

Image Segmentation

Shengnan Wang

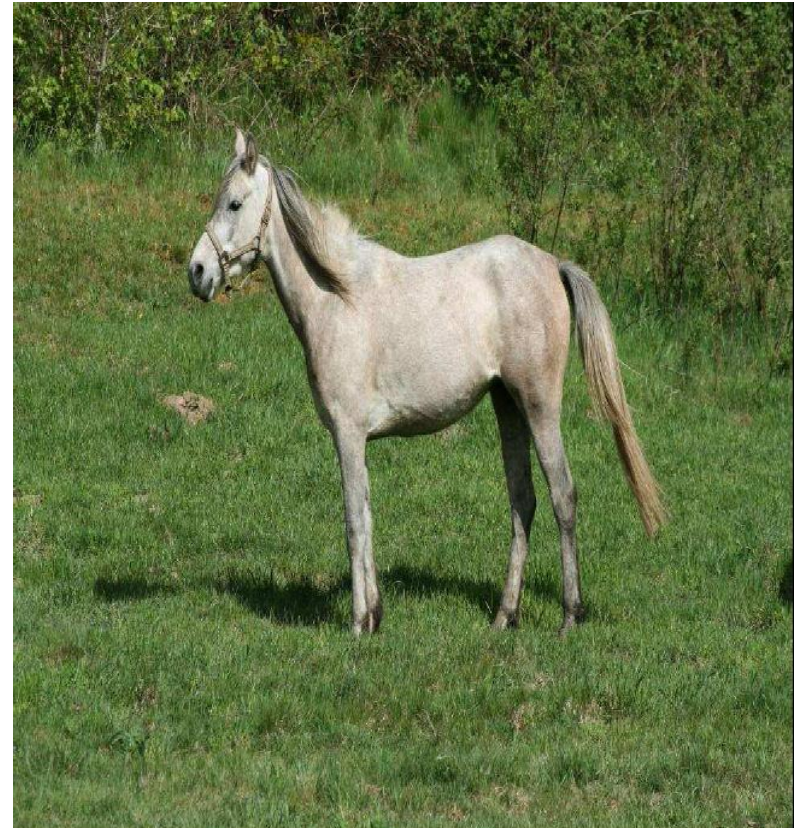
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 - 2. Density Estimation Methods
 - 3. Deriving the Mean Shift
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- III. Application
 - 1. Segmentation

Motivation

- What do we see in an image?
- How is the image represented?

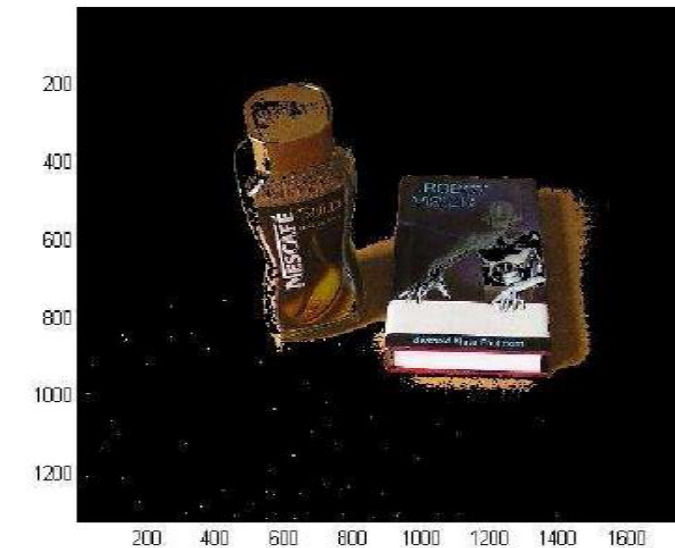
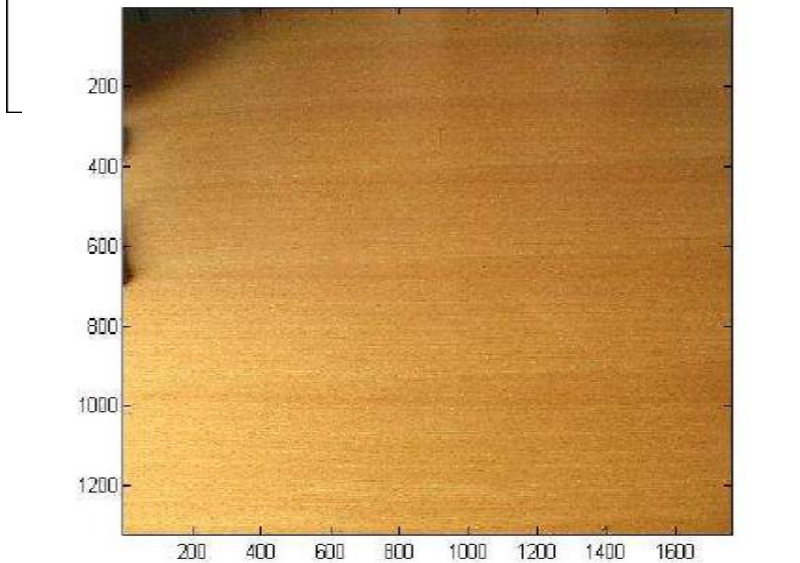
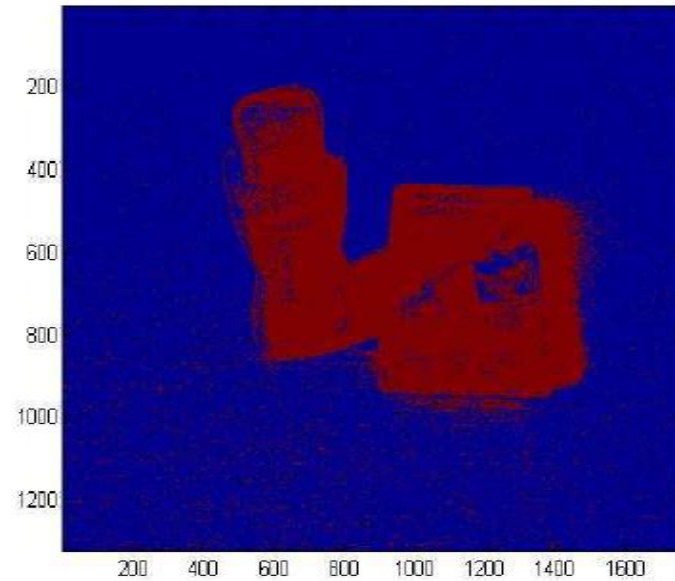
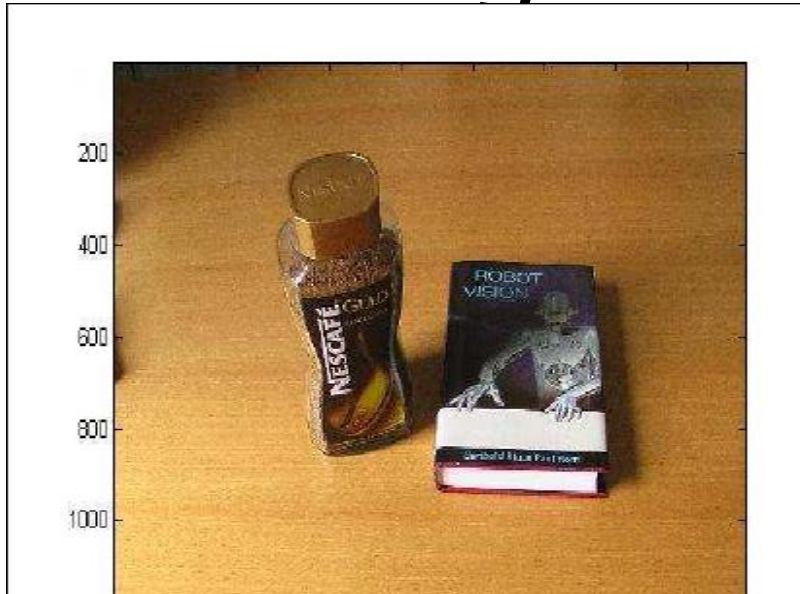


- Goal: Find relevant image regions for the objects we want to analyze

Image Segmentation

- Definition 1: Partition the image into connected subsets that maximize some “uniformity” criteria.
- Definition 2: Identify possibly overlapping but maximal connected subsets that satisfy some uniformity

Background Subtraction



Other Applications

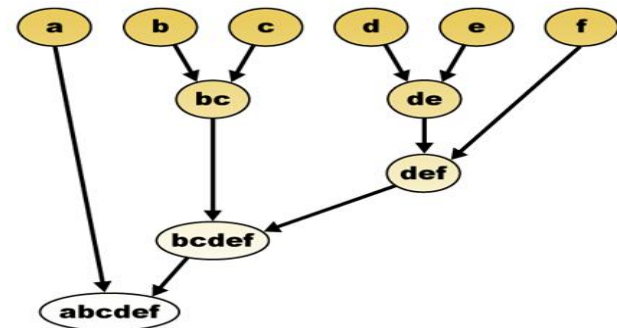
- Medical Imaging
 - Locate tumors and other pathologies
 - Measure tissue volumes
 - Computer-guided surgery
 - Diagnosis
 - Treatment planning
 - Study of anatomical structure
- Locate objects in satellite images (roads, forests, etc.)
- Face Detection
- Machine Vision
- Automatic traffic controlling systems

Methods

- Clustering Methods
 - K means, Mean Shift
- Graph Partitioning Methods
 - Normalized Cut
- Histogram-Based Methods
- Edge Detection Methods
- Model based Segmentation
- Multi-scale, Region Growing , Neural Networks, Watershed Transformation

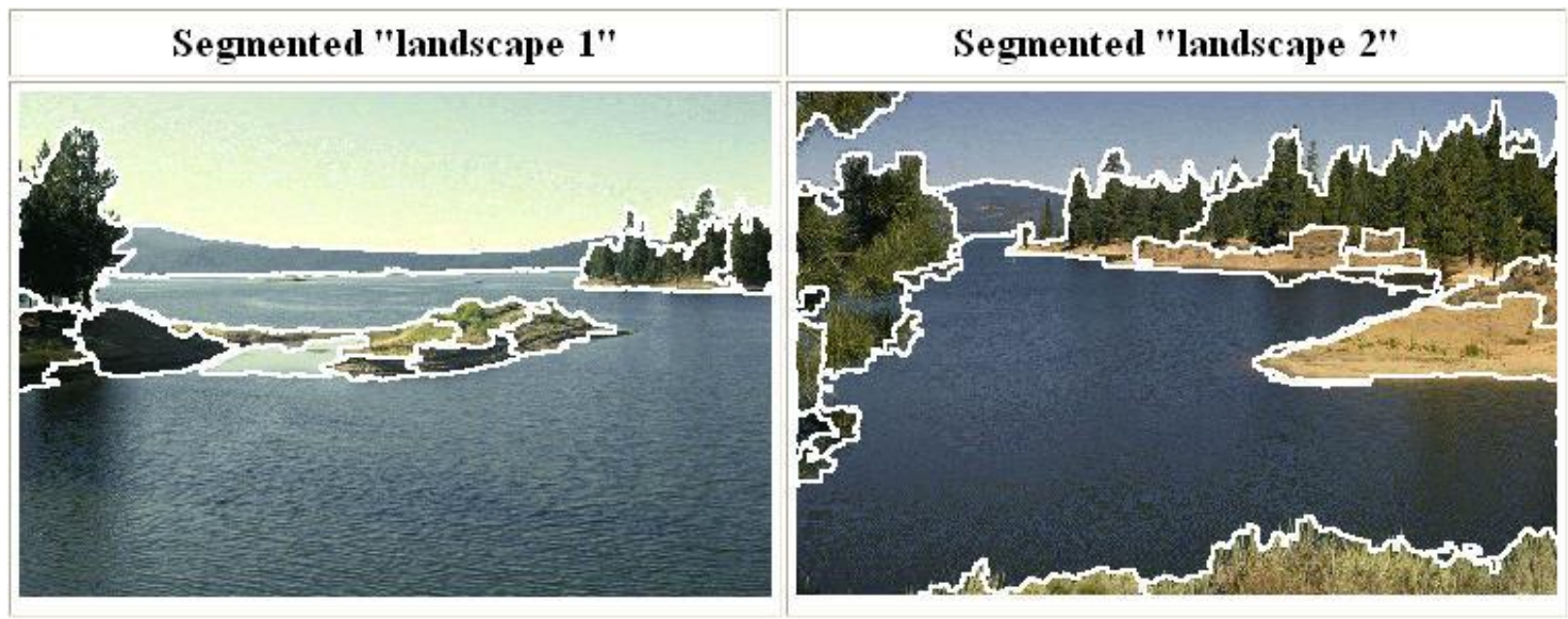
Segmentation as clustering

- Cluster together (pixels, tokens, etc.) that belong together
- Agglomerative clustering
 - attach pixel to cluster it is closest to
 - repeat
- Divisive clustering
 - split cluster along best boundary
 - repeat
- Point-Cluster distance
 - single-link clustering
 - complete-link clustering
 - group-average clustering
- Dendrograms(Tree)
 - yield a picture of output as clustering process continues



Mean Shift Segmentation

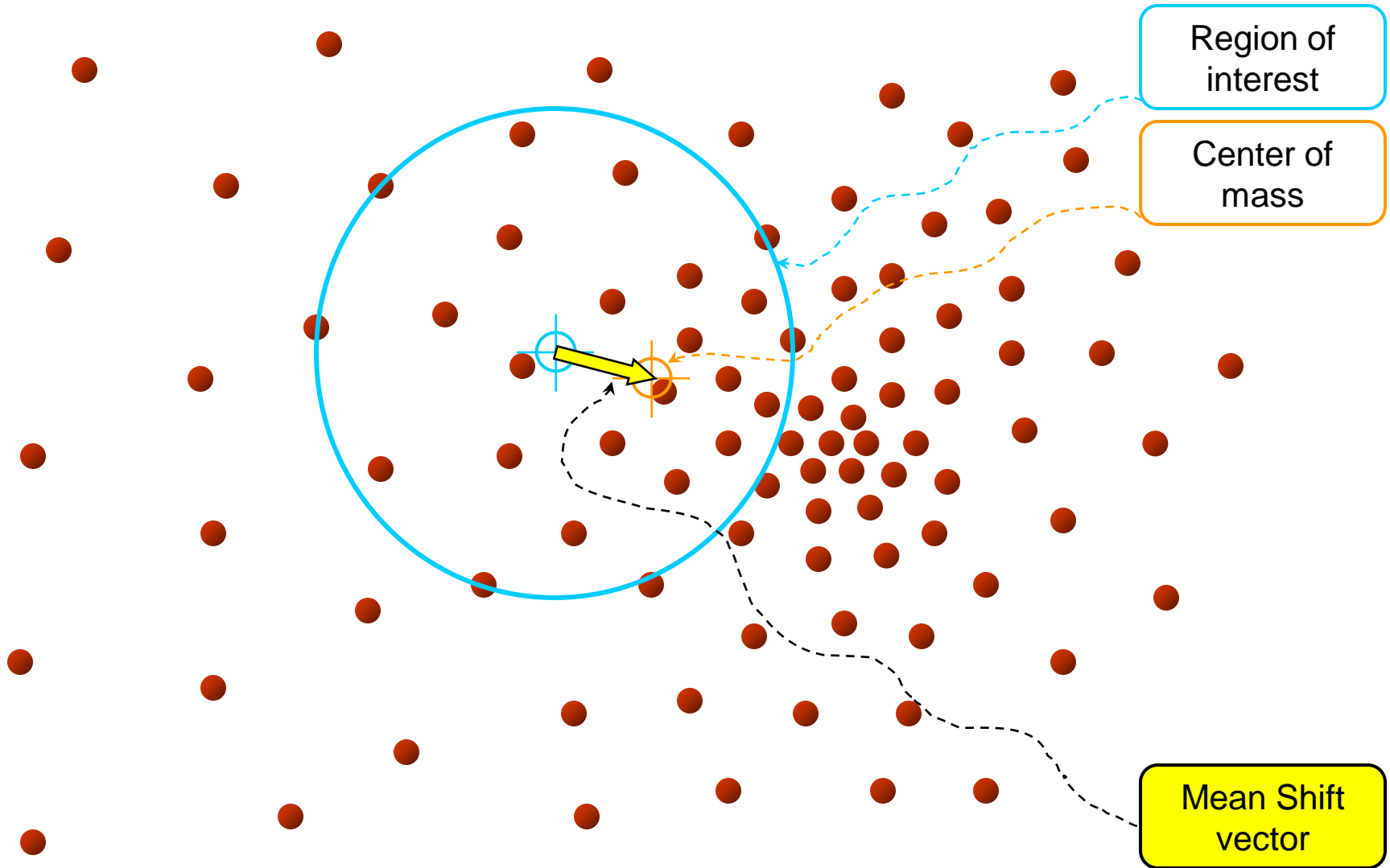
- Perhaps the best technique to date...



<http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html>

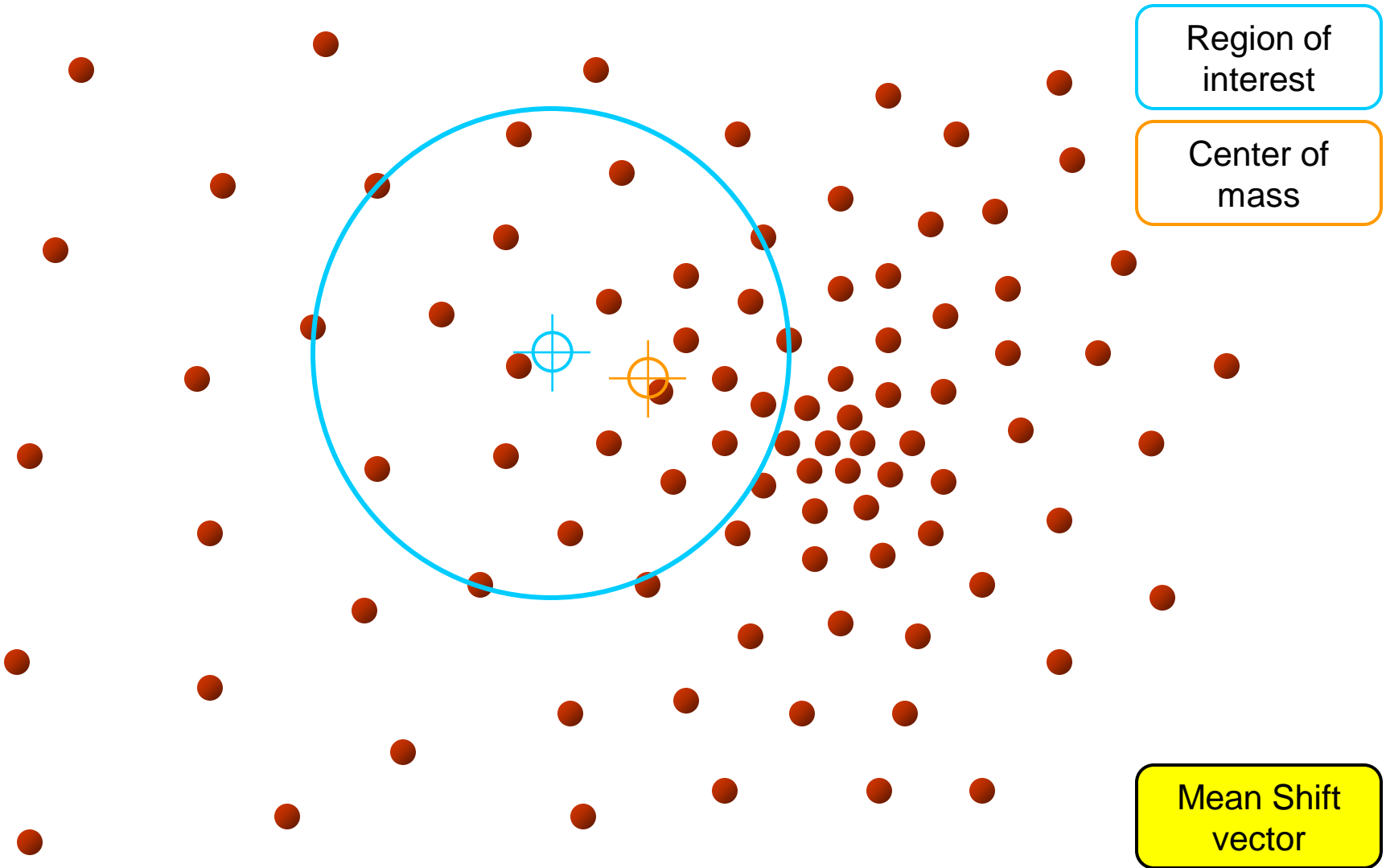
Mean Shift Theory

Intuitive Description



Objective : Find the densest region
Distribution of identical billiard balls

Intuitive Description



Objective : Find the densest region
Distribution of identical billiard balls

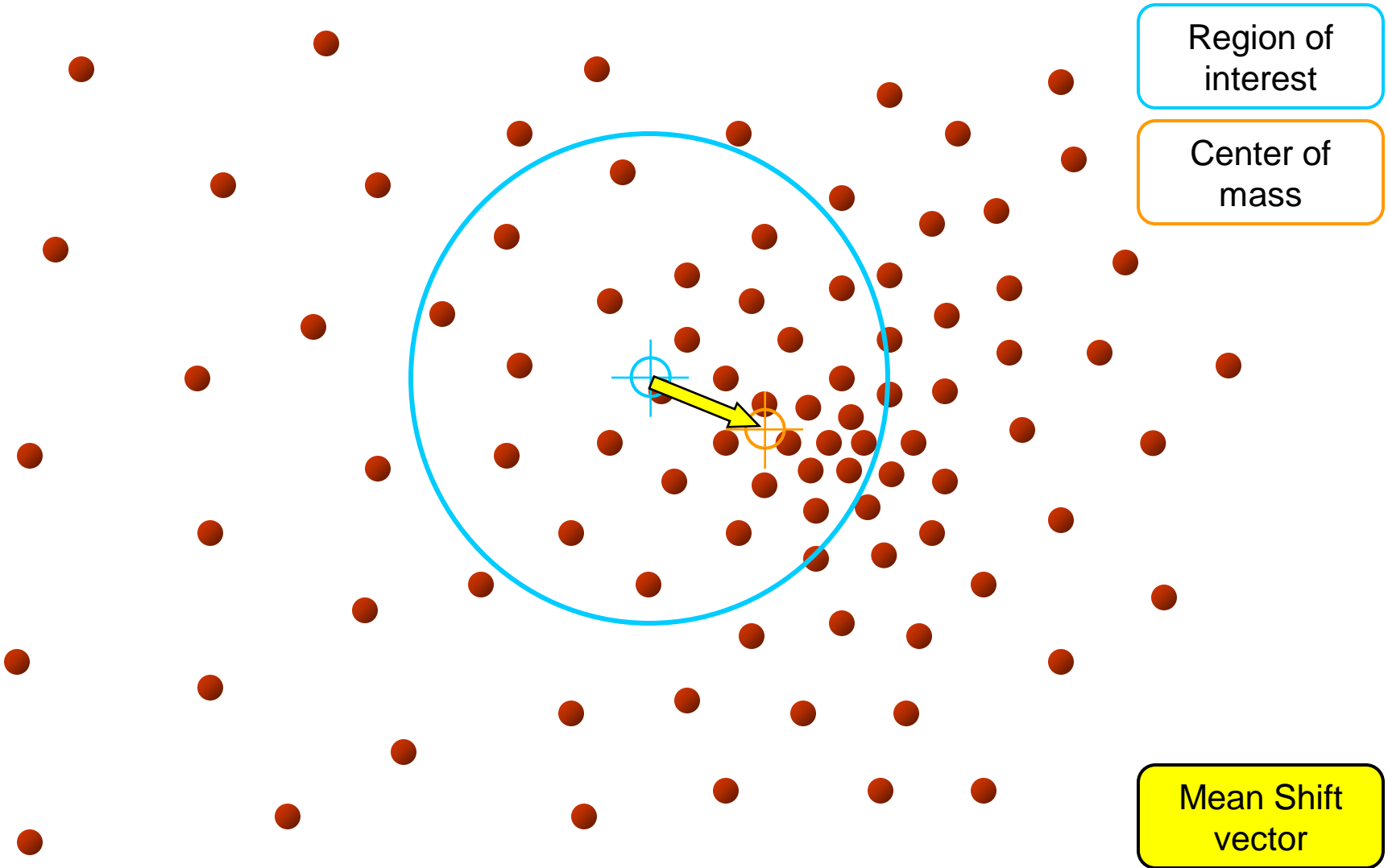
Intuitive Description

Region of
interest

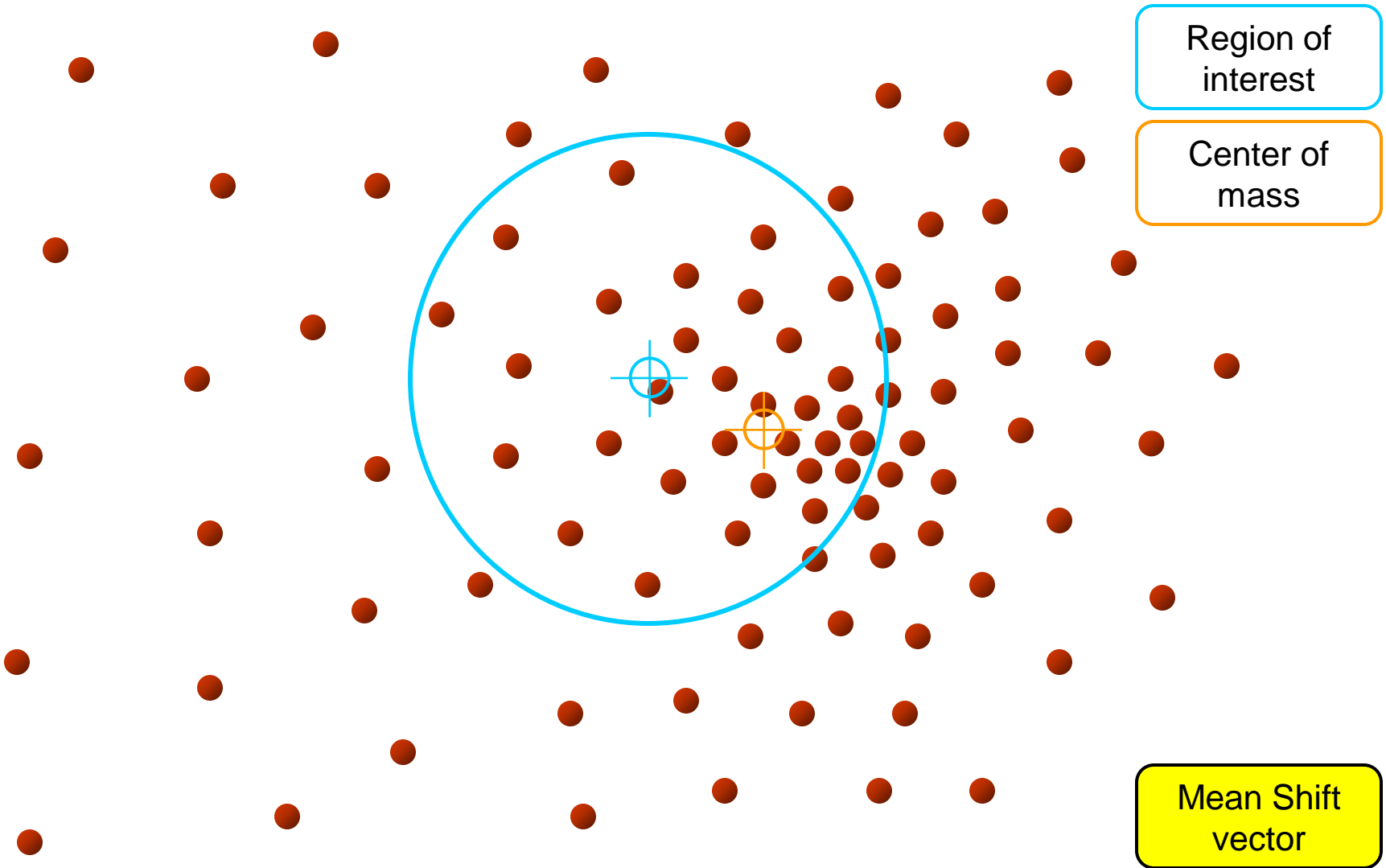
Center of
mass

Mean Shift
vector

Objective : Find the densest region
Distribution of identical billiard balls



Intuitive Description



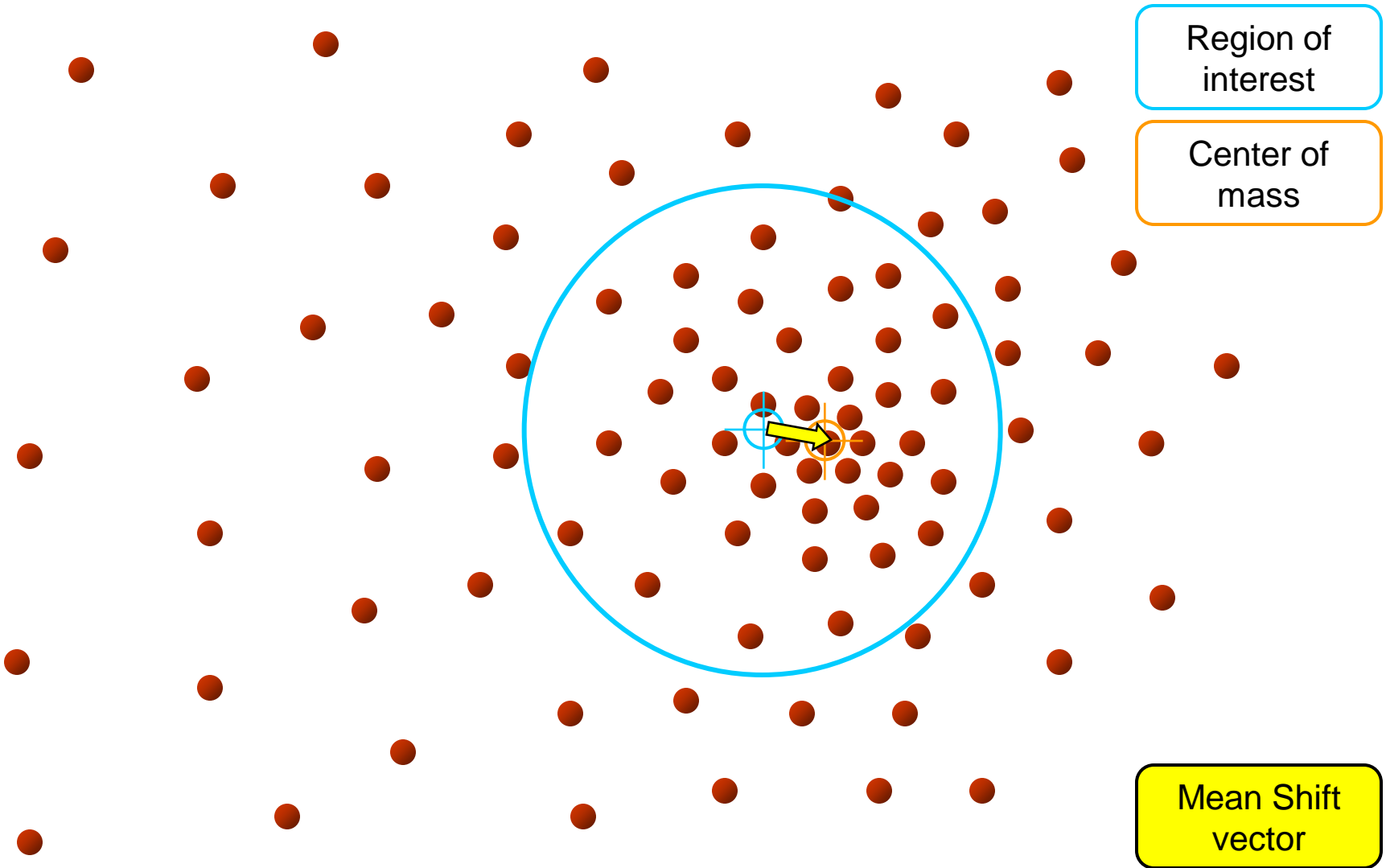
Region of interest

Center of mass

Mean Shift vector

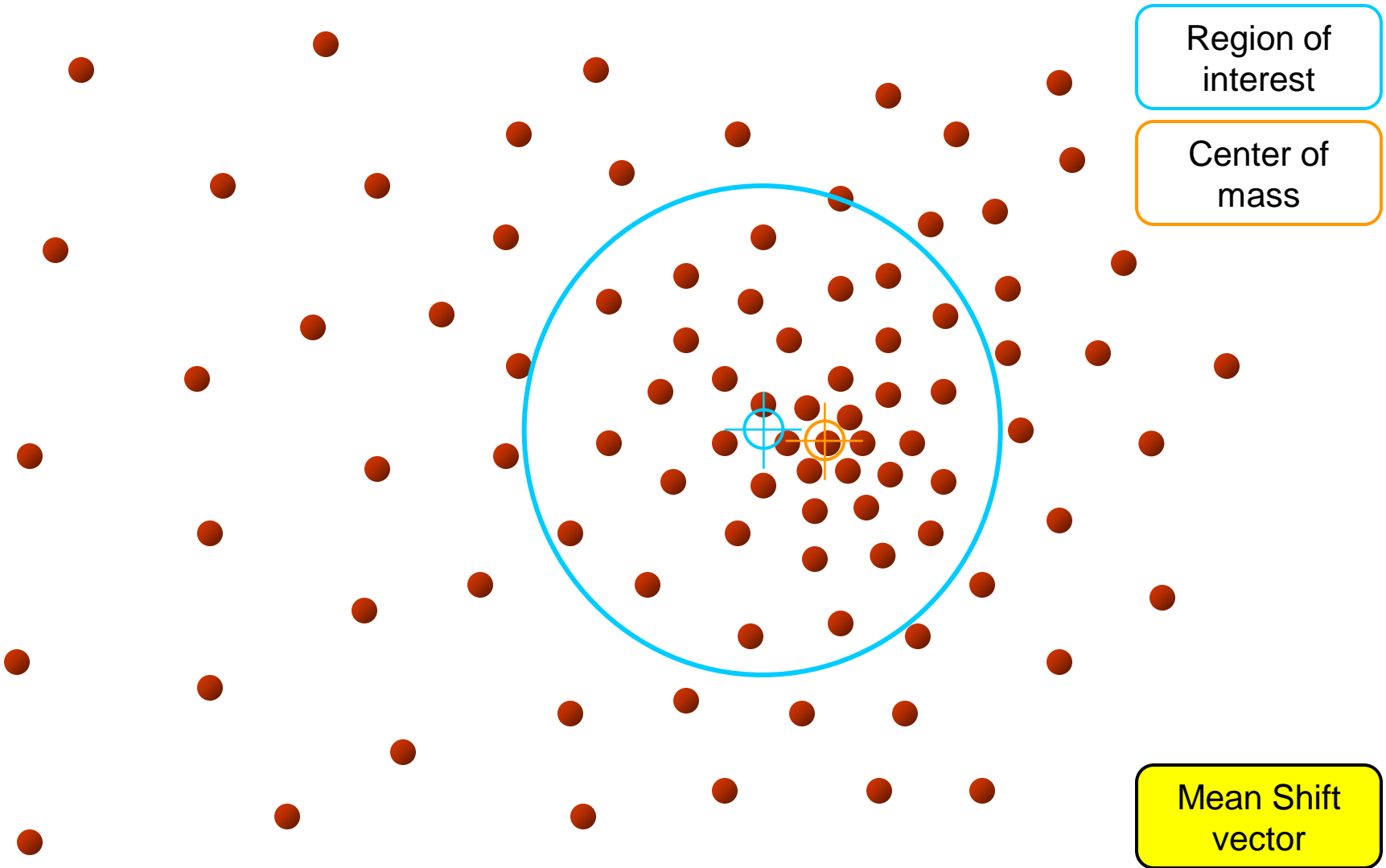
Objective : Find the densest region
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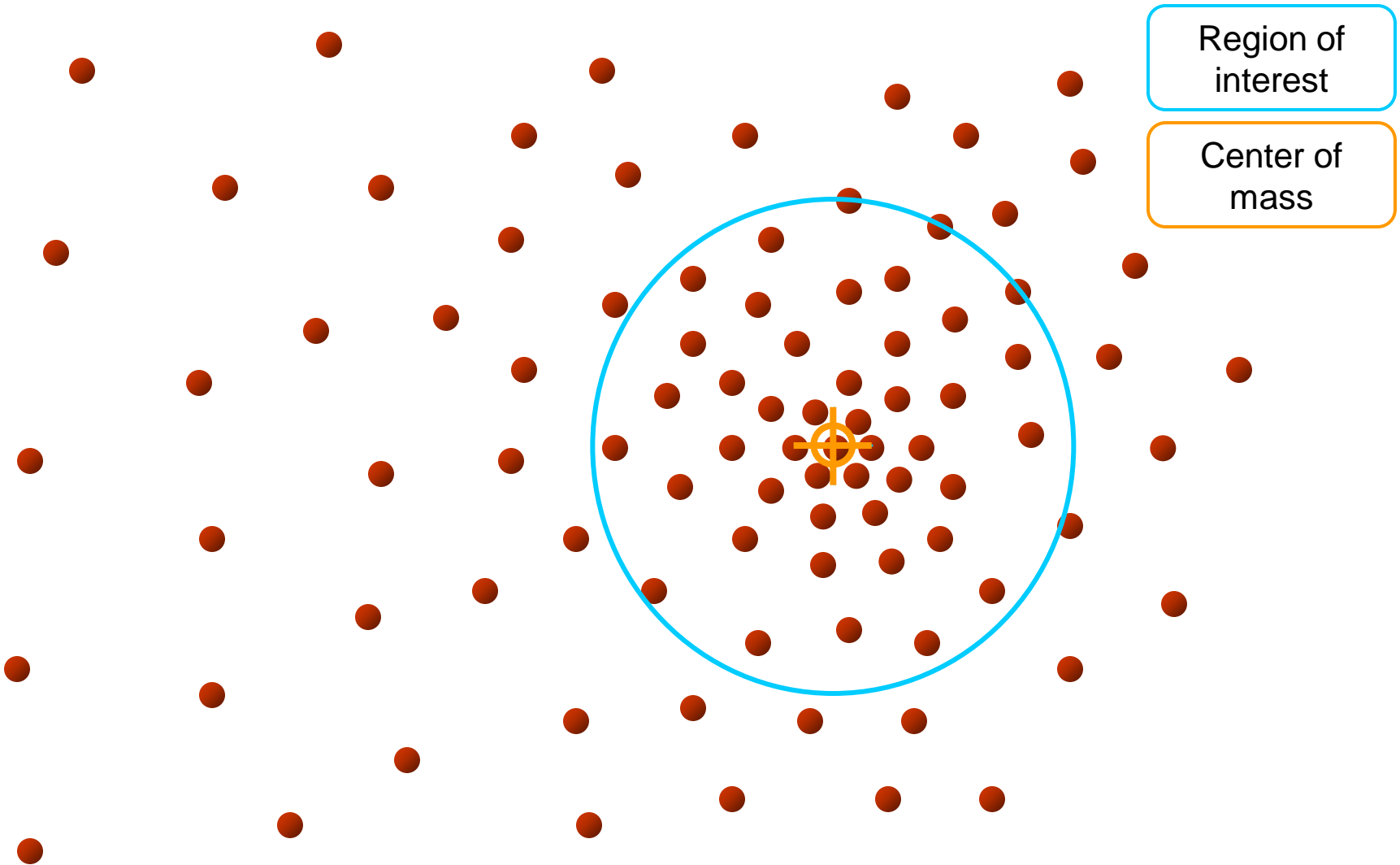
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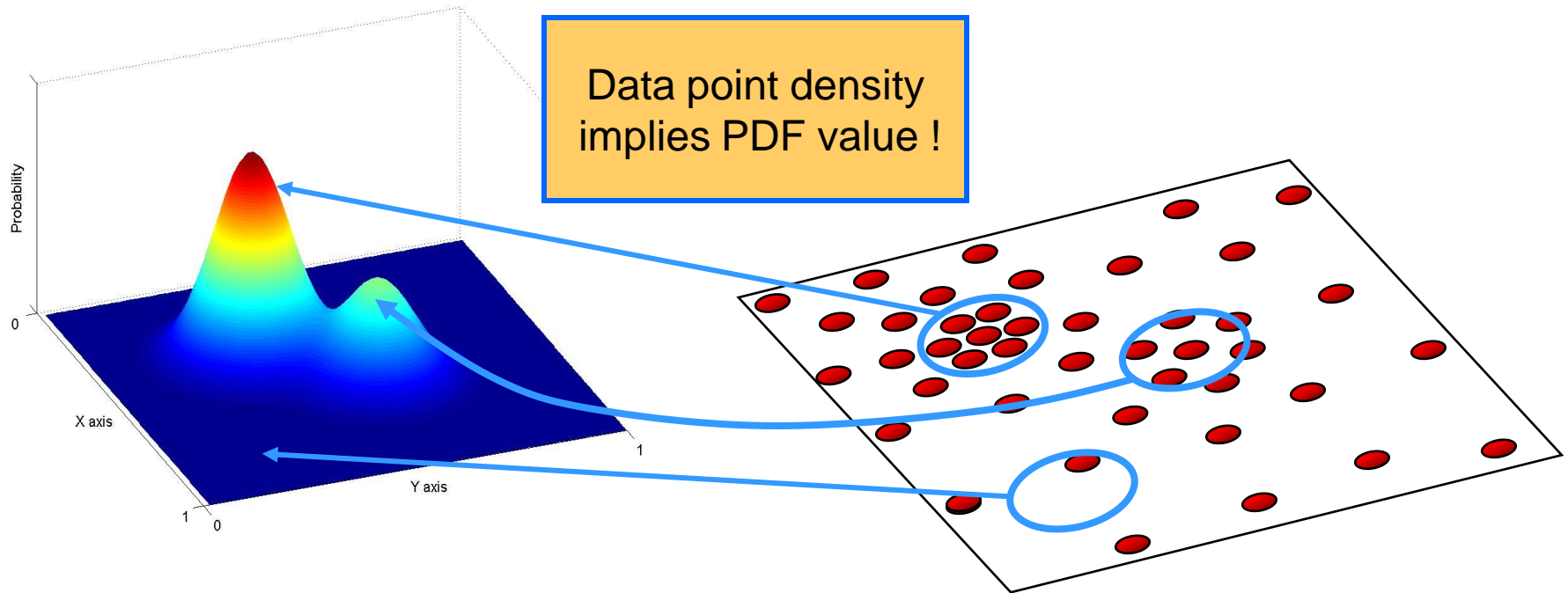


Objective : Find the densest region
Distribution of identical billiard balls

1. What is Mean Shift?

- **Non-parametric density estimation**

Assumption : The data points are sampled from an underlying PDF



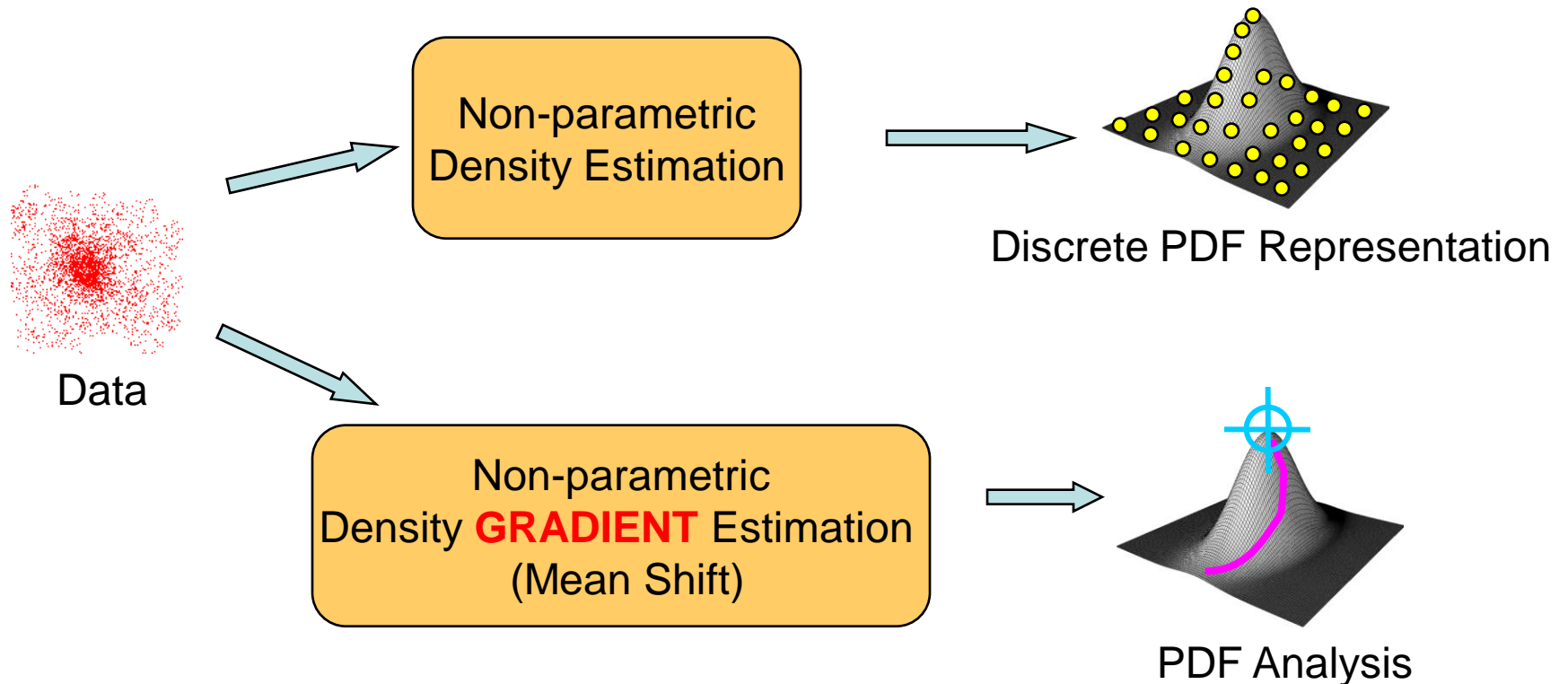
Assumed Underlying PDF

Real Data Samples

1. What is Mean Shift?

- A tool for:

Finding Modes in a set of data samples, manifesting an underlying **probability density function** (PDF) in \mathbb{R}^N



2. Density Estimation Method

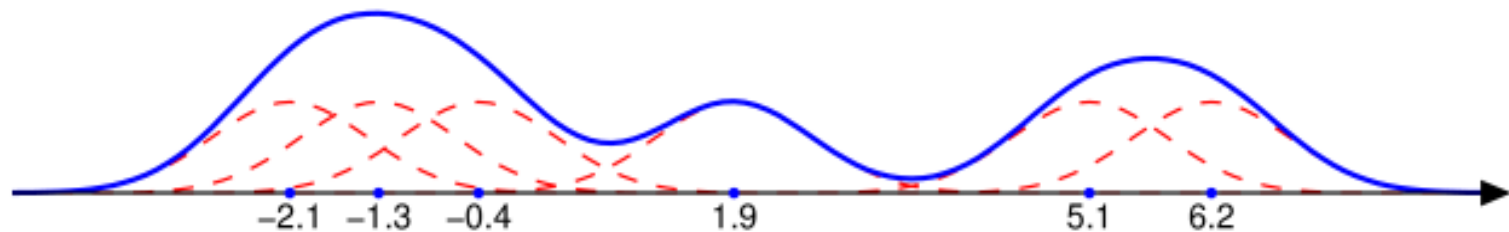
- Kernel Density Estimation

Univariate kernel density estimator

Given a random sample X_1, \dots, X_n with a continuous, univariate density f . The kernel density estimator is

$$\hat{f}(x, h) = \frac{1}{nh} \sum_{i=1}^n K\left(\frac{x - X_i}{h}\right)$$

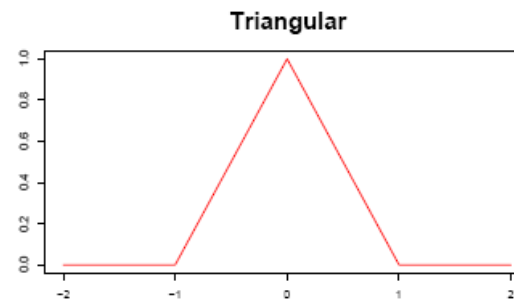
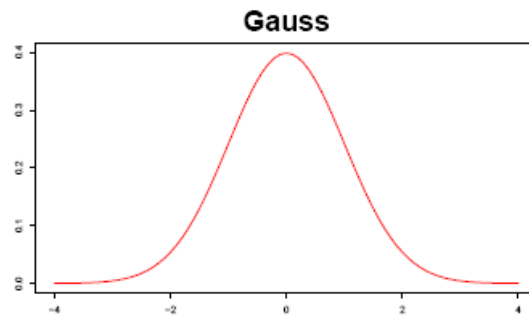
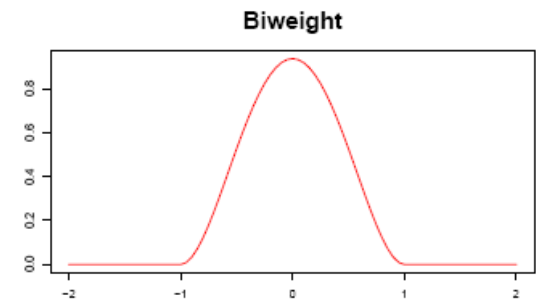
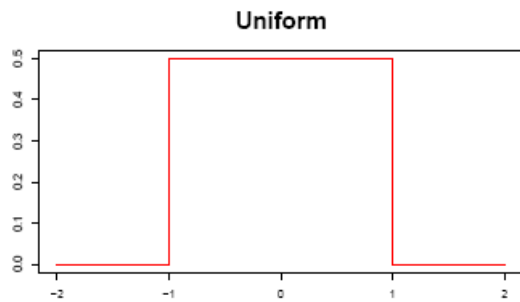
with **kernel K** and **bandwidth h** . Under mild conditions (h must decrease with increasing n) the kernel estimate converges in probability to the true density.



2. Density Estimation Method

- **Kernel Density Estimation**

 - Various kernels



3. Deriving the Mean Shift

Kernel Density Estimation

Gradient

$$\nabla P(\mathbf{x}) = \frac{1}{n} \sum_{i=1}^n \nabla K(\mathbf{x} - \mathbf{x}_i)$$

Give up estimating the PDF !
Estimate **ONLY** the gradient

Using the
Kernel form:

$$K(\mathbf{x} - \mathbf{x}_i) = ck \left(\left\| \frac{\mathbf{x} - \mathbf{x}_i}{h} \right\|^2 \right)$$

Function of vector length
only

Define :

$$g(\mathbf{x}) = -k'(\mathbf{x})$$

Size of window

$$\nabla P(\mathbf{x}) = \frac{c}{n} \sum_{i=1}^n \nabla k_i = \frac{c}{n} \left[\sum_{i=1}^n g_i \right] \square \left[\frac{\sum_{i=1}^n \mathbf{x}_i g_i}{\sum_{i=1}^n g_i} - \mathbf{x} \right]$$

Computing The Mean Shift

Gradient

$$\nabla P(\mathbf{x}) = \frac{c}{n} \sum_{i=1}^n \nabla k_i = \frac{c}{n} \left[\sum_{i=1}^n g_i \right] \square \left[\frac{\sum_{i=1}^n \mathbf{x}_i g_i}{\sum_{i=1}^n g_i} - \mathbf{x} \right]$$

$$g(\mathbf{x}) = -k'(\mathbf{x})$$

Computing The Mean Shift

$$\nabla P(\mathbf{x}) = \frac{c}{n} \sum_{i=1}^n \nabla k_i = \frac{c}{n} \left[\sum_{i=1}^n g_i \right] \left[\frac{\sum_{i=1}^n \mathbf{x}_i g_i}{\sum_{i=1}^n g_i} - \mathbf{x} \right]$$

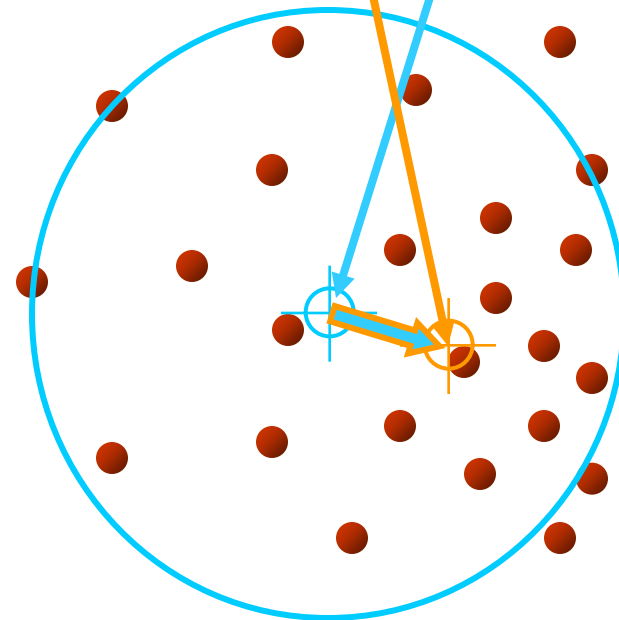
Yet another Kernel density estimation !

Simple Mean Shift procedure:

- Compute mean shift vector

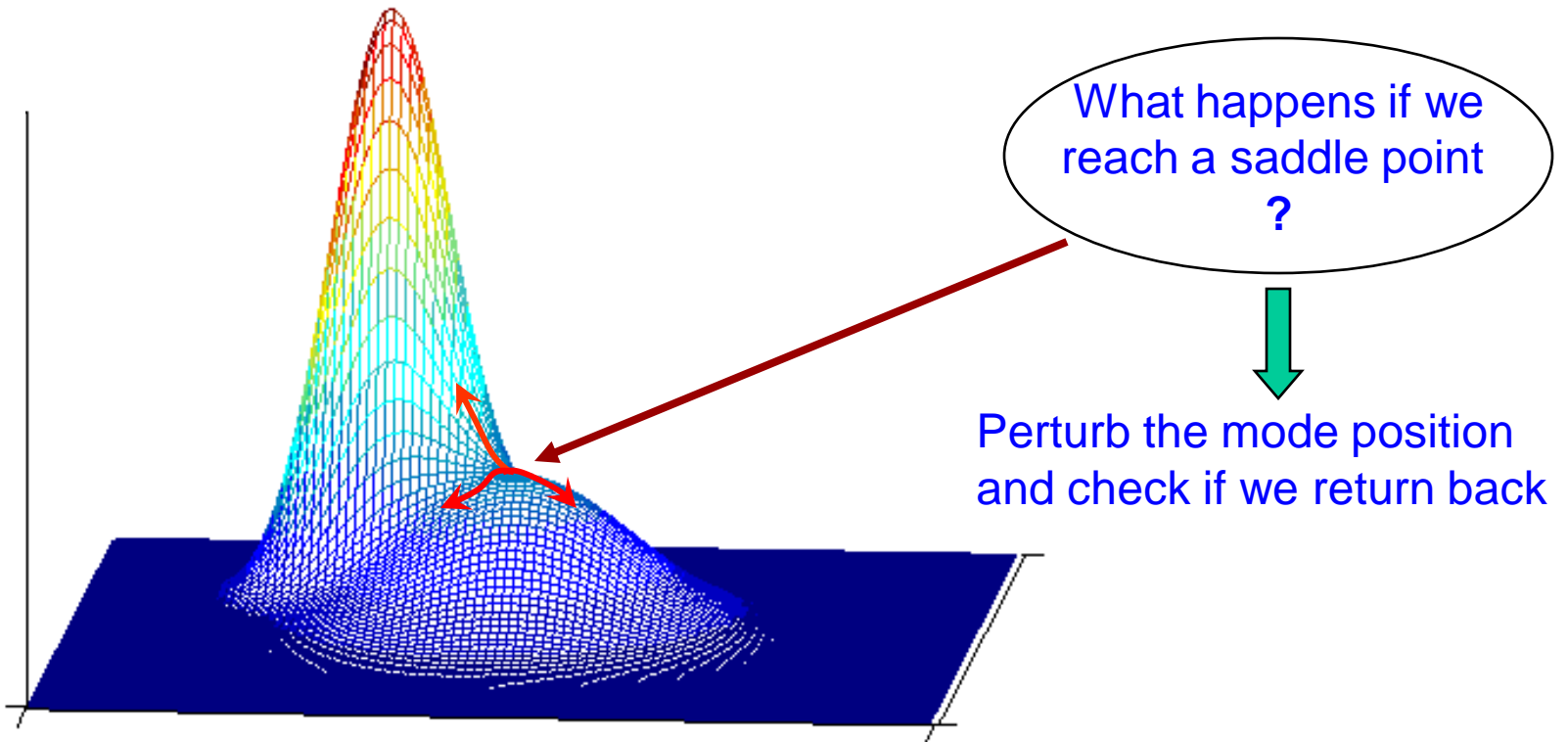
$$\mathbf{m}(\mathbf{x}) = \left[\frac{\sum_{i=1}^n \mathbf{x}_i g \left(\frac{\|\mathbf{x} - \mathbf{x}_i\|^2}{h} \right)}{\sum_{i=1}^n g \left(\frac{\|\mathbf{x} - \mathbf{x}_i\|^2}{h} \right)} - \mathbf{x} \right]$$

- Translate the Kernel window by $\mathbf{m}(\mathbf{x})$
- repeat



$$g(\mathbf{x}) = -k'(\mathbf{x})$$

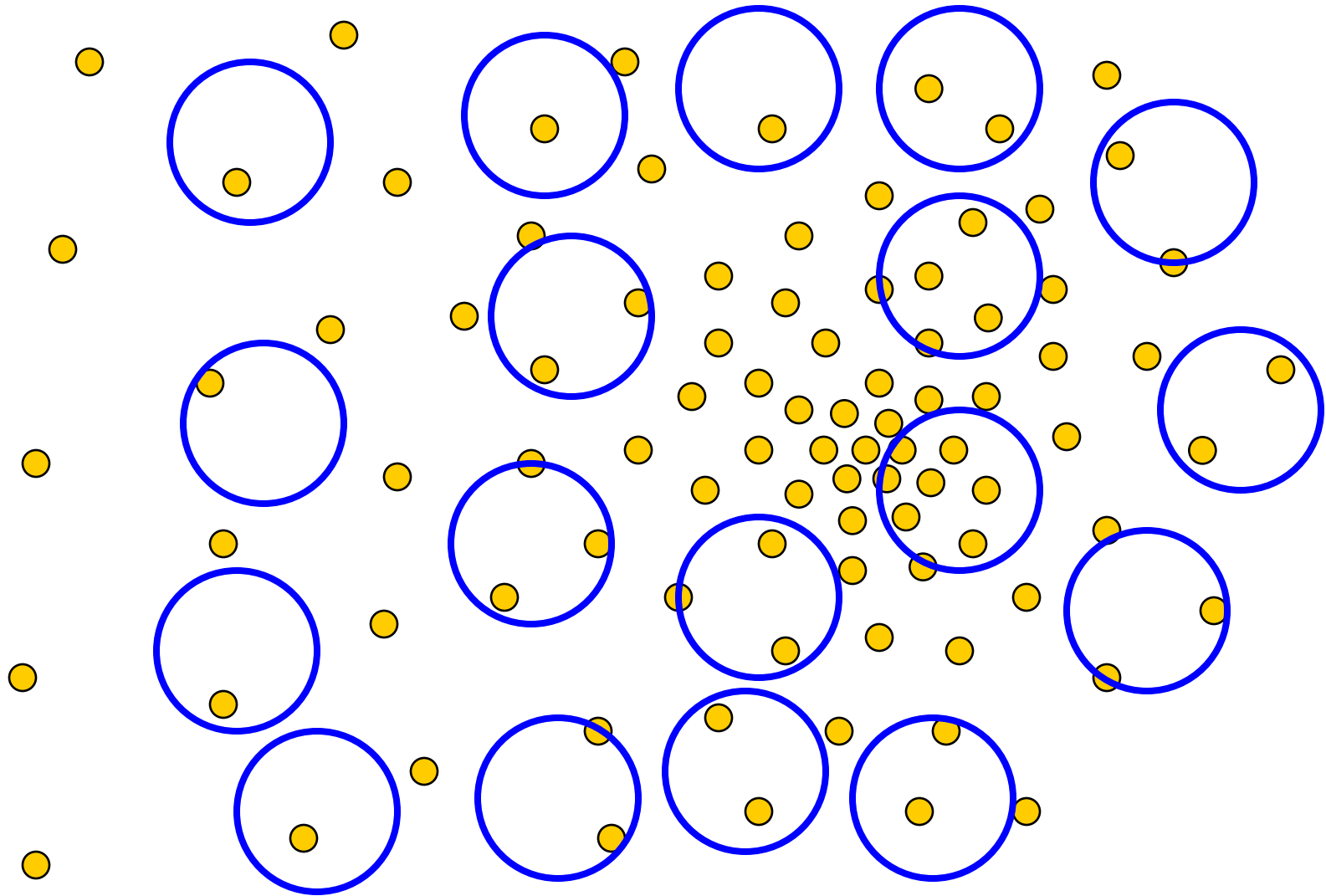
Mean Shift Mode Detection



Updated Mean Shift Procedure:

- Find all modes using the Simple Mean Shift Procedure
- Prune modes by perturbing them (find saddle points and plateaus)
- Prune nearby – take highest mode in the window

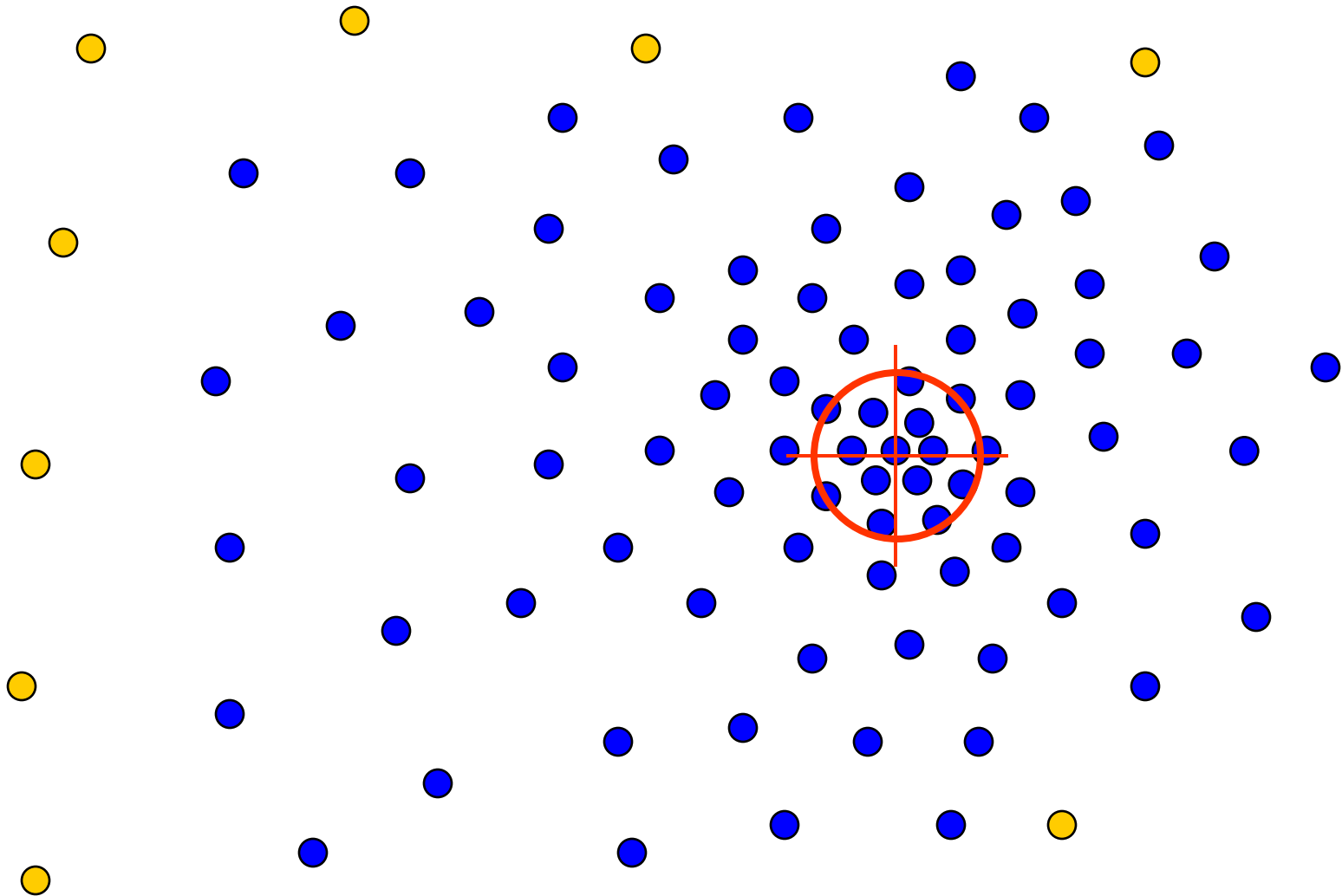
Real Modality Analysis



Tessellate the space
with windows

Run the procedure in parallel

Real Modality Analysis



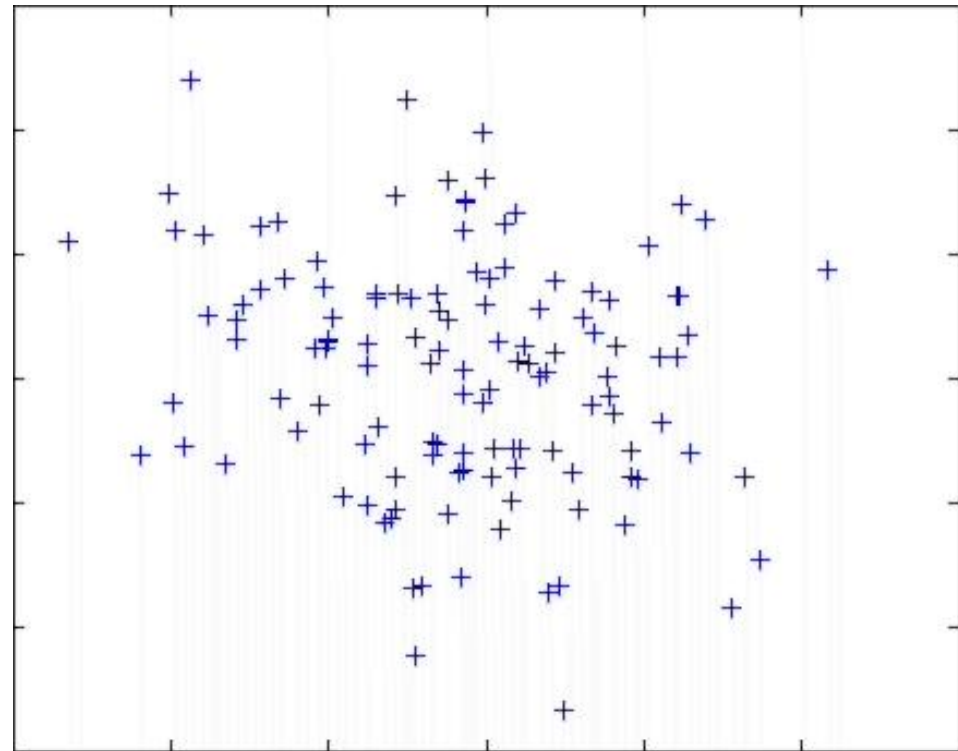
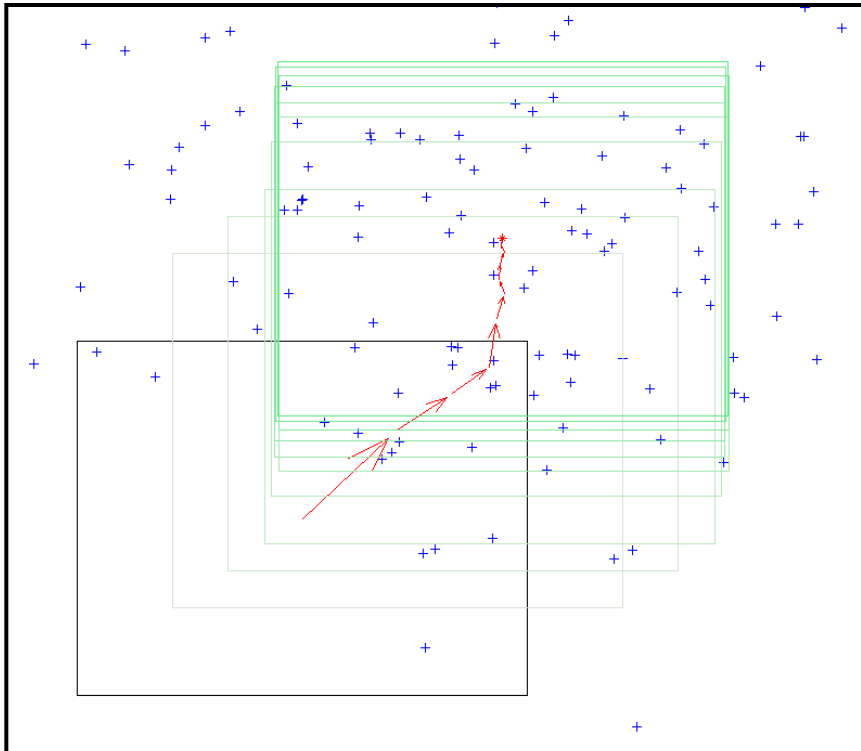
The blue data points were traversed by the windows towards the mode

Mean Shift Algorithm

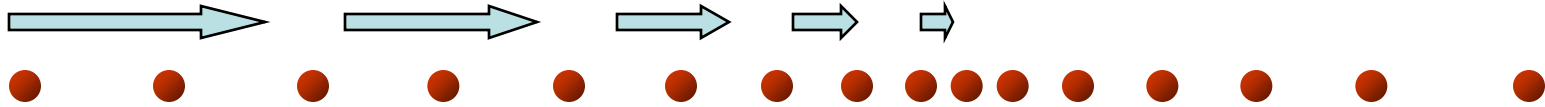
Mean Shift Algorithm

1. Choose a search window size.
2. Choose the initial location of the search window.
3. Compute the mean location (centroid of the data) in the search window.
4. Center the search window at the mean location computed in Step 3.
5. Repeat Steps 3 and 4 until convergence.

The mean shift algorithm seeks the “mode” or point of highest density of a data distribution:



4. Mean Shift Properties

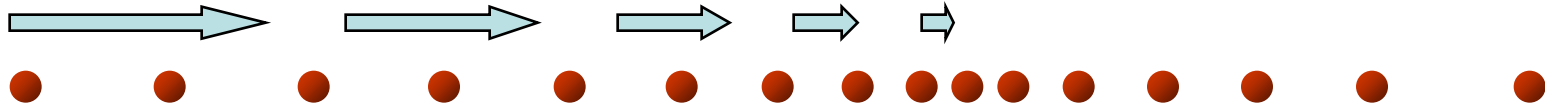


- Automatic convergence speed – the mean shift vector size depends on the gradient itself.
- Near maxima, the steps are small and refined
- Convergence is guaranteed for infinitesimal steps only → infinitely convergent, (therefore set a lower bound)

**Adaptive
Gradient
Ascent**



4. Mean Shift Properties



Advantages :

- Application independent tool
- Suitable for real data analysis
- Does not assume any prior shape (e.g. elliptical) on data clusters
- Can handle arbitrary feature spaces
- Only ONE parameter to choose
- h (window size) has a physical meaning, unlike K-Means

Disadvantages :

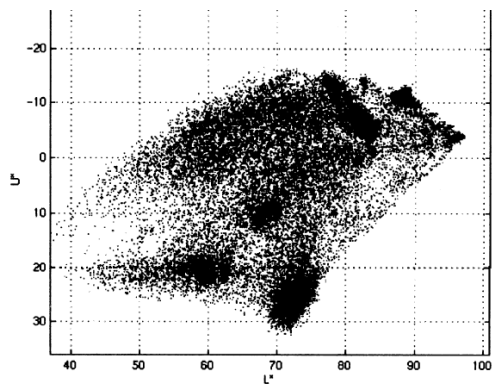
- The window size (bandwidth selection) is not trivial
- Inappropriate window size can cause modes to be merged, or generate additional “shallow” modes → Use adaptive window size



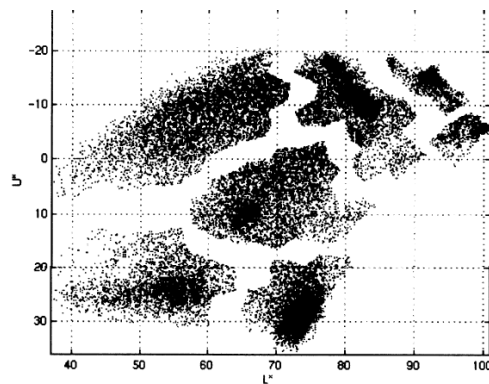
Mean Shift Segmentation

Mean Shift Segmentation Algorithm

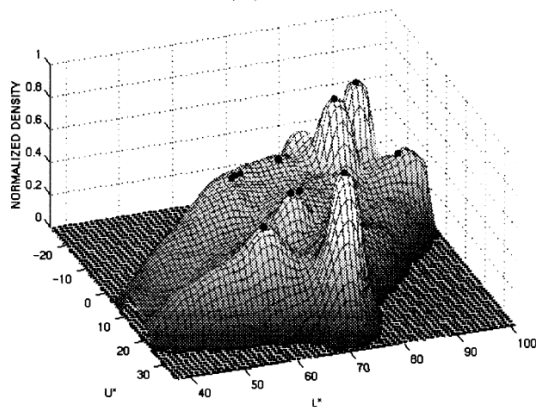
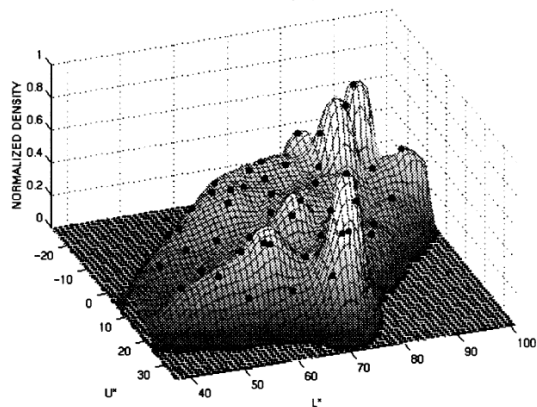
1. Convert the image into tokens (via color, gradients, texture measures etc).
2. Choose initial search window locations uniformly in the data.
3. Compute the mean shift window location for each initial position.
4. Merge windows that end up on the same “peak” or mode.
5. The data these merged windows traversed are clustered together.



(a)



(b)



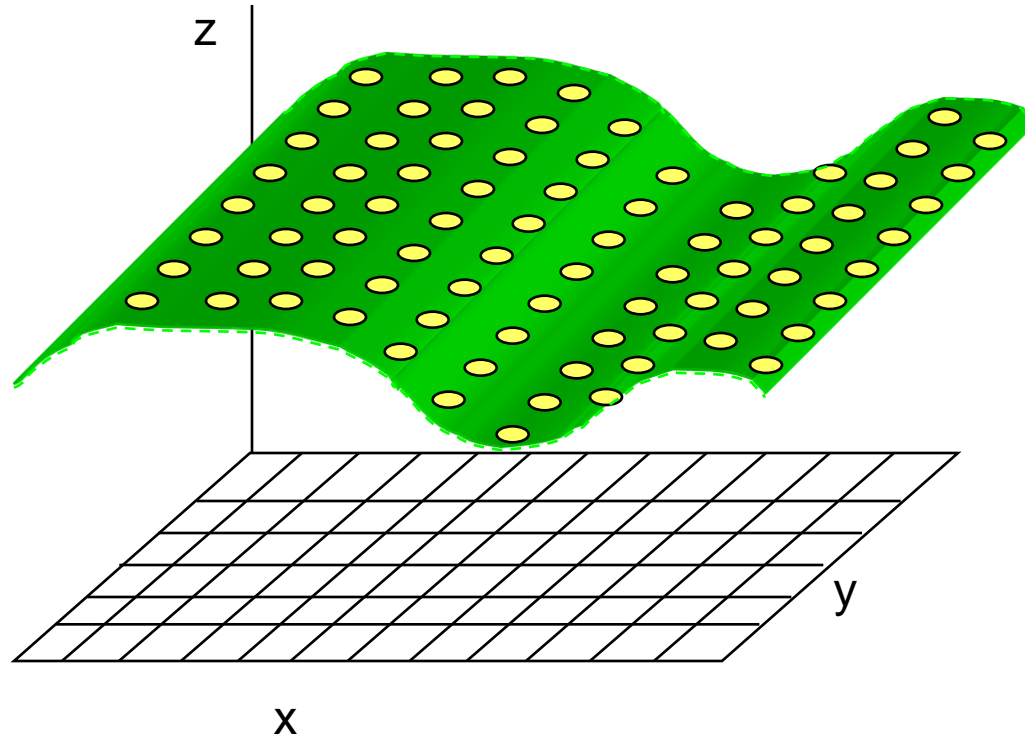
Discontinuity Preserving Smoothing

Feature space : Joint domain = spatial coordinates + color space

$$K(\mathbf{x}) = C \cdot k_s \left(\left\| \frac{\mathbf{x}^s}{h_s} \right\| \right) \cdot k_r \left(\left\| \frac{\mathbf{x}^r}{h_r} \right\| \right)$$

Meaning : treat the image as data points in the spatial and range (value) domain

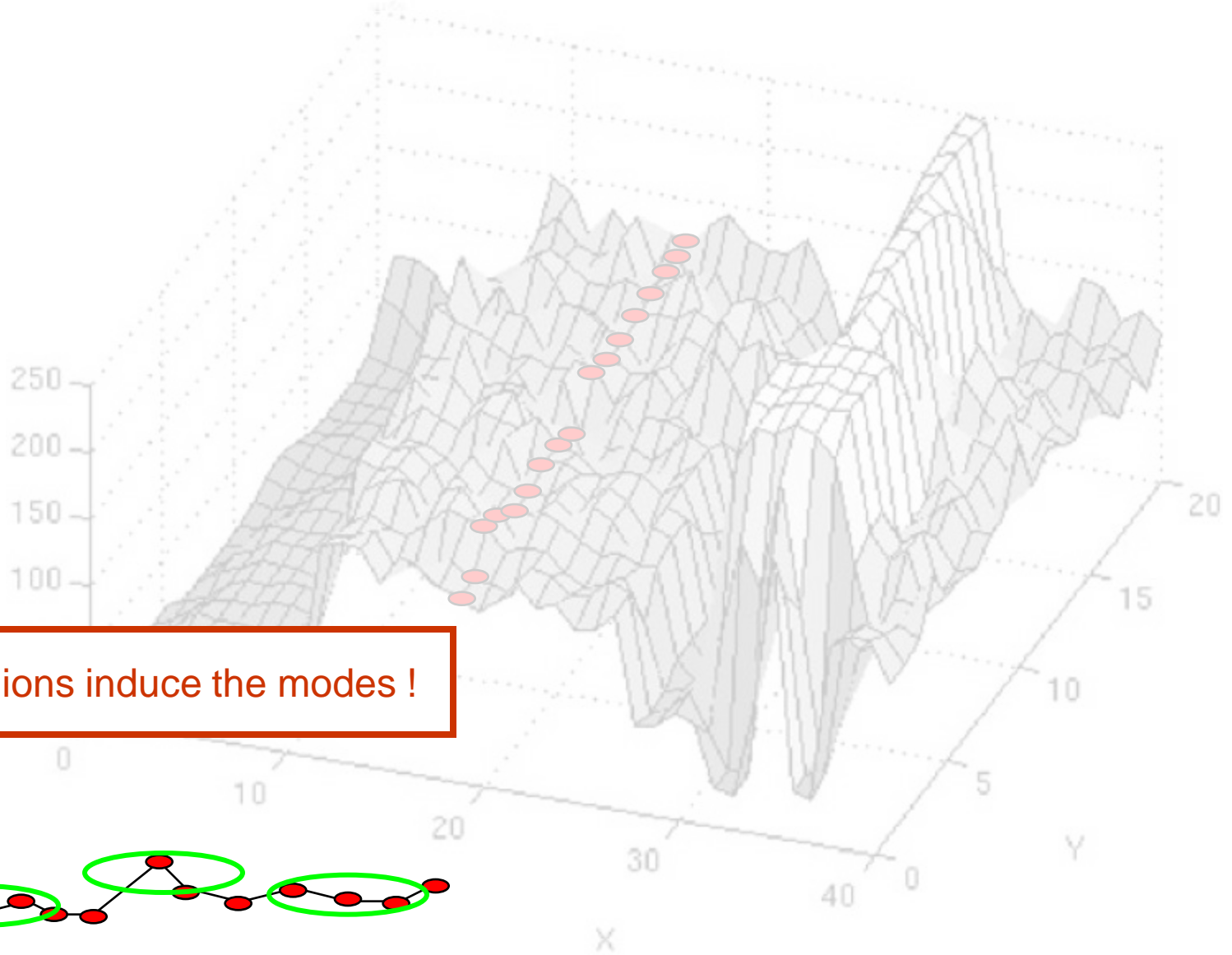
Discontinuity Preserving Smoothing



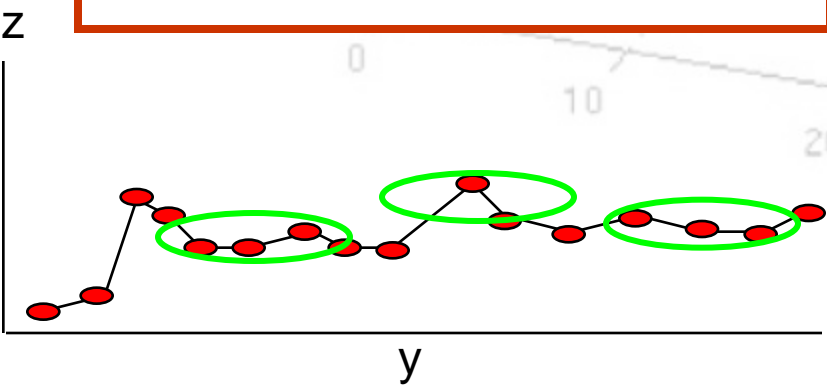
The image gray levels...

... can be viewed as data points
in the x, y, z space (joined spatial
And color space)

Discontinuity Preserving Smoothing



Flat regions induce the modes !



Discontinuity Preserving Smoothing

The effect of window size in spatial and range spaces



Original



$(h_s, h_r) = (8, 8)$



$(h_s, h_r) = (8, 16)$



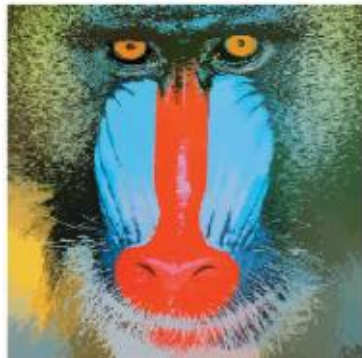
$(h_s, h_r) = (16, 4)$



$(h_s, h_r) = (16, 8)$



$(h_s, h_r) = (16, 16)$



$(h_s, h_r) = (32, 4)$



$(h_s, h_r) = (32, 8)$



$(h_s, h_r) = (32, 16)$

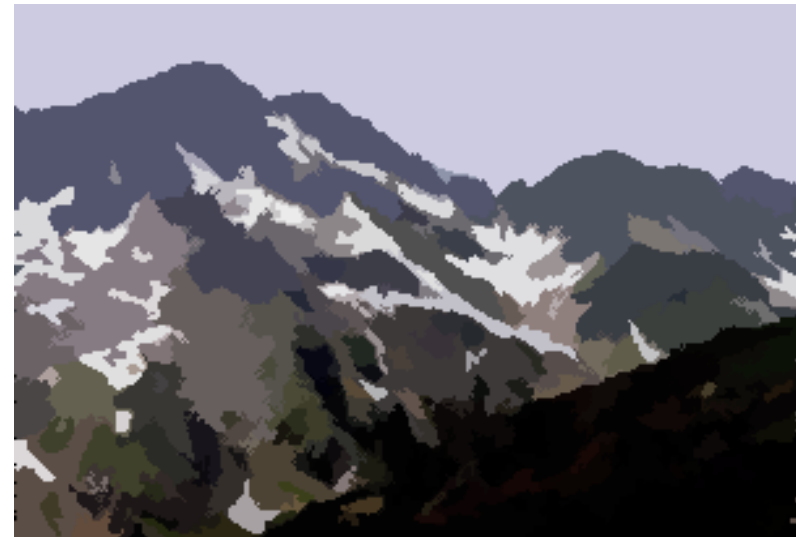
Segmentation

Segment = Cluster, or Cluster of Clusters

Algorithm:

- Run Filtering (*discontinuity preserving smoothing*)
- Cluster the clusters which are closer than window size

Mean Shift Segmentation Results:











K-means



Mean Shift



Normalized
Cut



Max
Entropy
Threshold

Questions?