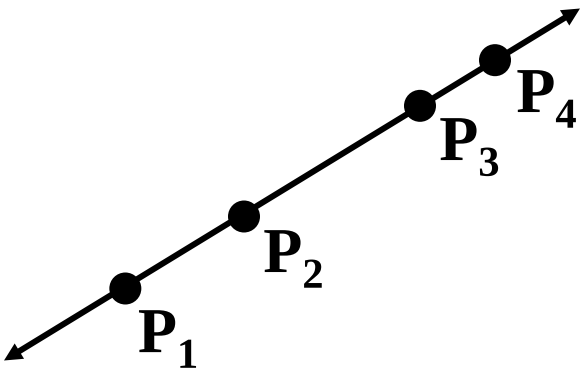
Compute Z from image measurements

- Need more than vanishing points to do this

The cross ratio

- A Projective Invariant
  - Something that does not change under projective transformations (including perspective projection)

The cross-ratio of 4 collinear points



$$\frac{\|\mathbf{P}_3 - \mathbf{P}_1\| \, \|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_3 - \mathbf{P}_2\| \, \|\mathbf{P}_4 - \mathbf{P}_1\|}$$

$$\mathbf{P}_i = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

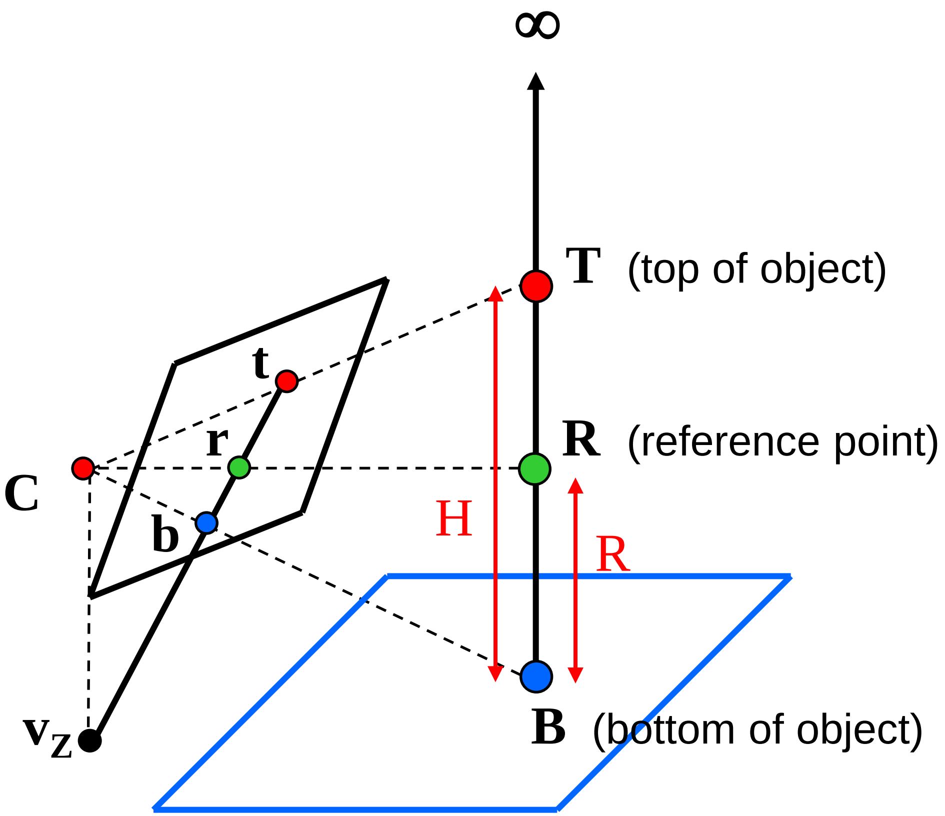
Can permute the point ordering

$$\frac{\|\mathbf{P}_1 - \mathbf{P}_3\| \, \|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_1 - \mathbf{P}_2\| \, \|\mathbf{P}_4 - \mathbf{P}_3\|}$$

- 4! = 24 different orders (but only 6 distinct values)

This is the fundamental invariant of projective geometry

Measuring height



$$\frac{\|\mathbf{T}-\mathbf{B}\|\,\|\infty-\mathbf{R}\|}{\|\mathbf{R}-\mathbf{B}\|\,\|\infty-\mathbf{T}\|}=\frac{H}{R}$$

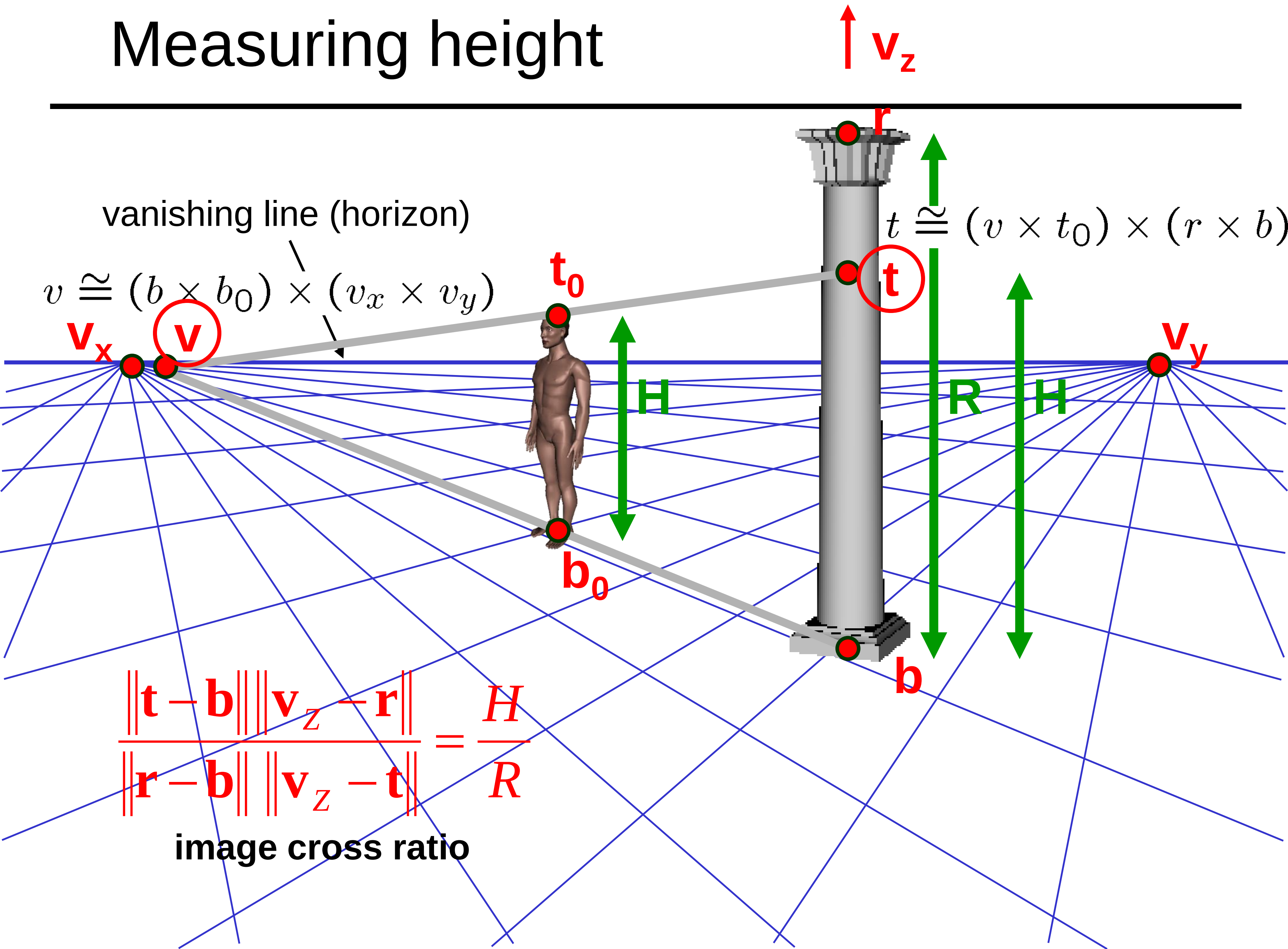
scene cross ratio

$$\frac{\|\mathbf{t}-\mathbf{b}\|\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{t}\|}=\frac{H}{R}$$

image cross ratio

scene points represented as $\mathbf{P}=\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix}$ image points as $\mathbf{p}=\begin{bmatrix}x\\y\\1\end{bmatrix}$

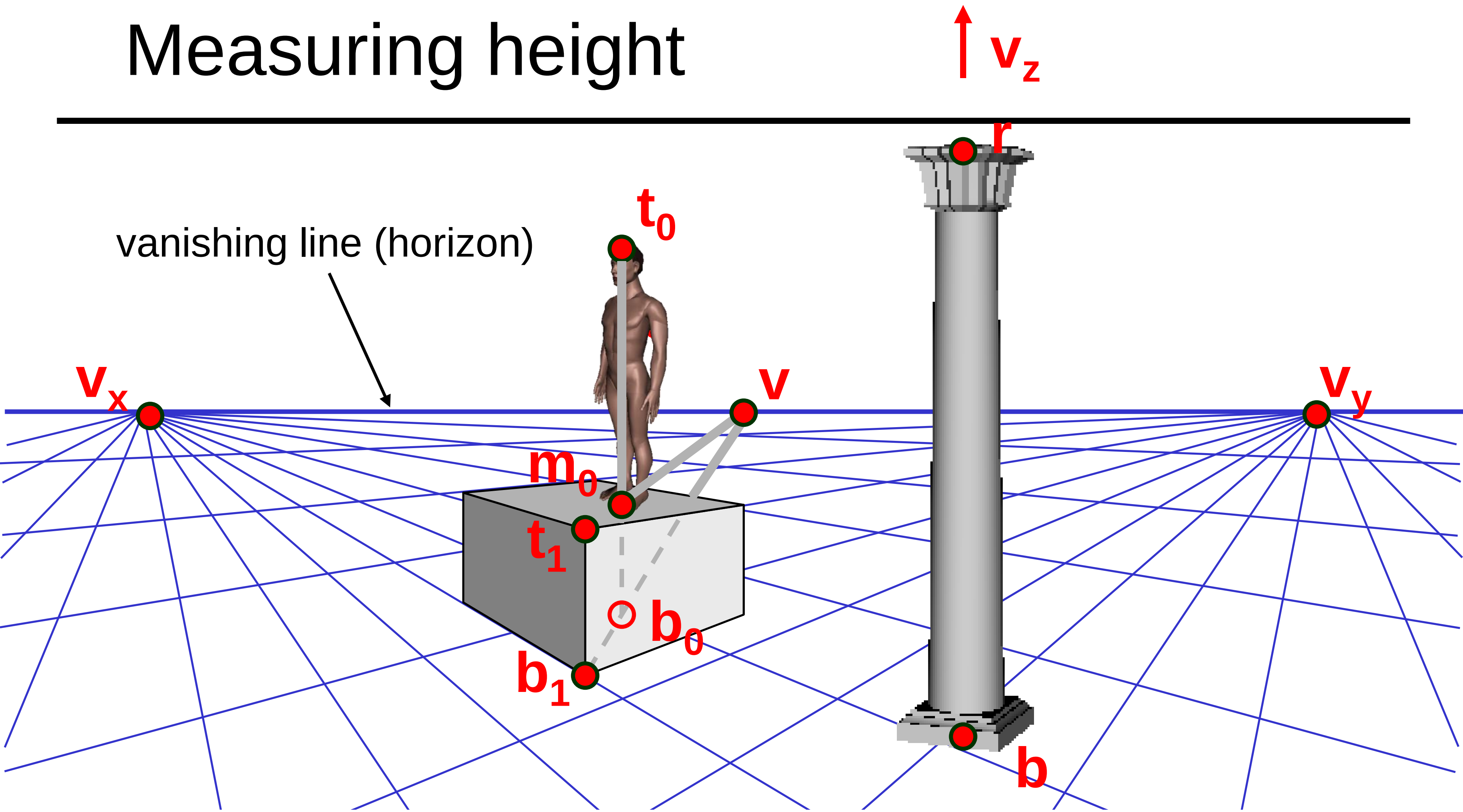
Measuring height



$$\frac{\|\mathbf{t}-\mathbf{b}\|\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{t}\|}=\frac{H}{R}$$

image cross ratio

Measuring height



What if the point on the ground plane $\mathbf{b_0}$ is not known?

- Here the guy is standing on the box, height of box is known
- Use one side of the box to help find $\mathbf{b_0}$ as shown above

Computing (X,Y,Z) coordinates

- Okay, we know how to compute height (Z coords)
  - how can we compute X, Y?

Camera calibration

- Goal: estimate the camera parameters
  - Version 1: solve for projection matrix

$$\mathbf{x} = \begin{bmatrix} wx \\ wy \\ w \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{\Pi X}$$

- Version 2: solve for camera parameters separately
  - intrinsics (focal length, principle point, pixel size)
  - extrinsics (rotation angles, translation)
  - radial distortion

Vanishing points and projection matrix

$$\boldsymbol{\Pi} = \begin{array}{cccc} \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \\ \boldsymbol{\pi}_1 \quad \boldsymbol{\pi}_2 \quad \boldsymbol{\pi}_3 \quad \boldsymbol{\pi}_4 \end{array} = \begin{bmatrix} \boldsymbol{\pi}_1 & \boldsymbol{\pi}_2 & \boldsymbol{\pi}_3 & \boldsymbol{\pi}_4 \end{bmatrix}$$

- $\boldsymbol{\pi}_1 = \boldsymbol{\Pi}\begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}^T$ = $\mathbf{v}_x$ (X vanishing point)
- similarly, $\boldsymbol{\pi}_2 = \mathbf{v}_Y, \ \boldsymbol{\pi}_3 = \mathbf{v}_Z$
- $\boldsymbol{\pi}_4 = \boldsymbol{\Pi}\begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}^T = \text{projection of world origin}$

$$\boldsymbol{\Pi} = \begin{bmatrix} \mathbf{v}_X & \mathbf{v}_Y & \mathbf{v}_Z & \mathbf{o} \end{bmatrix}$$

Not So Fast! We only know **v**'s up to a scale factor

$$\boldsymbol{\Pi} = \begin{bmatrix} a\,\mathbf{v}_X & b\mathbf{v}_Y & c\mathbf{v}_Z & \mathbf{o} \end{bmatrix}$$

- Can fully specify by providing 3 reference points

3D Modeling from a photograph

https://research.microsoft.com/vision/cambridge/3d/3dart.htm