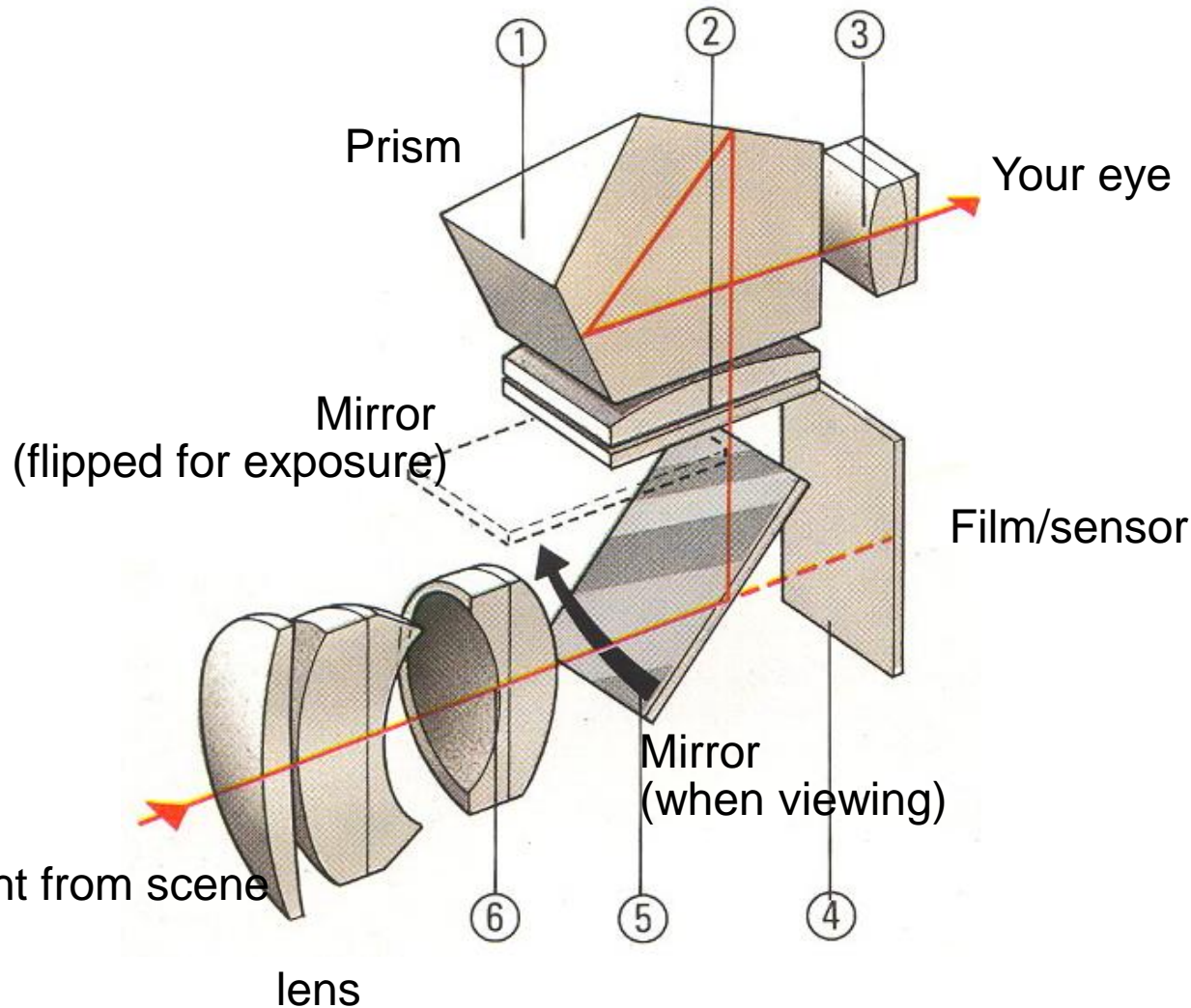
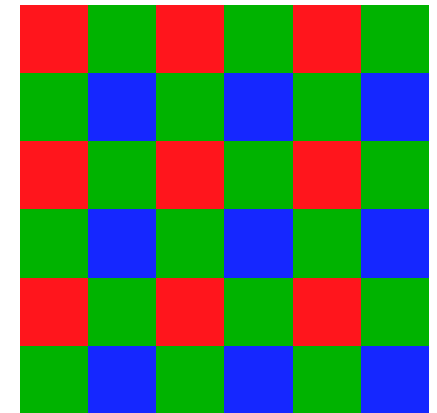


# Last Lecture

---

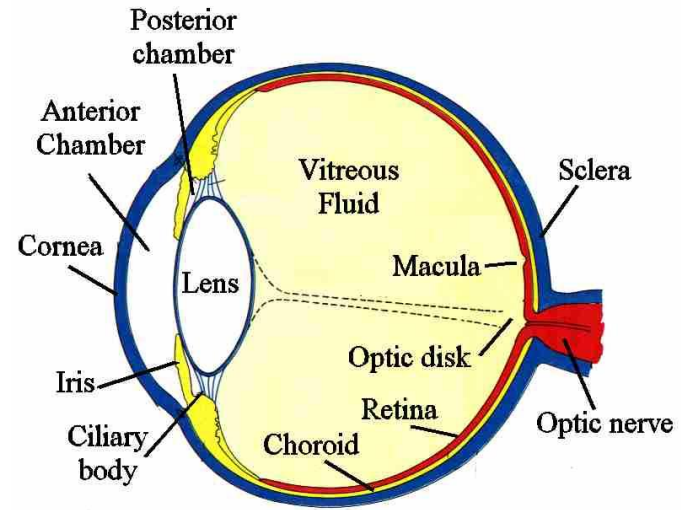


- Focal Length
- F-stop
- Depth of Field
- Color Capture

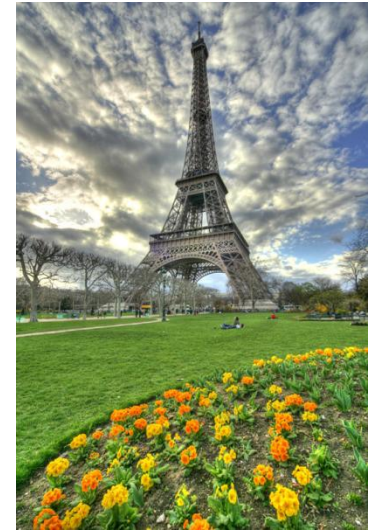
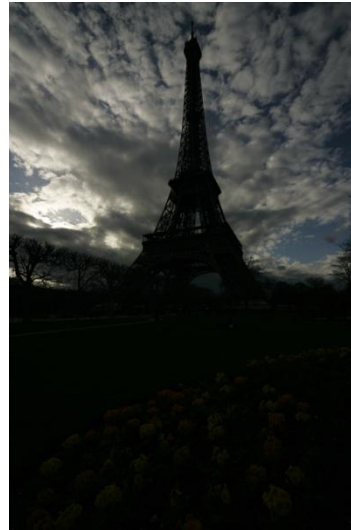
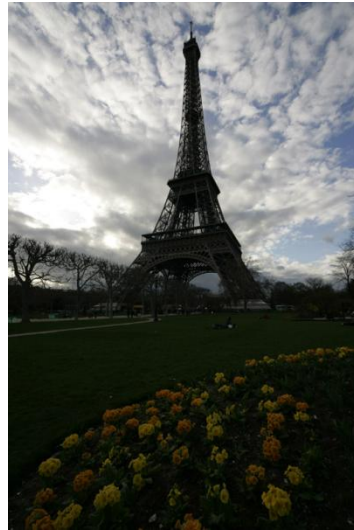


Bayer pattern

# Today

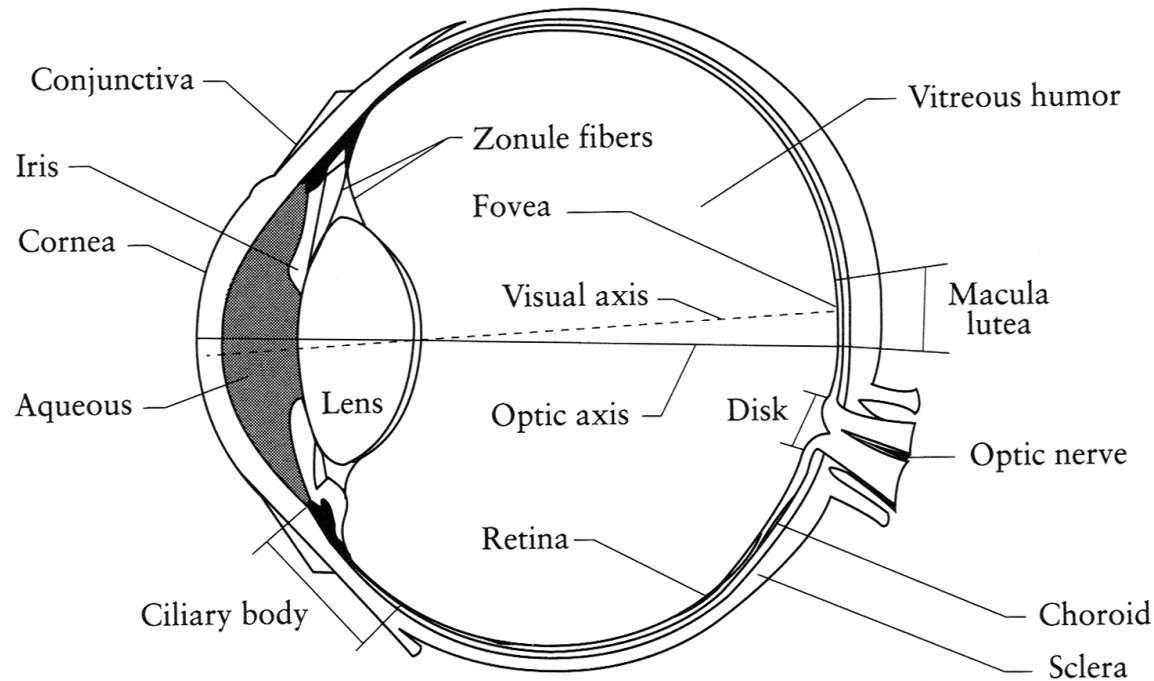


Kimber, D.C.; C.E. Gray, and C.E. Stackpole. (1966).  
*Anatomy and Physiology*. MacMillan Co., NY. pg.335.



# The Eye

---



- The human eye is a camera!
  - **Iris** - colored annulus with radial muscles
  - **Pupil** - the hole (aperture) whose size is controlled by the iris
  - What's the “film”?
- photoreceptor cells (rods and cones) in the **retina**

# Two types of light-sensitive receptors

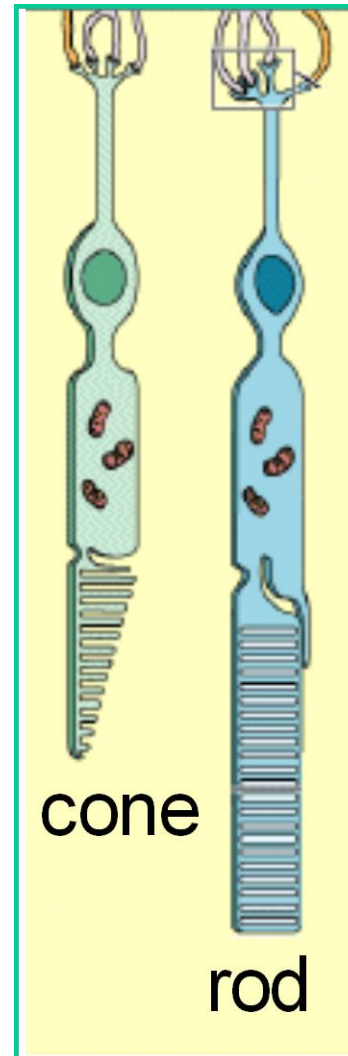
---

## Rods

rod-shaped  
highly sensitive  
operate at night  
gray-scale vision

## Cones

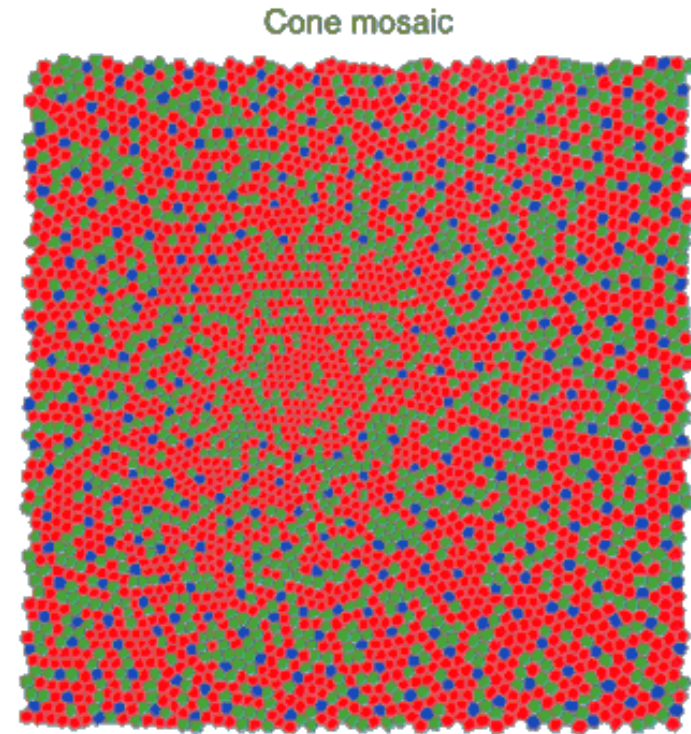
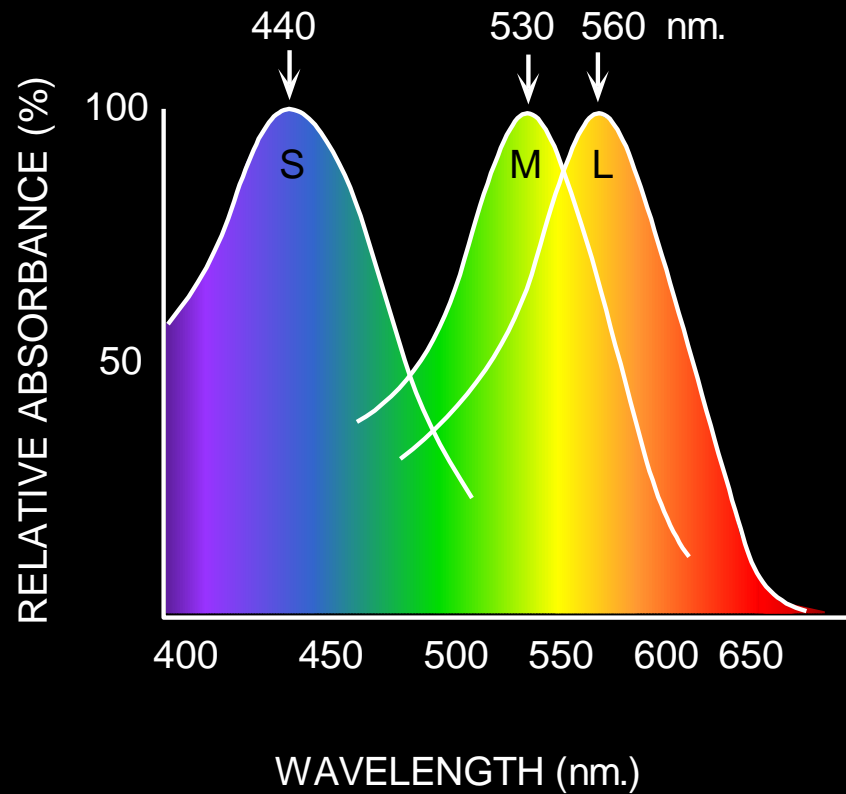
cone-shaped  
less sensitive  
operate in high light  
color vision



# Physiology of Color Vision

---

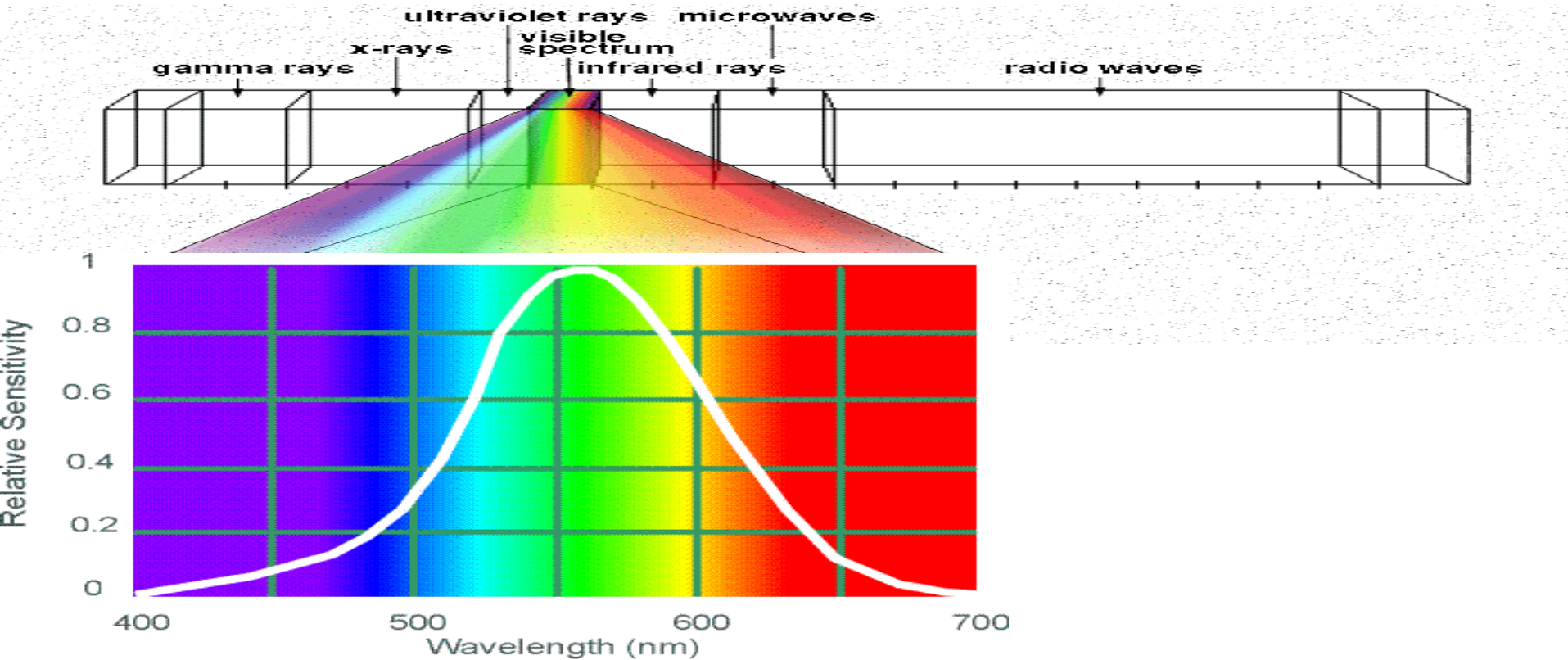
Three kinds of cones:





# Electromagnetic Spectrum

---



Human Luminance Sensitivity Function

# References

---

- <http://www.howstuffworks.com/digital-camera.htm>
- <http://electronics.howstuffworks.com/autofocus.htm>
- Ramanath, Snyder, Bilbro, and Sander. [Demosaicking Methods for Bayer Color Arrays](#), Journal of Electronic Imaging, 11(3), pp306-315.
- Rajeev Ramanath, Wesley E. Snyder, Youngjun Yoo, Mark S. Drew, [Color Image Processing Pipeline in Digital Still Cameras](#), IEEE Signal Processing Magazine Special Issue on Color Image Processing, vol. 22, no. 1, pp. 34-43, 2005.
- <http://www.worldatwar.org/photos/whitebalance/index.mhtml>
- <http://www.100fps.com/>

# The World in an Eye

Ko Nishino    Shree K. Nayar

Columbia University

[http://www1.cs.columbia.edu/CAVE/projects/world\\_eye/world\\_eye.php](http://www1.cs.columbia.edu/CAVE/projects/world_eye/world_eye.php)

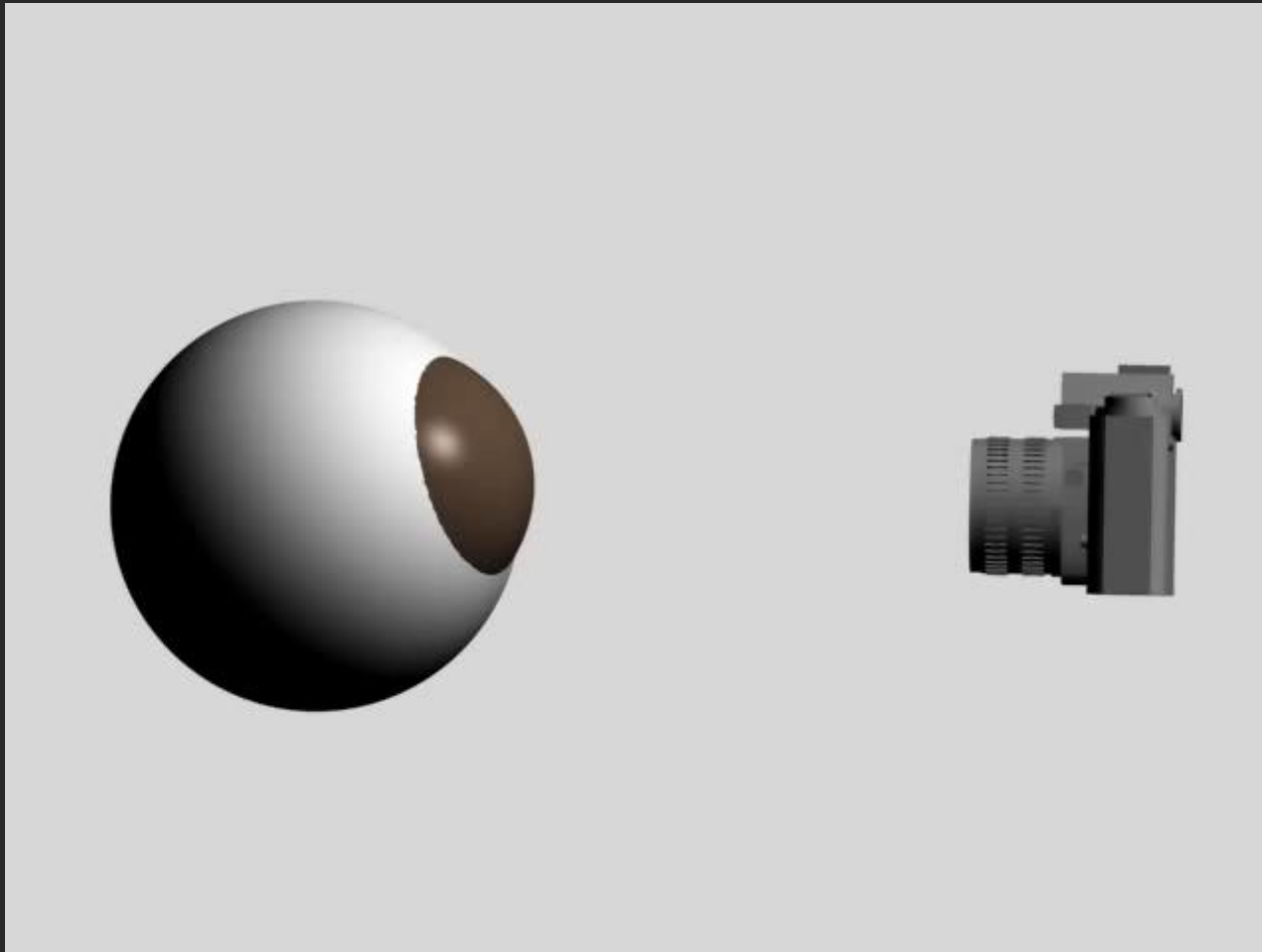
IEEE CVPR Conference  
June 2004, Washington DC, USA

Supported by NSF

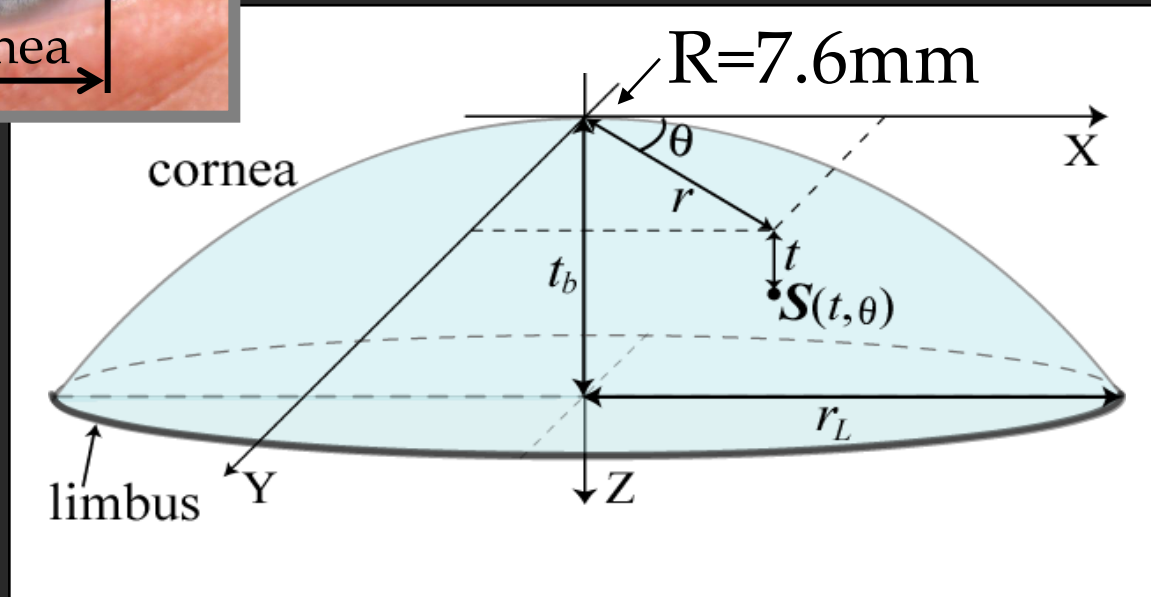
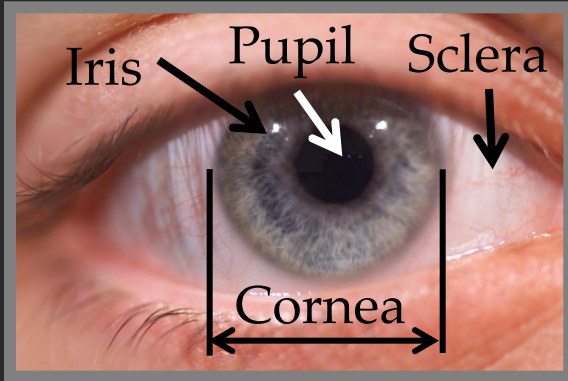




# Corneal Imaging System

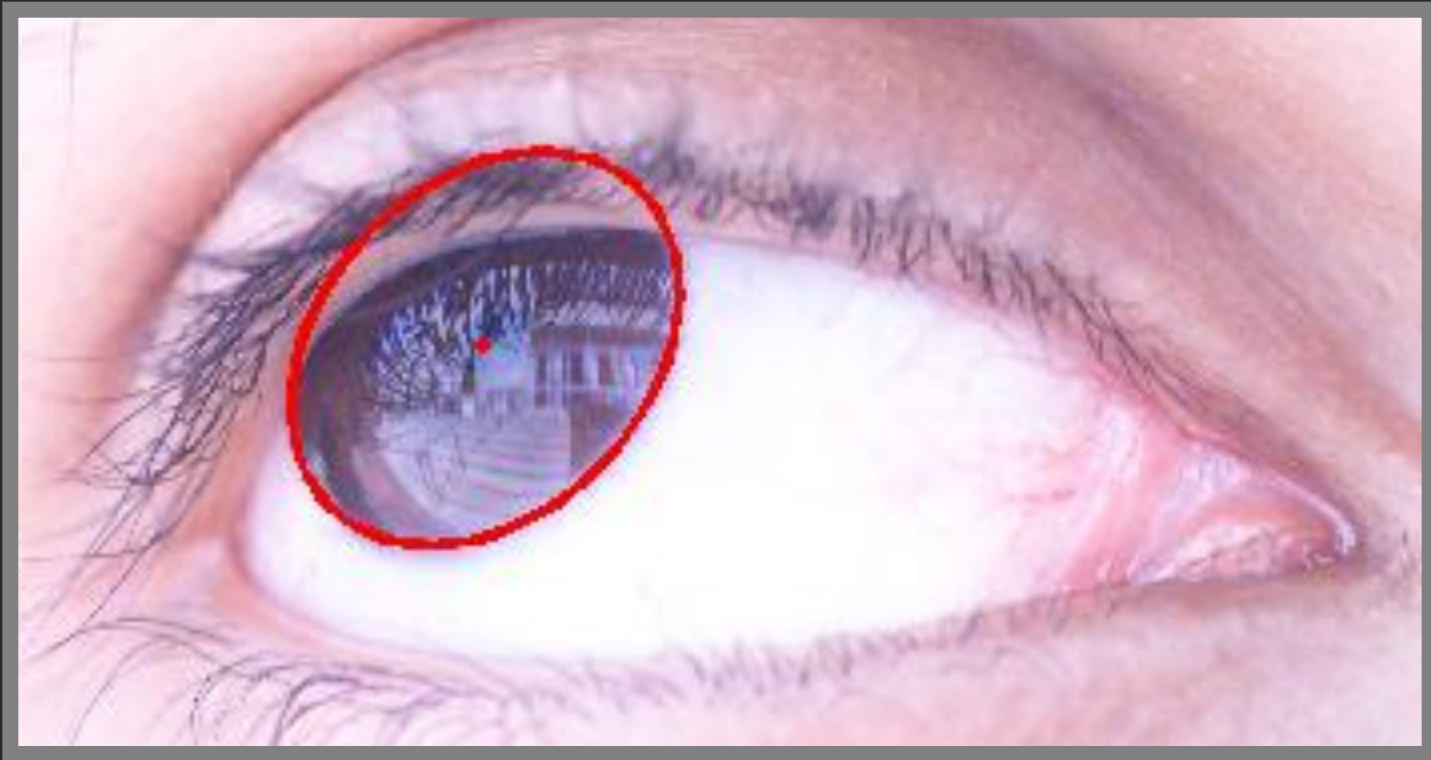


# Geometric Model of the Cornea



$$t_b = 2.18\text{mm} \quad r_L = 5.5\text{mm}$$

# Finding the Limbus



limbus parameters  $e$  : radii  $(r_x, r_y)$  center  $(c_x, c_y)$  tilt  $\theta$

$$\max_e \left| \underbrace{g_\sigma \otimes \begin{bmatrix} x \\ y \end{bmatrix}}_{\text{Gaussian}} * \underbrace{\frac{\partial}{\partial r_x} \oint_e I(x, y) ds}_{\text{intensity value}} + g_\sigma \otimes \begin{bmatrix} x \\ y \end{bmatrix} * \frac{\partial}{\partial r_y} \oint_e I(x, y) ds \right|$$



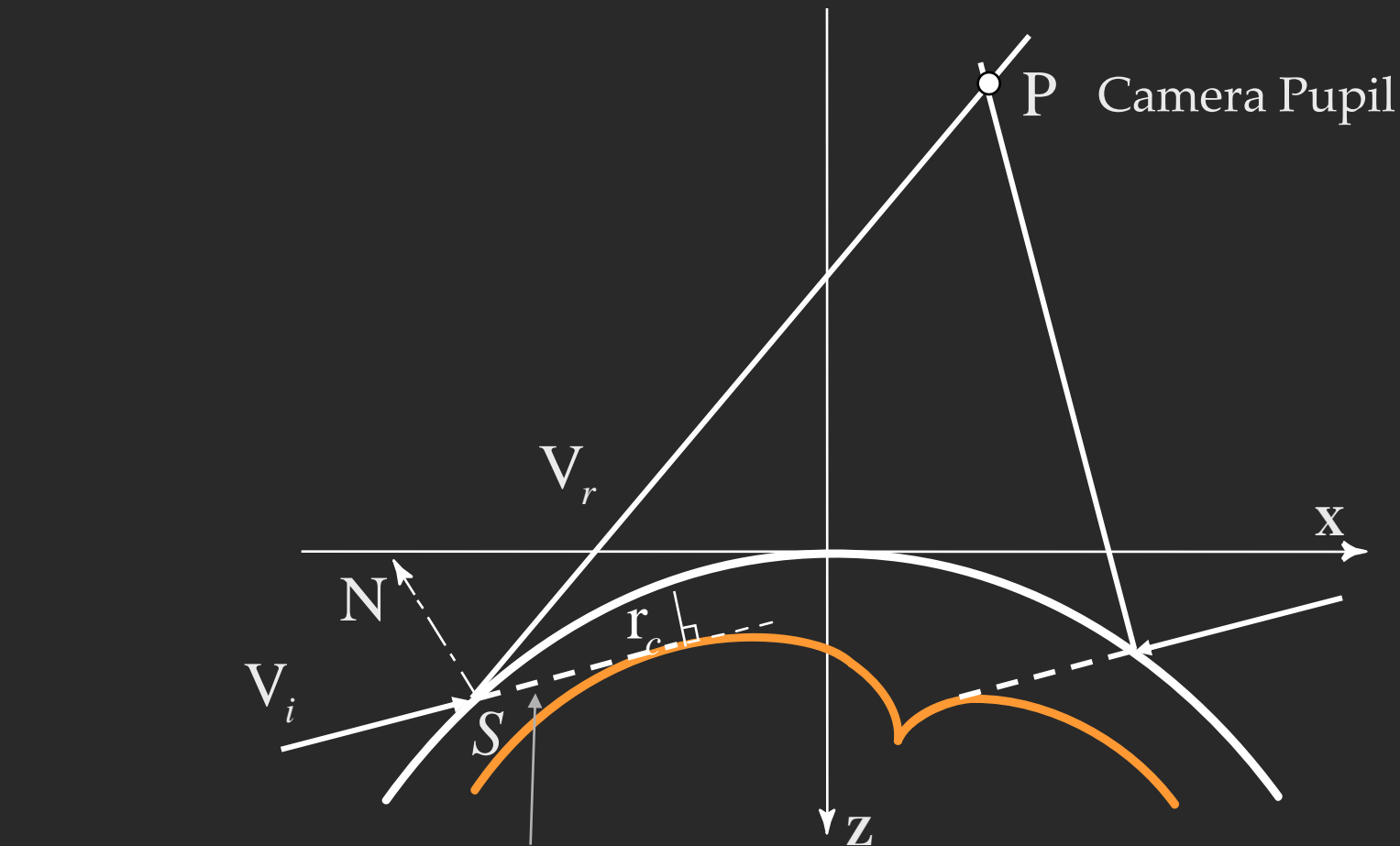


Self-calibration:  
3D Coordinates, 3D Orientation

How does the World Appear in an Eye?



# Imaging Characteristics: Viewpoint Locus

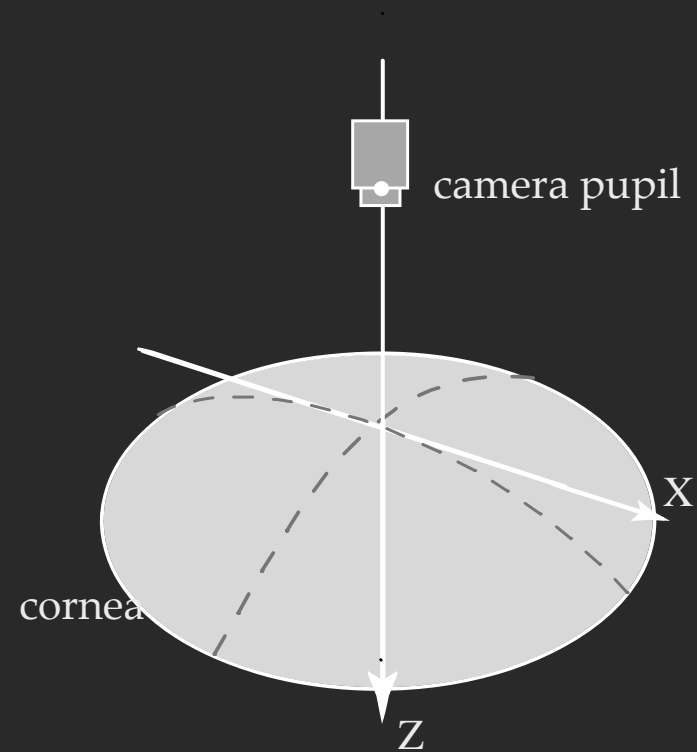


$$\mathbf{V}(\theta, r) = \mathbf{S}(\theta) + r\mathbf{V}_i(\theta)$$

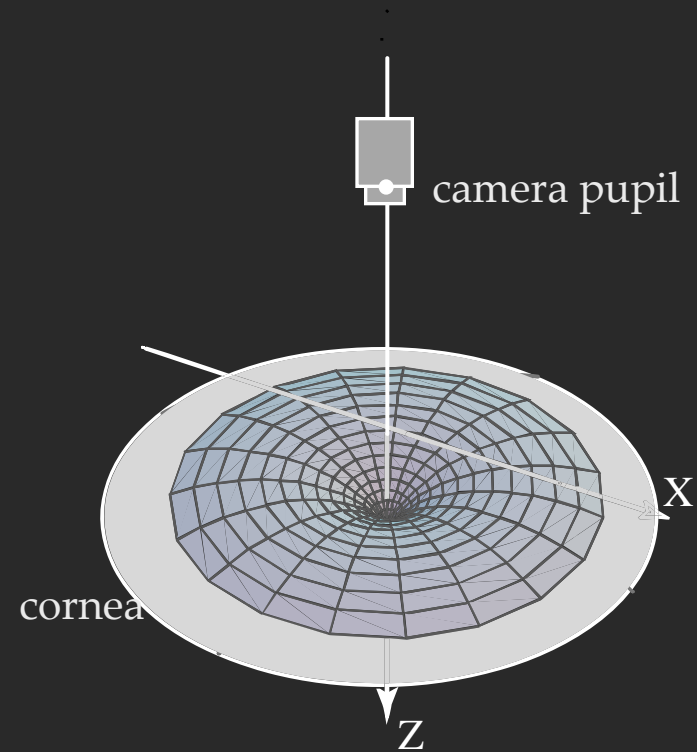
$$\det J \mathbf{V}(\theta, r_c) = 0$$

[Burkhard and Shealy 73; Swaminathan et al. 01]

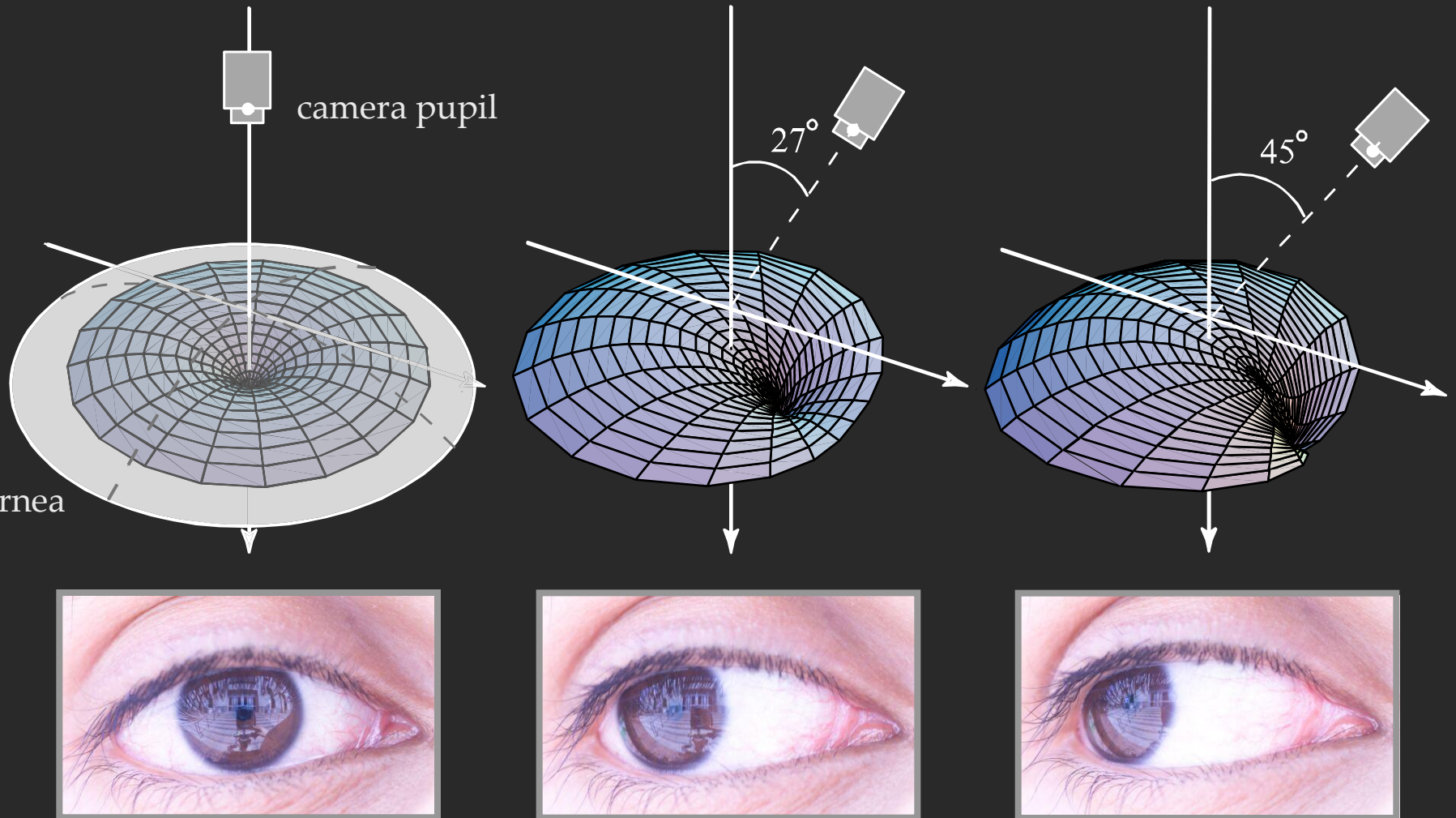
# Viewpoint Loci



# Viewpoint Loci



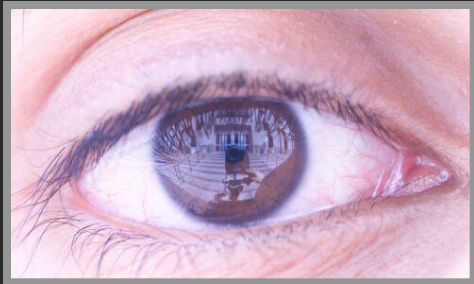
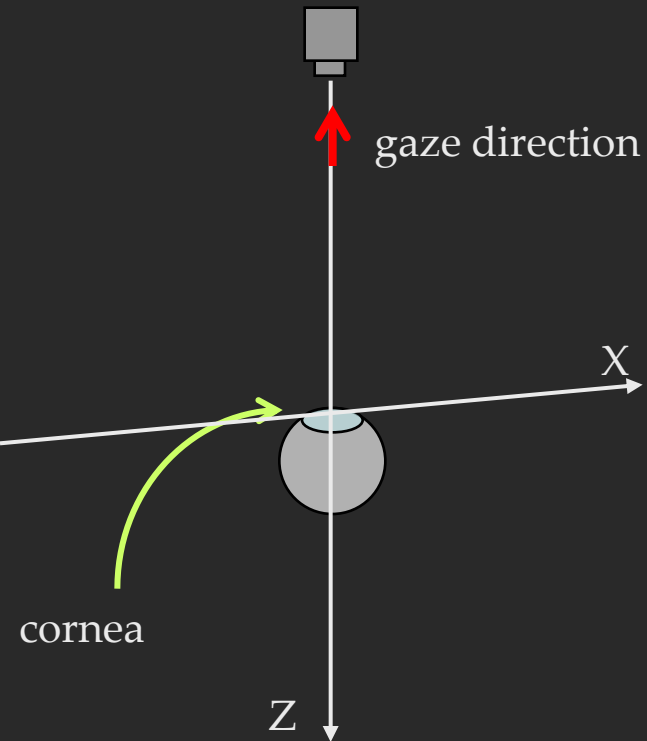
# Viewpoint Loci



A diagram illustrating the geometry of a camera pupil. A large circle represents the pupil, with a central point labeled "camera pupil". A light source, labeled  $V^i$ , is shown outside the pupil. A line connects the light source to the camera pupil. A dashed line represents the light path from the light source to the pupil. The diagram also shows a coordinate system with axes and a curved line representing the pupil's boundary.

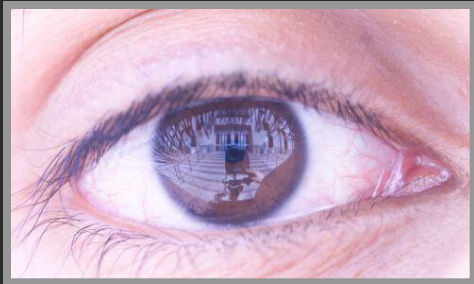
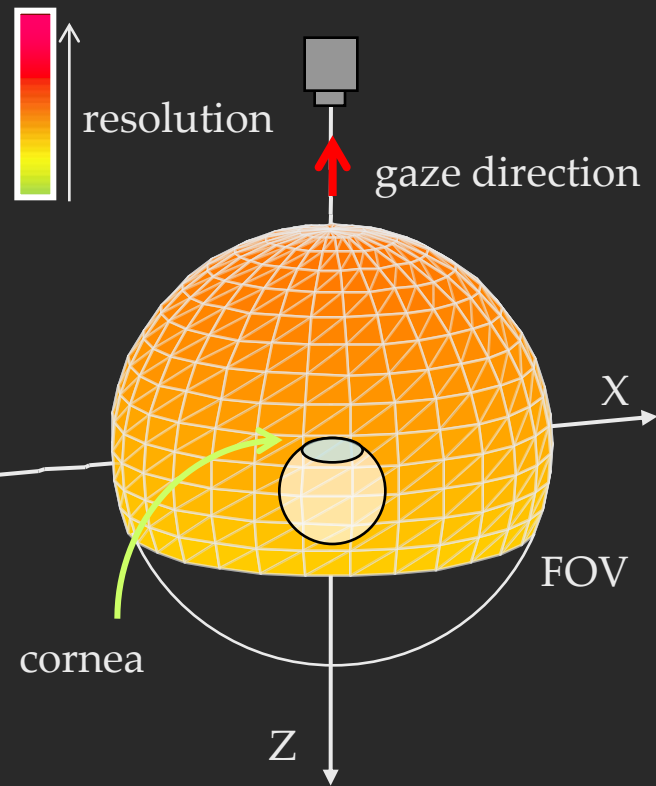
$$\text{FOV} = \int_0^{2\pi} \underbrace{\left( \underbrace{V_z^i}_{\text{closed loop of limbus}} \underbrace{(\theta)}_{\text{incident light rays}} + 1 \right) d\theta}$$

# Resolution and Field of View

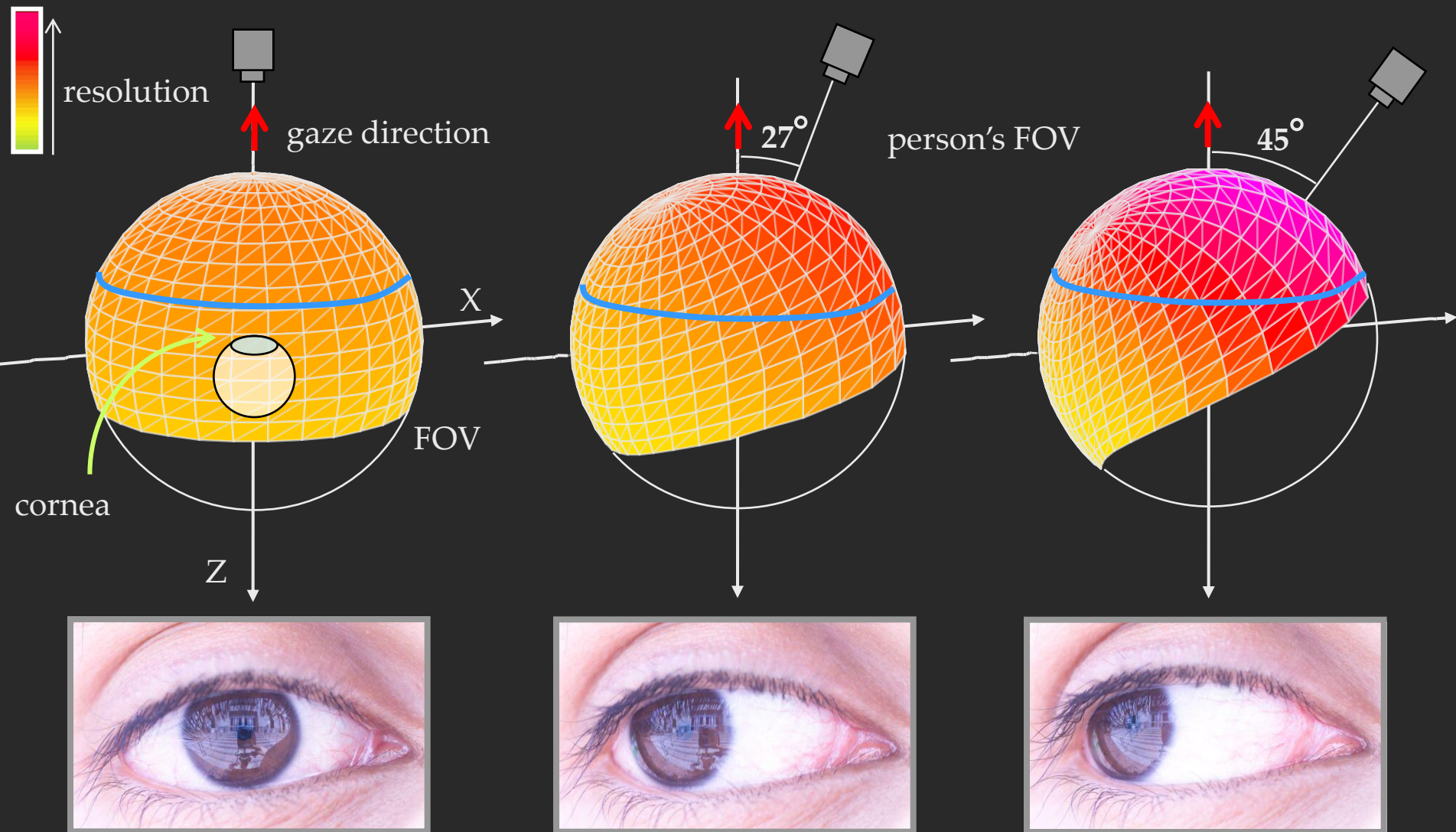




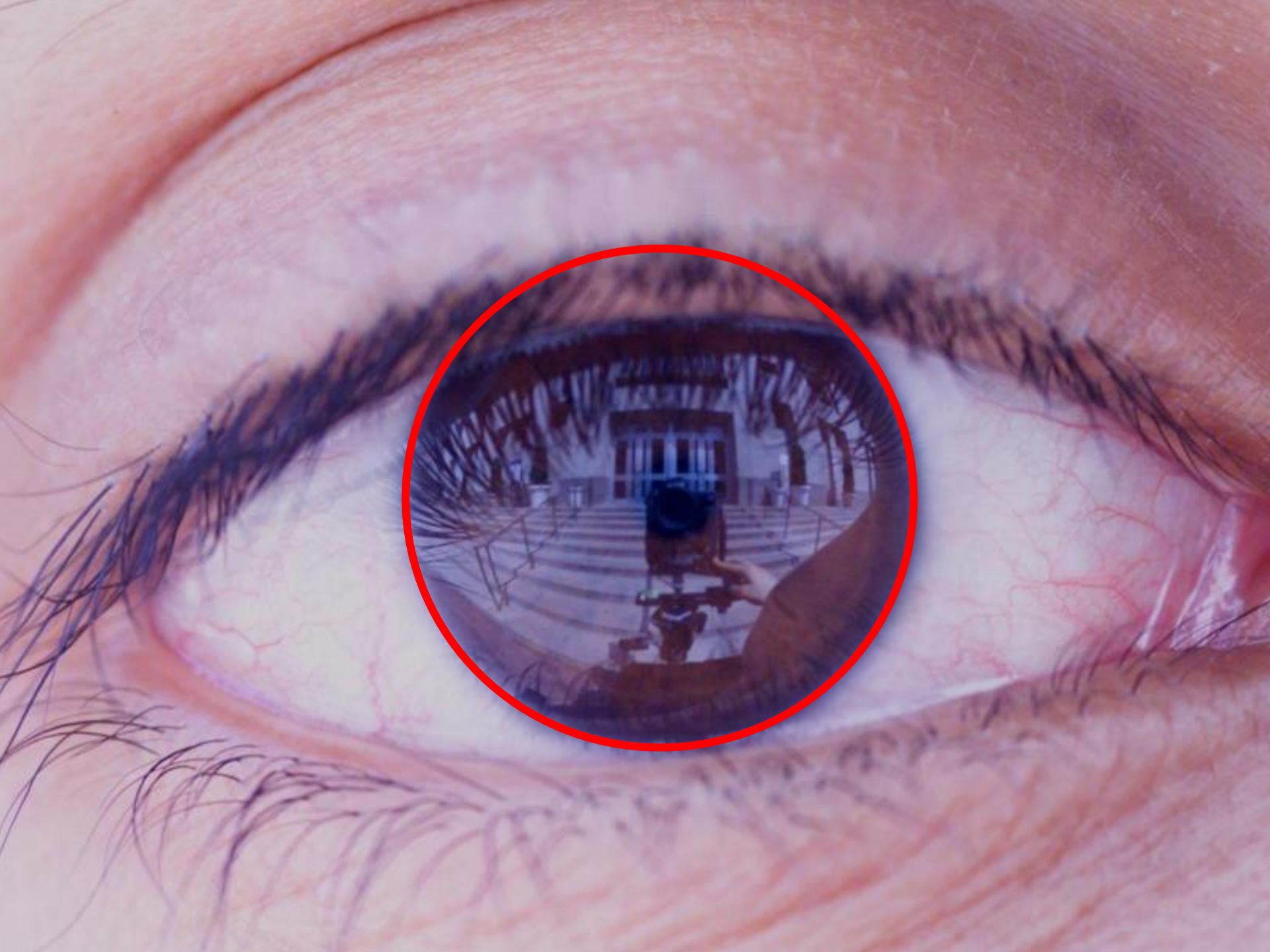
# Resolution and Field of View



# Resolution and Field of View



What does the Eye Reveal?



# Environment Map from an Eye



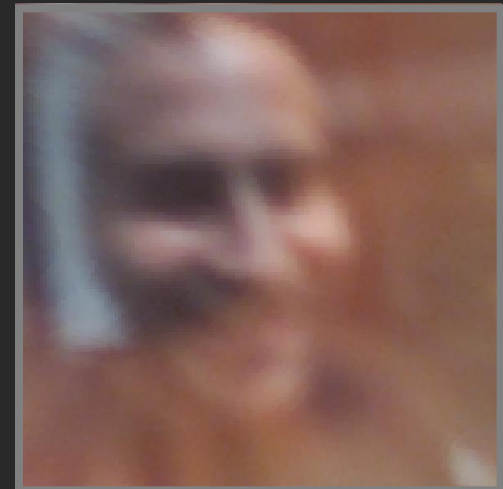


# What Exactly You are Looking At

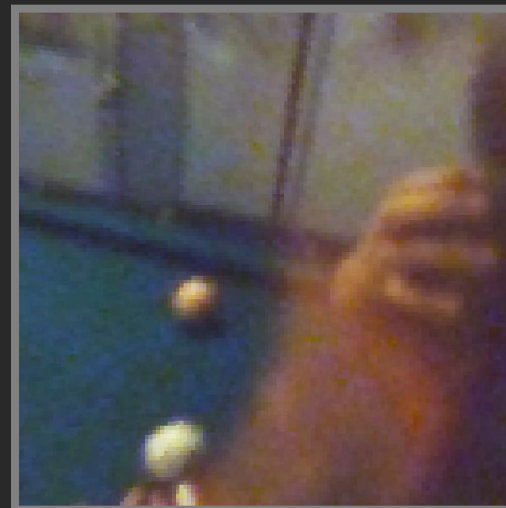
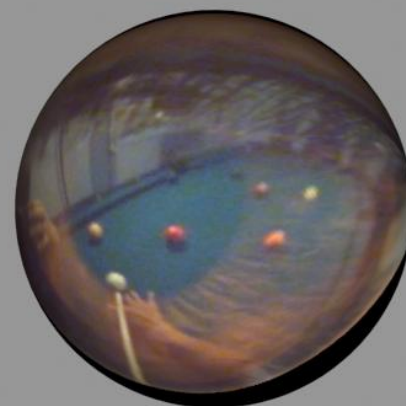
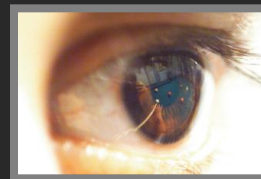
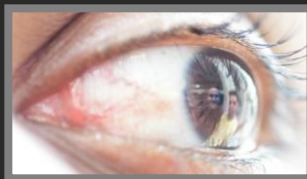
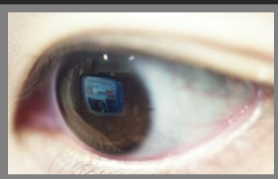
Eye Image:



Computed Retinal Image:

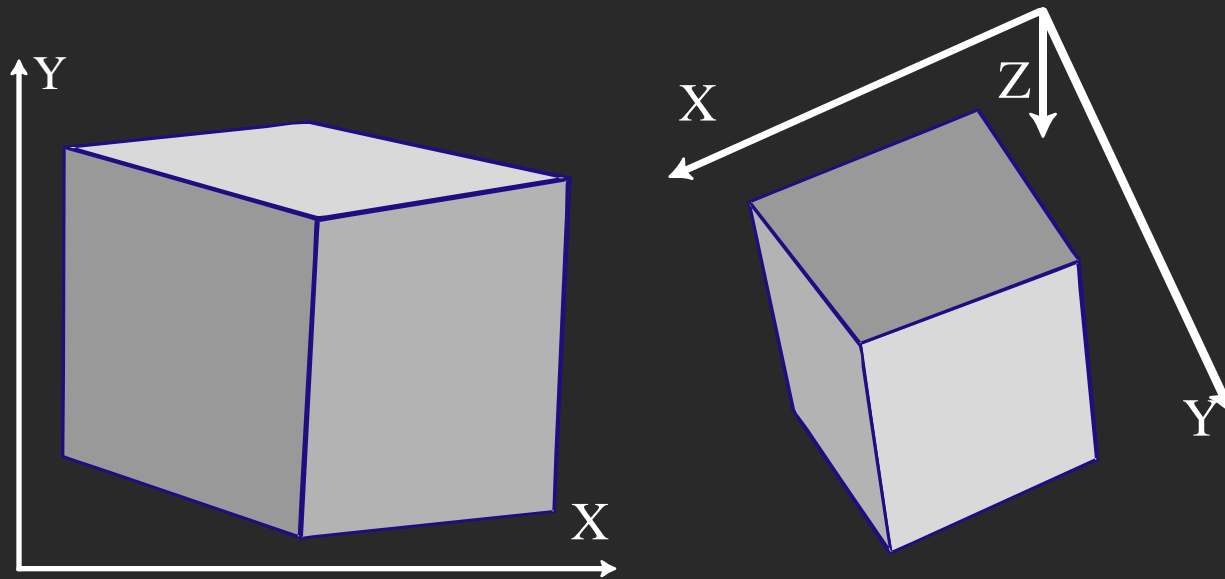






# Watching a Bus

# From Two Eyes in an Image ...



Reconstructed Structure (frontal and side view)

# Eyes Reveal ...

- Where the person is
- What the person is looking at
- The structure of objects

# Implications

Human Affect Studies: Social Networks

Security: Human Localization

Advanced Interfaces: Robots, Computers

Computer Graphics: Relighting [SIGGRAPH 04]

Questions?

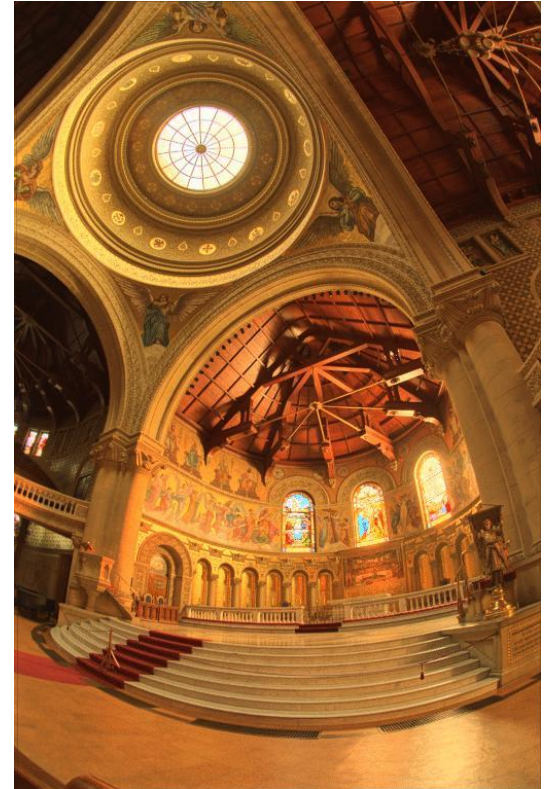


# What do we see?

---

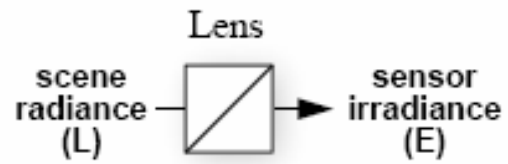


Vs.



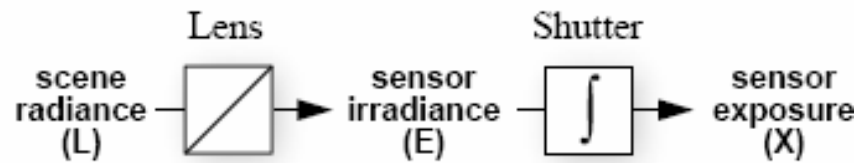
# Camera pipeline

---



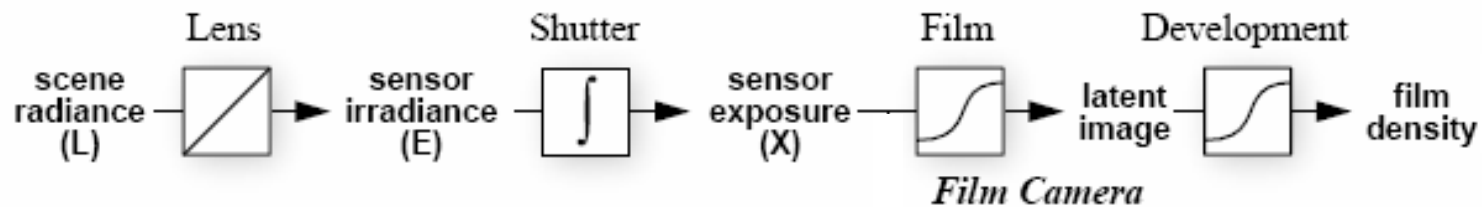
# Camera pipeline

---



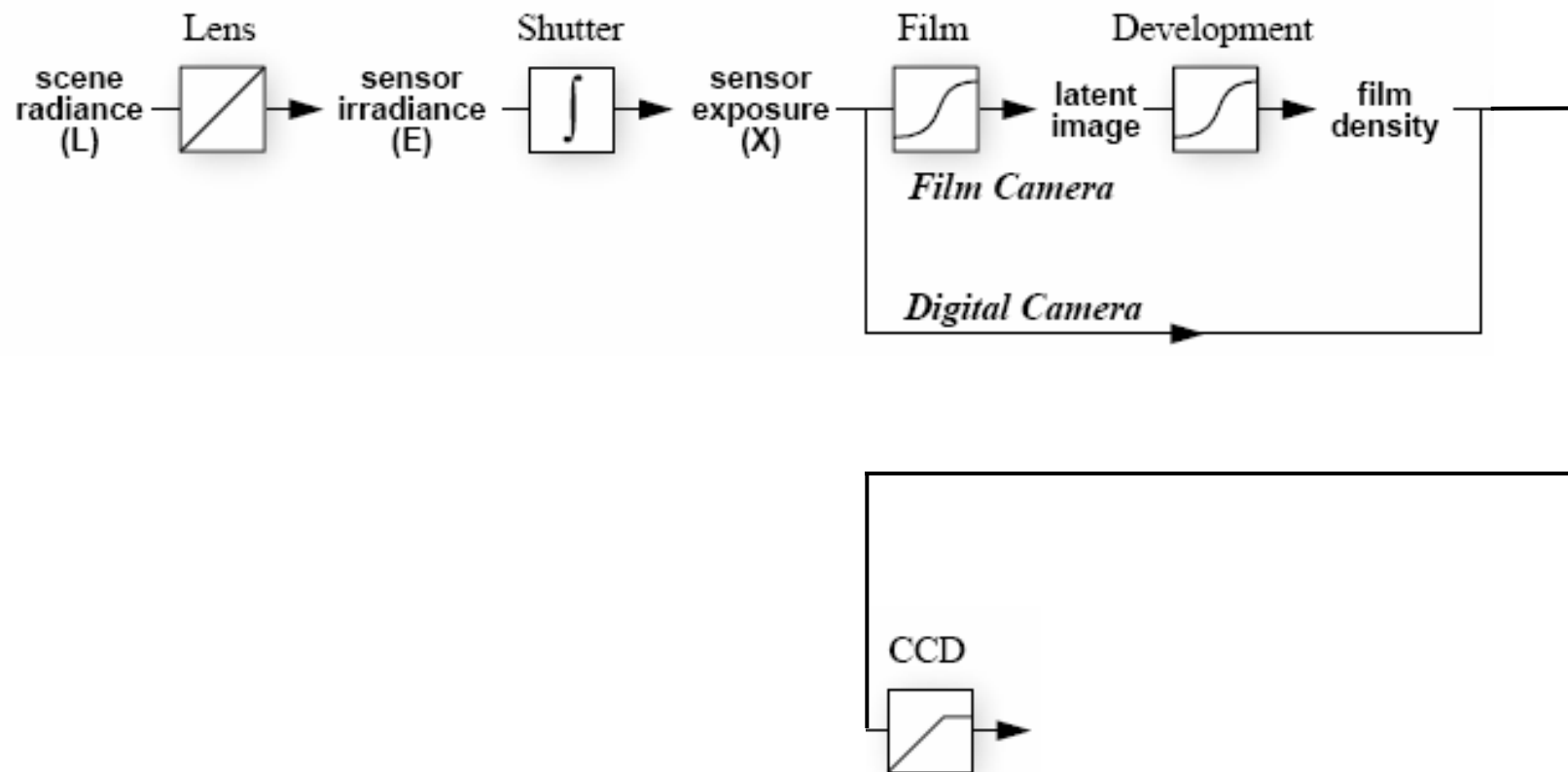
# Camera pipeline

---



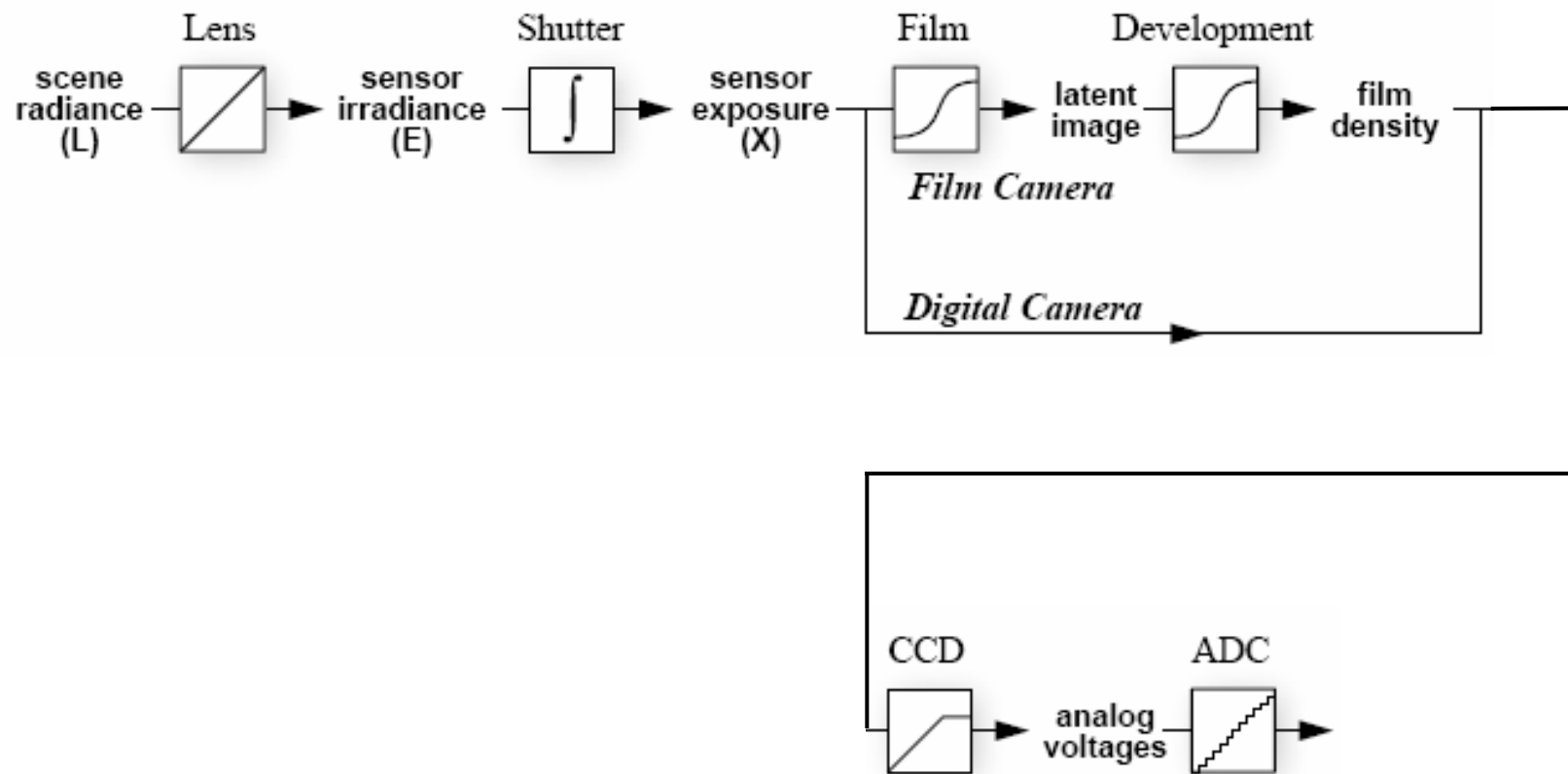
# Camera pipeline

---



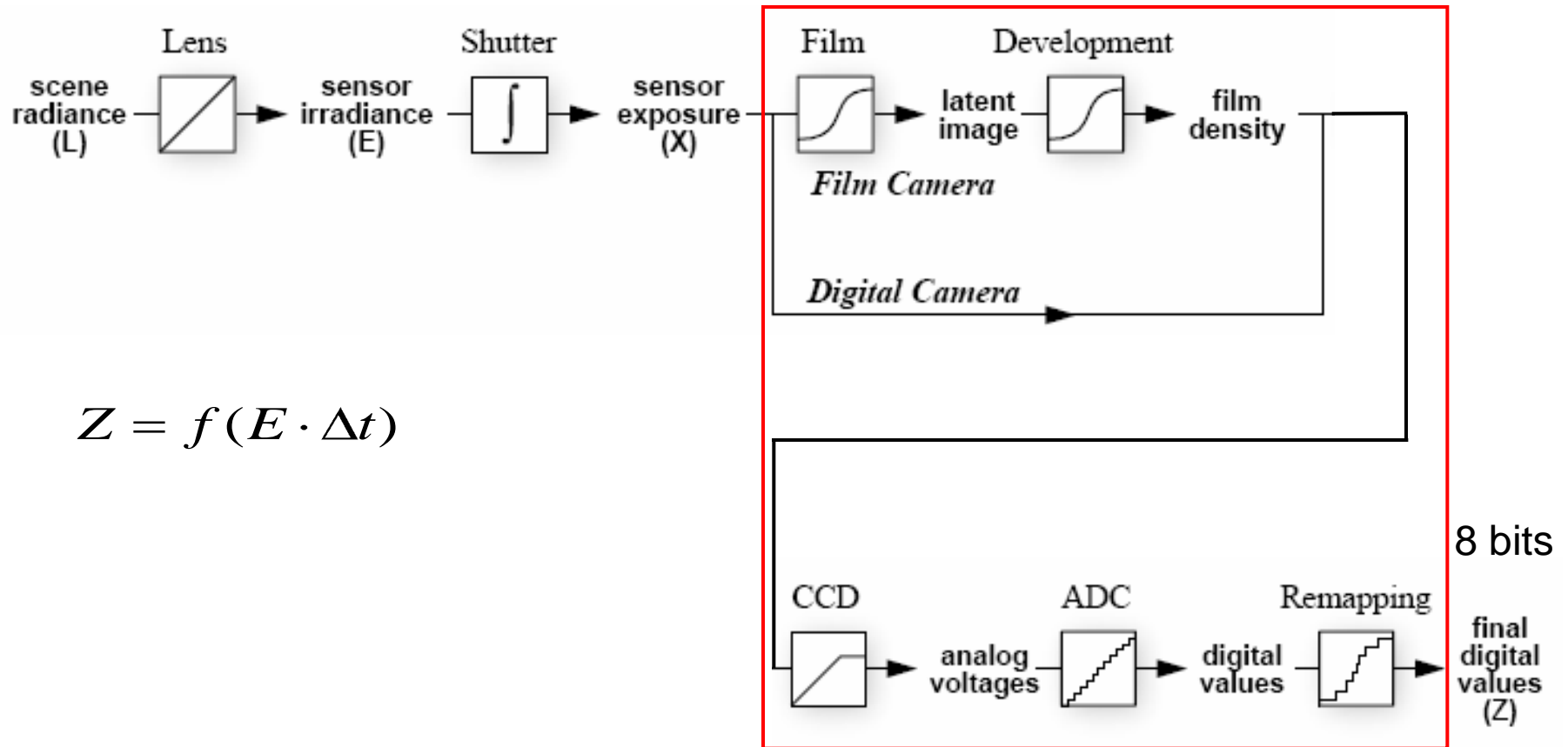
# Camera pipeline

---



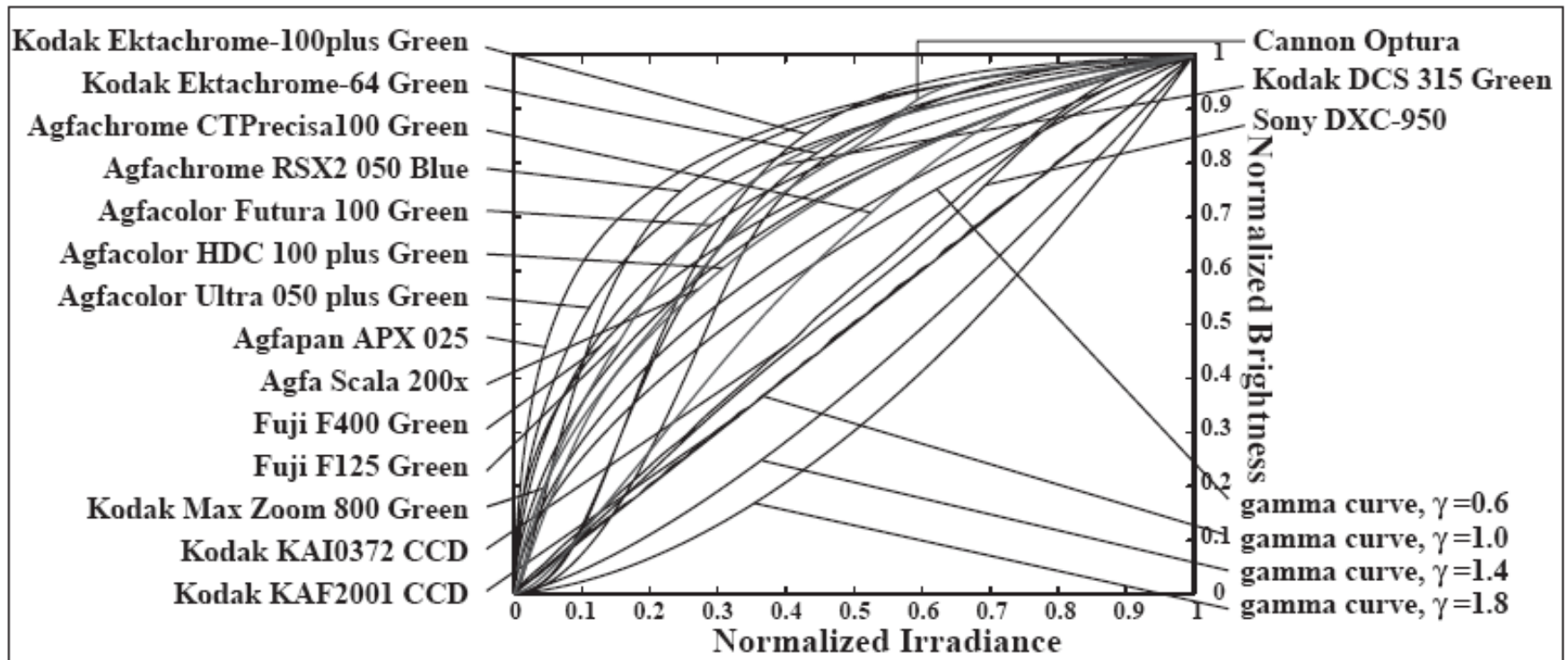


# Camera pipeline



# Real-world response functions

In general, the response function is not provided by camera makers who consider it part of their proprietary product differentiation. In addition, they are beyond the standard gamma curves.



# Camera is not a photometer

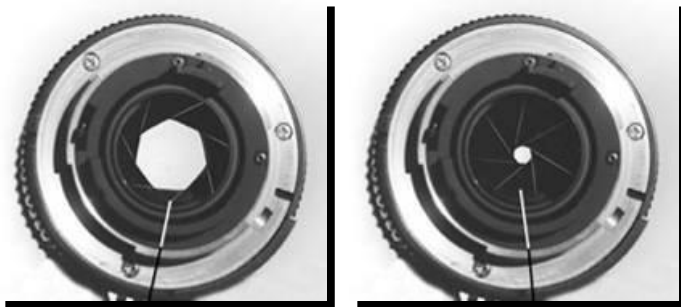
---

- Limited dynamic range
  - ⇒ Perhaps use multiple exposures?
- Unknown, nonlinear response
  - ⇒ Not possible to convert pixel values to radiance
- Solution:
  - Recover response curve from multiple exposures, then reconstruct the ***radiance map***

# Varying exposure

---

- Ways to change exposure
  - Shutter speed
  - Aperture
  - Neutral density filters



# Shutter speed

---

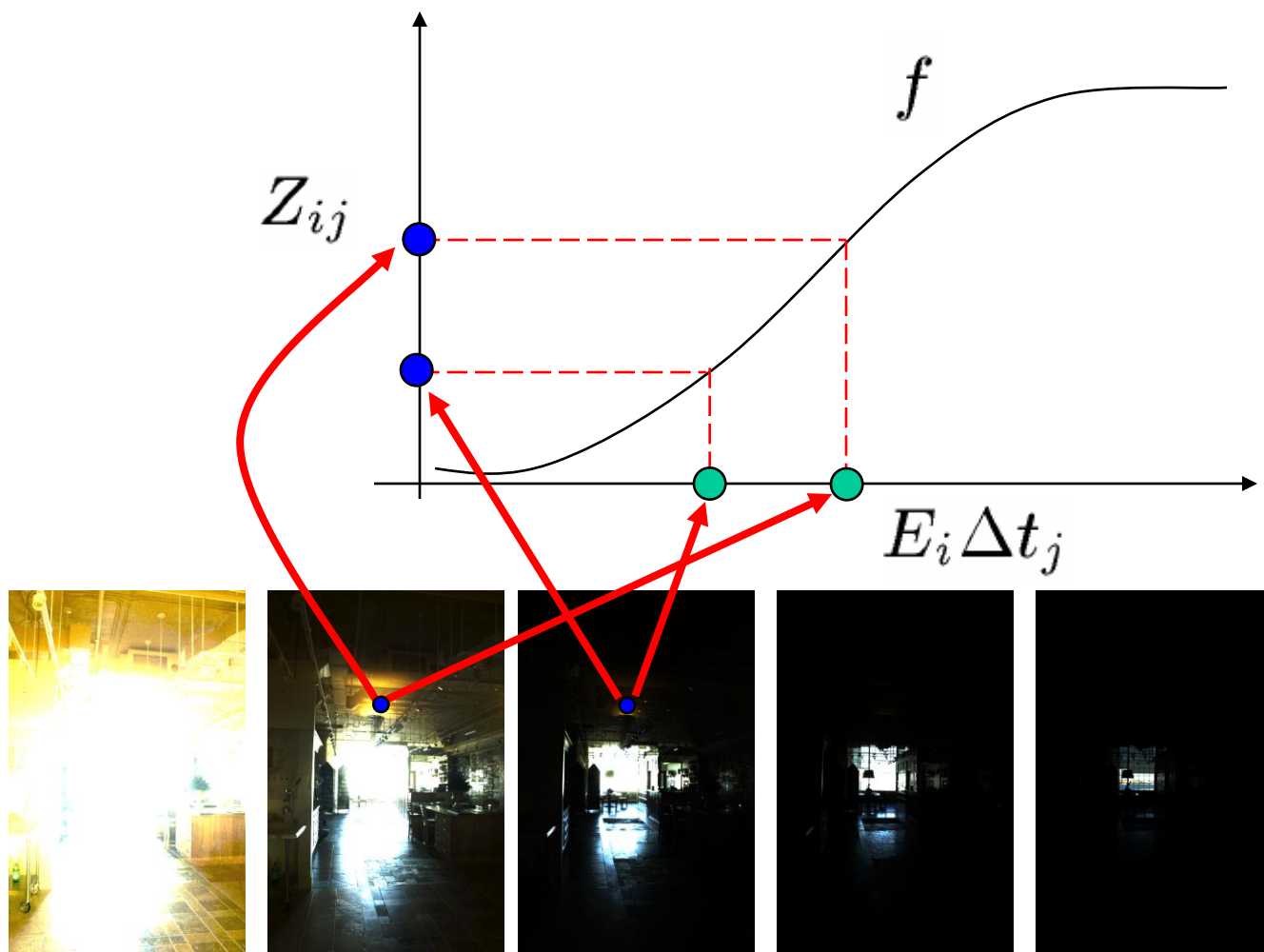
- Note: shutter times usually obey a power series – each “stop” is a factor of 2
- $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{1}{15}$ ,  $\frac{1}{30}$ ,  $\frac{1}{60}$ ,  $\frac{1}{125}$ ,  $\frac{1}{250}$ ,  $\frac{1}{500}$ ,  $\frac{1}{1000}$  sec

Usually really is:

$\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{1}{16}$ ,  $\frac{1}{32}$ ,  $\frac{1}{64}$ ,  $\frac{1}{128}$ ,  $\frac{1}{256}$ ,  $\frac{1}{512}$ ,  $\frac{1}{1024}$  sec

# HDRI capturing from multiple exposures

- We want to obtain the response curve

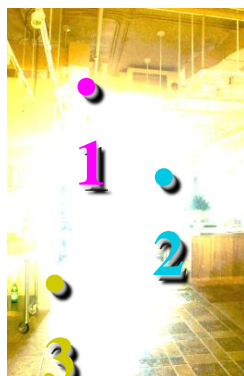




# HDRI capturing from multiple exposures

---

## Image series



$\Delta t =$   
2 sec



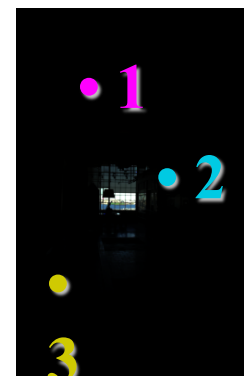
$\Delta t =$   
1 sec



$\Delta t =$   
1/2 sec



$\Delta t =$   
1/4 sec



$\Delta t =$   
1/8 sec

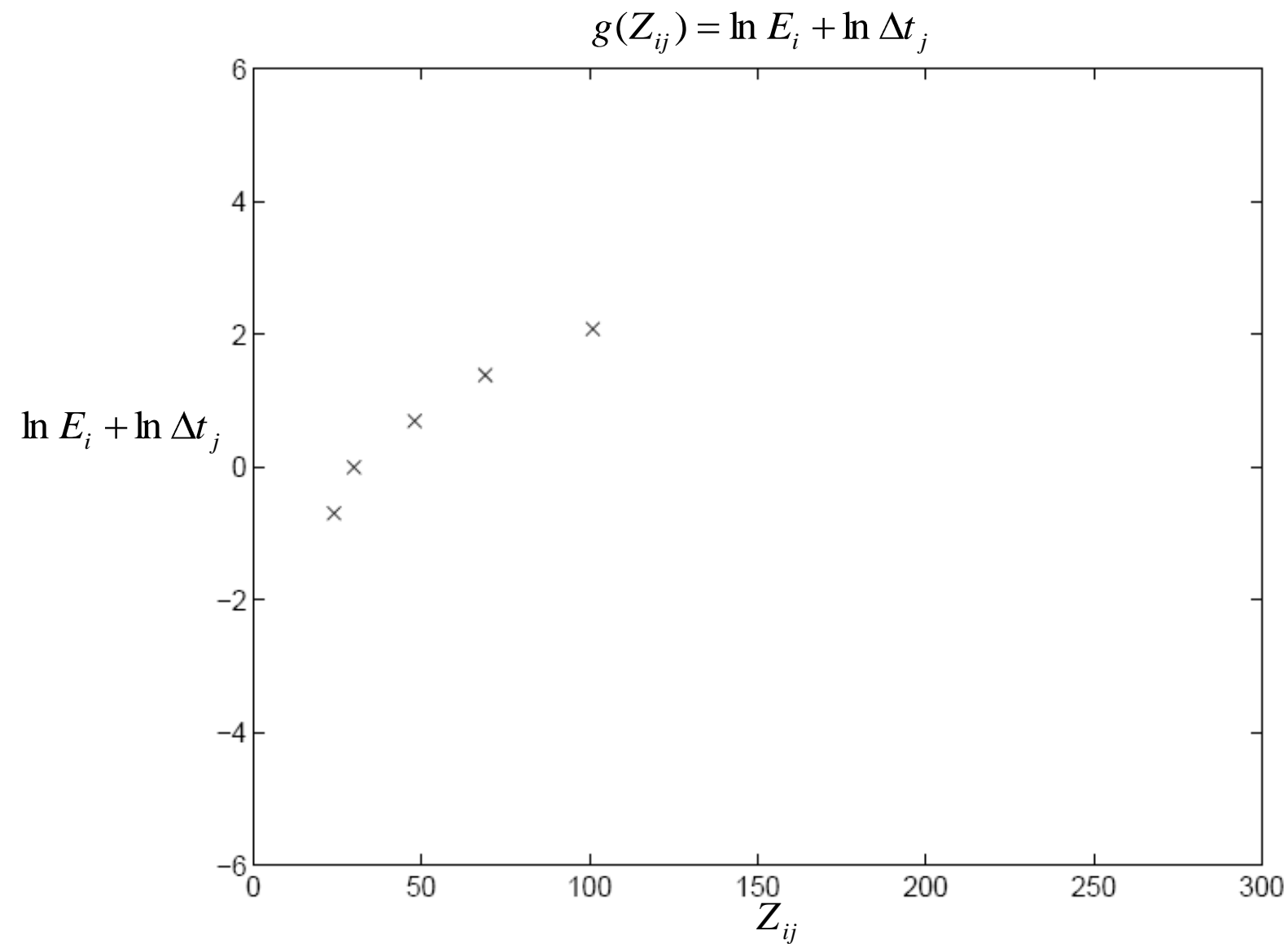
$$Z_{ij} = f(E_i \Delta t_j)$$

$$f^{-1}(Z_{ij}) = E_i \Delta t_j$$

$$g(Z_{ij}) = \ln E_i + \ln \Delta t_j, \text{ where } g = \ln f^{-1}$$

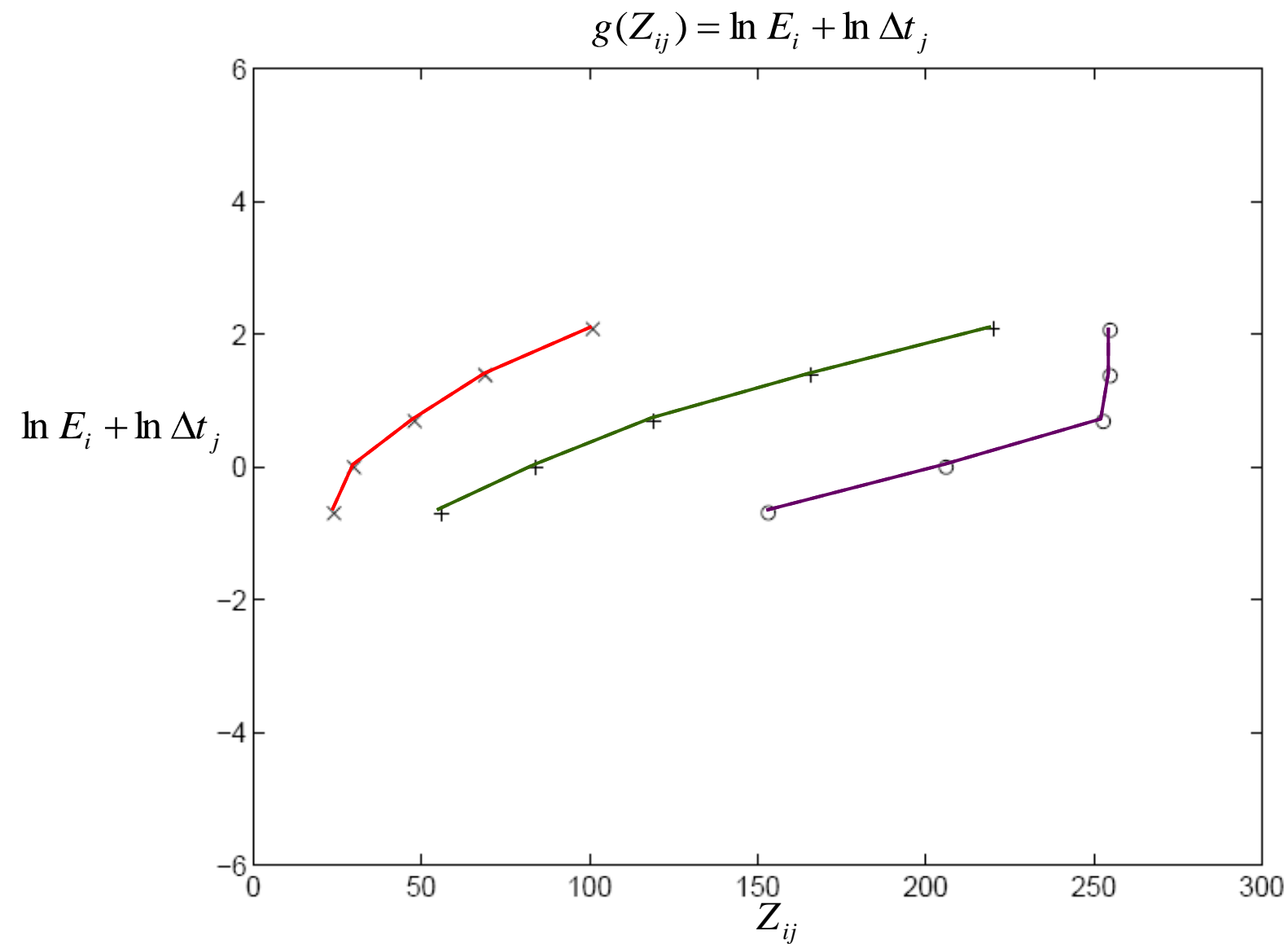
# Idea behind the math

---



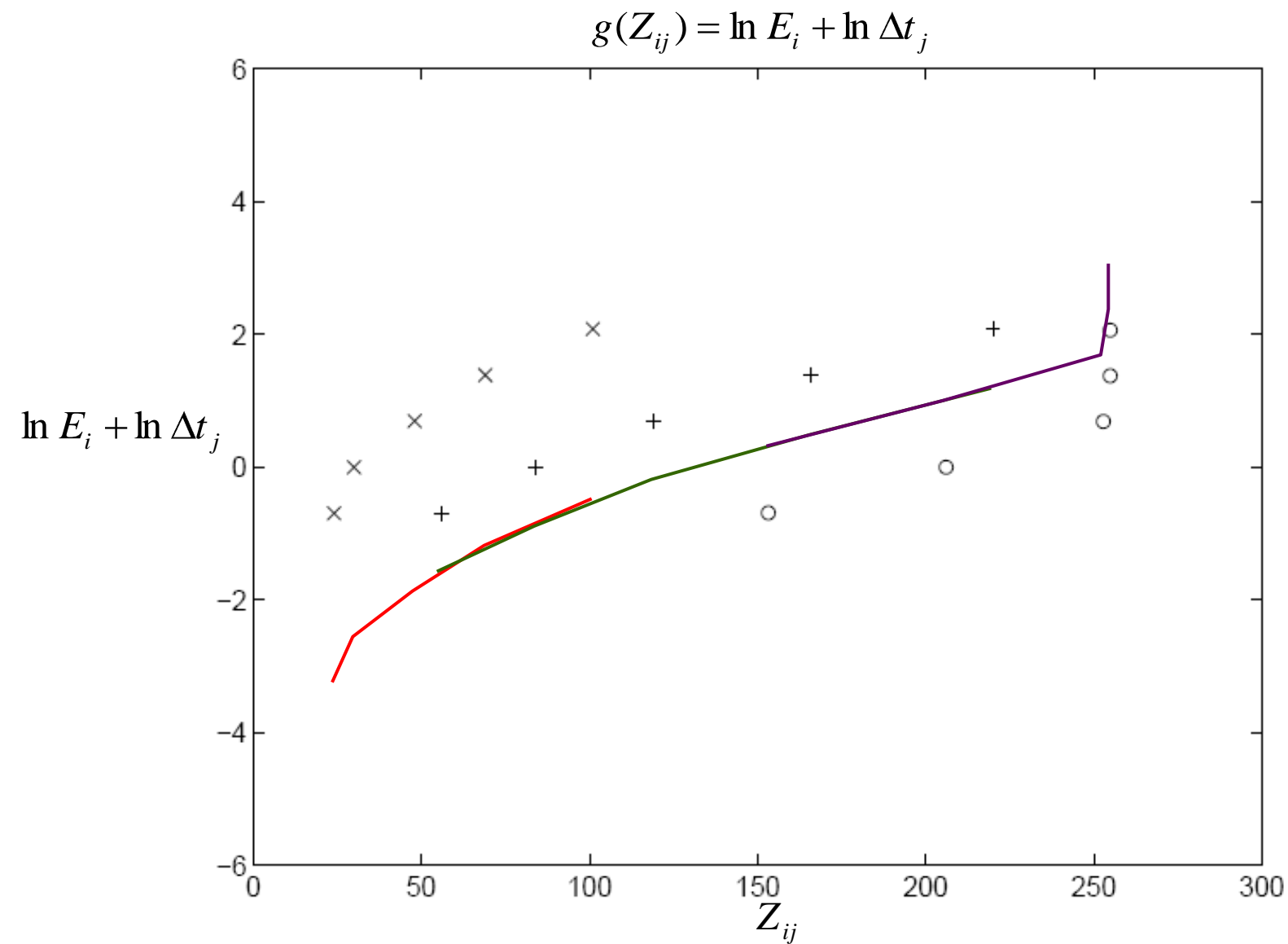
# Idea behind the math

---



# Idea behind the math

---



# Math for recovering response curve

---

$$Z_{ij} = f(E_i \Delta t_j)$$

$f$  is monotonic, it is invertible

$$\ln f^{-1}(Z_{ij}) = \ln E_i + \ln \Delta t_j$$

let us define function  $g = \ln f^{-1}$

$$g(Z_{ij}) = \ln E_i + \ln \Delta t_j$$

minimize the following

$$\mathcal{O} = \sum_{i=1}^N \sum_{j=1}^P [g(Z_{ij}) - \ln E_i - \ln \Delta t_j]^2 + \lambda \sum_{z=Z_{min}+1}^{Z_{max}-1} g''(z)^2$$

$$g''(z) = g(z-1) - 2g(z) + g(z+1)$$

# Recovering response curve

---

- The solution can be only up to a scale, add a constraint

$$g(Z_{mid}) = 0, \text{ where } Z_{mid} = \frac{1}{2}(Z_{min} + Z_{max})$$

- Add a hat weighting function

$$w(z) = \begin{cases} z - Z_{min} & \text{for } z \leq \frac{1}{2}(Z_{min} + Z_{max}) \\ Z_{max} - z & \text{for } z > \frac{1}{2}(Z_{min} + Z_{max}) \end{cases}$$

$$\mathcal{O} = \sum_{i=1}^N \sum_{j=1}^P \{w(Z_{ij}) [g(Z_{ij}) - \ln E_i - \ln \Delta t_j]\}^2 +$$

$$\lambda \sum_{z=Z_{min}+1}^{Z_{max}-1} [w(z)g''(z)]^2$$



# Recovering response curve

---

$$N(P - 1) > (Z_{max} - Z_{min})$$

- We want  
If  $P=11$ ,  $N \sim 25$  (typically 50 is used)
- We prefer that selected pixels are well distributed and sampled from constant regions. They picked points by hand.
- It is an overdetermined system of linear equations and can be solved using SVD

# How to optimize?

---

$$\mathcal{O} = \sum_{i=1}^N \sum_{j=1}^P \{w(Z_{ij}) [g(Z_{ij}) - \ln E_i - \ln \Delta t_j]\}^2 +$$

$$\lambda \sum_{z=Z_{min}+1}^{Z_{max}-1} [w(z)g''(z)]^2$$

1. Set partial derivatives zero
- 2.

$$\min \sum_{i=1}^M (\mathbf{a}_i \mathbf{x} - \mathbf{b}_i)^2 \rightarrow \text{least - square solution of } \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_N \end{bmatrix} \mathbf{x} = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_N \end{bmatrix}$$

# Sparse linear system

---

$$\begin{array}{c}
 \begin{array}{cc}
 & \begin{array}{c} 256 \qquad n \end{array} \\
 \begin{array}{c} n \times p \\ 1 \\ \text{---} \\ 254 \end{array} \left[ \begin{array}{c} \\ \\ \\ \end{array} \right] & \begin{array}{c} \\ \\ \\ \end{array} \\
 \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} & \begin{array}{c} \\ \\ \\ \end{array}
 \end{array}
 \end{array}
 \begin{array}{c}
 \begin{array}{c} \left[ \begin{array}{c} g(0) \\ \vdots \\ g(255) \\ \text{---} \\ \ln E_1 \\ \vdots \\ \ln E_n \end{array} \right] \\ \text{---} \end{array} \\
 \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array}
 \end{array}
 =
 \begin{array}{c}
 \begin{array}{c} \left[ \begin{array}{c} \\ \\ \\ \end{array} \right] \\ \text{---} \end{array}
 \end{array}
 \end{array}
 \quad \text{---} \quad \begin{array}{c} \left[ \begin{array}{c} \\ \\ \\ \end{array} \right] \\ \text{---} \end{array}
 \end{array}$$

$Ax=b$

# Questions

---

- Will  $g(127)=0$  always be satisfied? Why and why not?
- How to find the least-square solution for an over-determined system?

# Least-square solution for a linear system

---

$$\begin{array}{ccc} \mathbf{A} \mathbf{x} = \mathbf{b} \\ m \times n & n & m \\ m > n \end{array}$$

The are often mutually incompatible. We instead find  $\mathbf{x}$  to minimize the norm  $\|\mathbf{A}\mathbf{x} - \mathbf{b}\|$  of the residual vector  $\mathbf{A}\mathbf{x} - \mathbf{b}$ . If there are multiple solutions, we prefer the one with the minimal length  $\|\mathbf{x}\|$ .

# Least-square solution for a linear system

---

If we perform SVD on  $\mathbf{A}$  and rewrite it as

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T \quad \mathbf{\Sigma} = \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_n \end{bmatrix}$$

then  $\hat{\mathbf{x}} = \mathbf{V}\mathbf{\Sigma}^{-1}\mathbf{U}^T\mathbf{b}$  is the least-square solution.  
pseudo inverse



# Libraries for SVD

---

- Matlab
- GSL
- Boost
- LAPACK (recommended)
- ATLAS

# Matlab code

---

```
%
% gsolve.m - Solve for imaging system response function
%
% Given a set of pixel values observed for several pixels in several
% images with different exposure times, this function returns the
% imaging system's response function g as well as the log film irradiance
% values for the observed pixels.
%
% Assumes:
%
%   Zmin = 0
%   Zmax = 255
%
% Arguments:
%
%   Z(i,j) is the pixel values of pixel location number i in image j
%   B(j)   is the log delta t, or log shutter speed, for image j
%   l      is lamdba, the constant that determines the amount of smoothness
%   w(z)   is the weighting function value for pixel value z
%
% Returns:
%
%   g(z)   is the log exposure corresponding to pixel value z
%   lE(i)  is the log film irradiance at pixel location i
%
```

# Matlab code

---

```
function [g,lE]=gsolve(Z,B,l,w)

n = 256;
A = zeros(size(Z,1)*size(Z,2)+n+1,n+size(Z,1));
b = zeros(size(A,1),1);

k = 1;                                %% Include the data-fitting equations
for i=1:size(Z,1)
    for j=1:size(Z,2)
        wij = w(Z(i,j)+1);
        A(k,Z(i,j)+1) = wij; A(k,n+i) = -wij; b(k,1) = wij * B(i,j);
        k=k+1;
    end
end

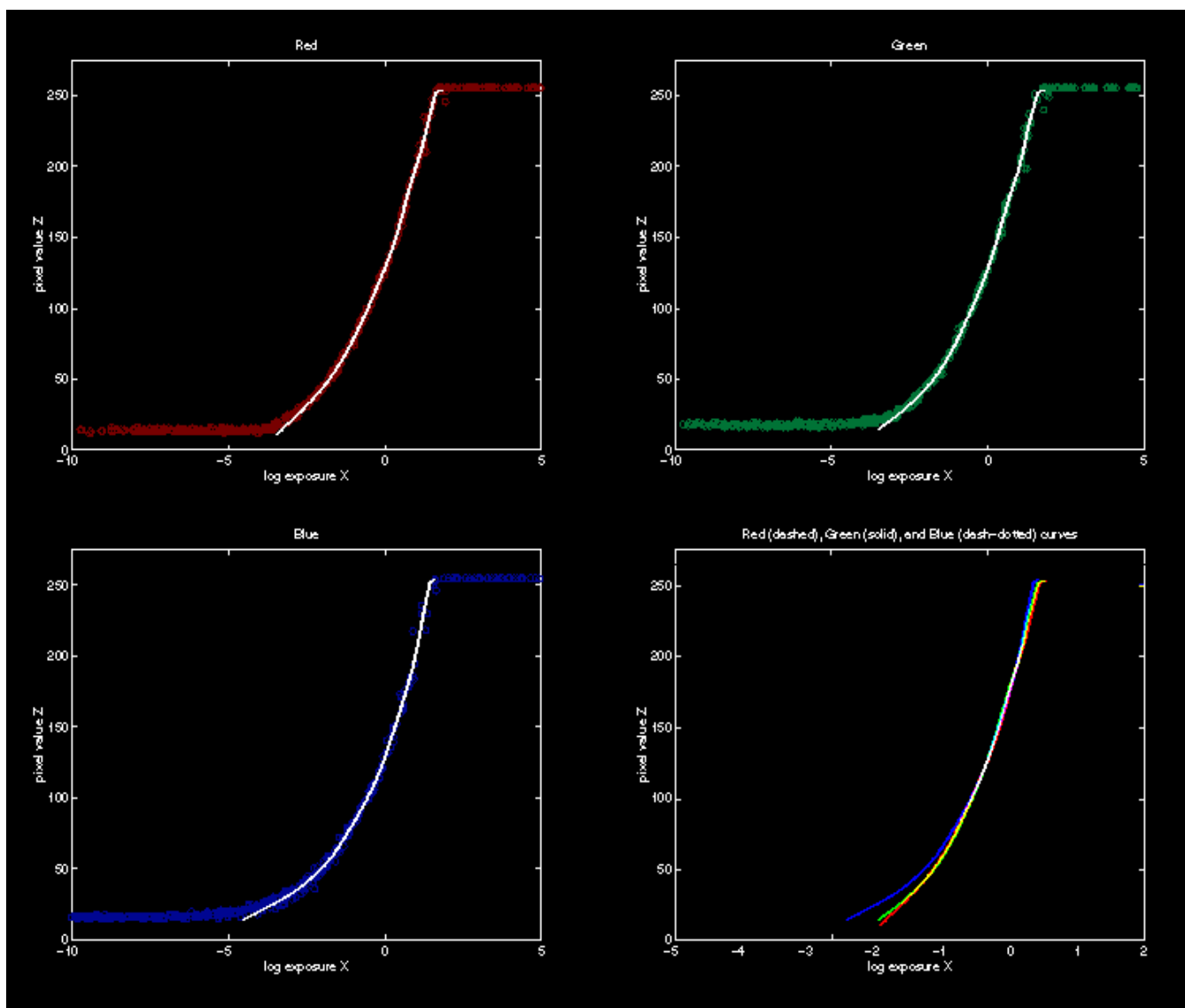
A(k,129) = 1;                          %% Fix the curve by setting its middle value to 0
k=k+1;

for i=1:n-2                             %% Include the smoothness equations
    A(k,i)=l*w(i+1); A(k,i+1)=-2*l*w(i+1); A(k,i+2)=l*w(i+1);
    k=k+1;
end

x = A\b;                                %% Solve the system using SVD

g = x(1:n);
lE = x(n+1:size(x,1));
```

# Recovered response function



# Constructing HDR radiance map

---

$$\ln E_i = g(Z_{ij}) - \ln \Delta t_j$$

combine pixels to reduce noise and obtain a more reliable estimation

$$\ln E_i = \frac{\sum_{j=1}^P w(Z_{ij})(g(Z_{ij}) - \ln \Delta t_j)}{\sum_{j=1}^P w(Z_{ij})}$$

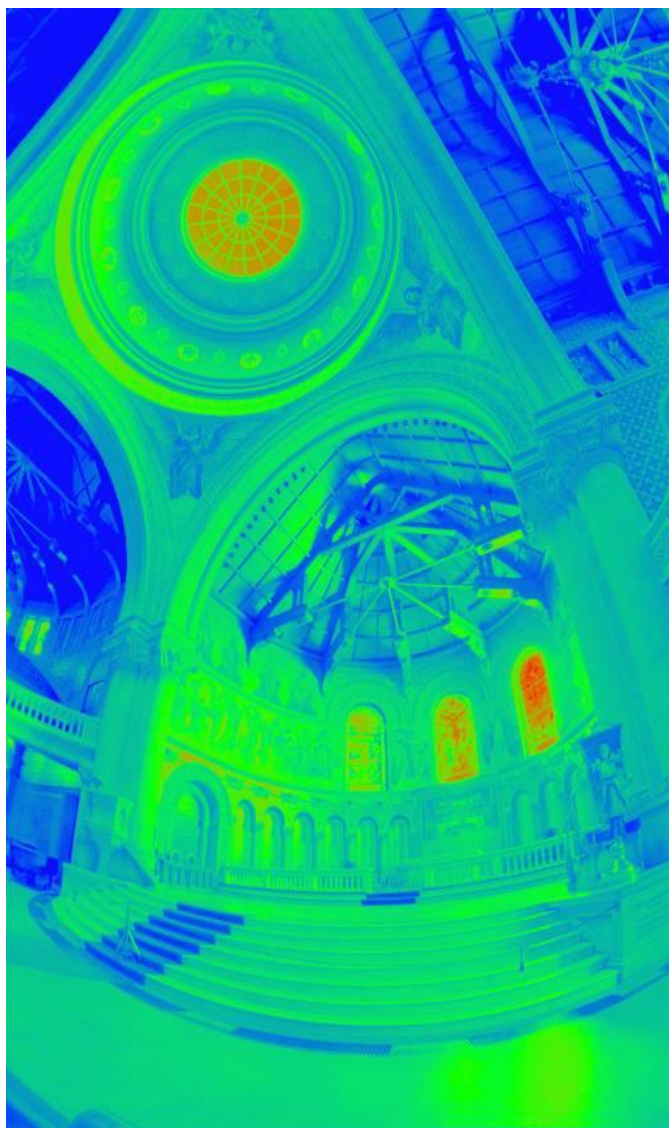
# Varying shutter speeds

---



# Reconstructed radiance map

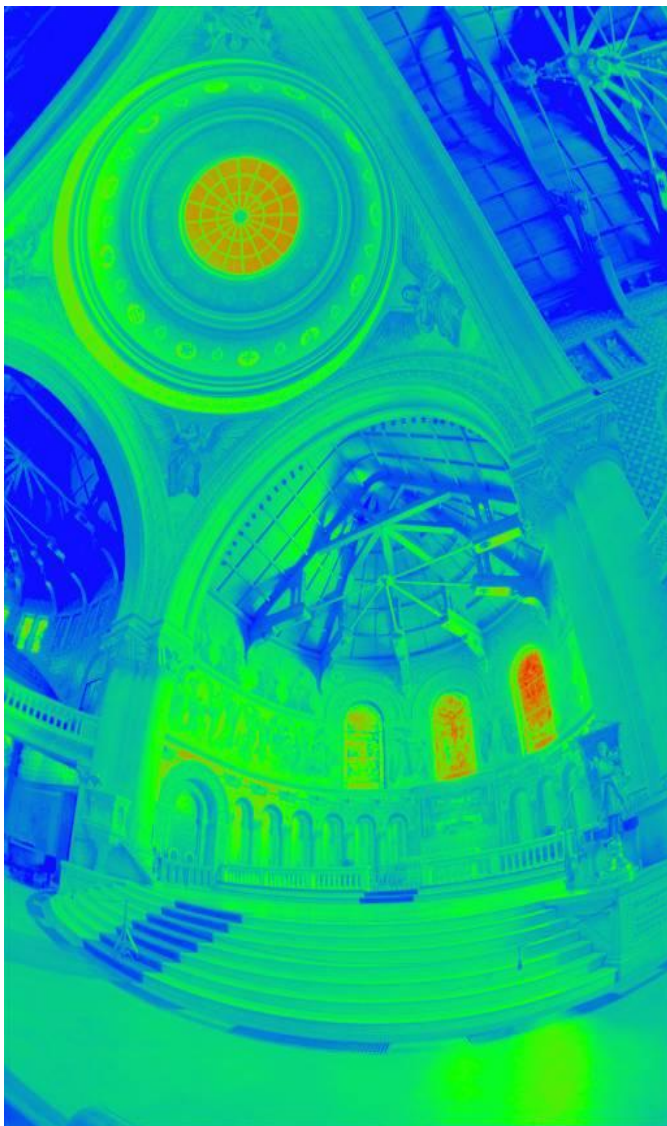
---





# What is this for?

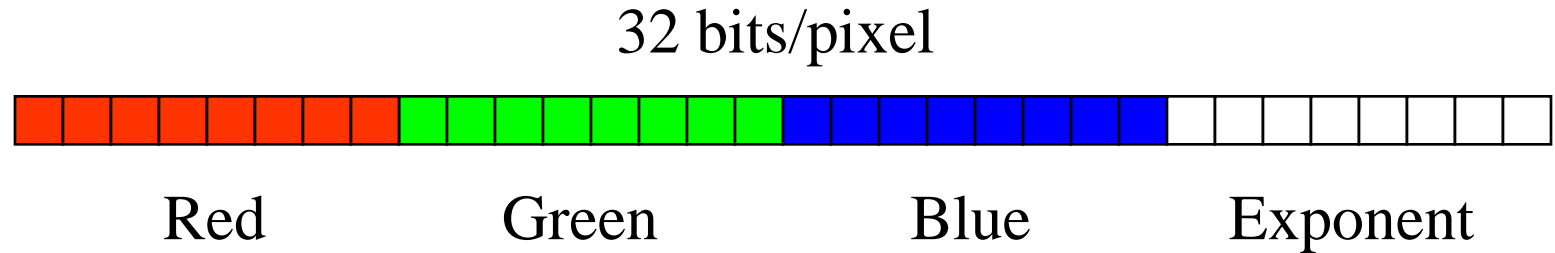
---



- Human perception
- Vision/graphics applications

# Radiance format (.pic, .hdr, .rad)

---



(145, 215, 87, 149) =  
(145, 215, 87) \*  $2^{(149-128)}$  =  
1190000 1760000 713000

(145, 215, 87, 103) =  
(145, 215, 87) \*  $2^{(103-128)}$  =  
0.00000432 0.00000641 0.00000259

# Demo

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<http://www.hdrsoft.com/examples.html>

# Image alignment

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# Median Threshold Bitmap (MTB) alignment

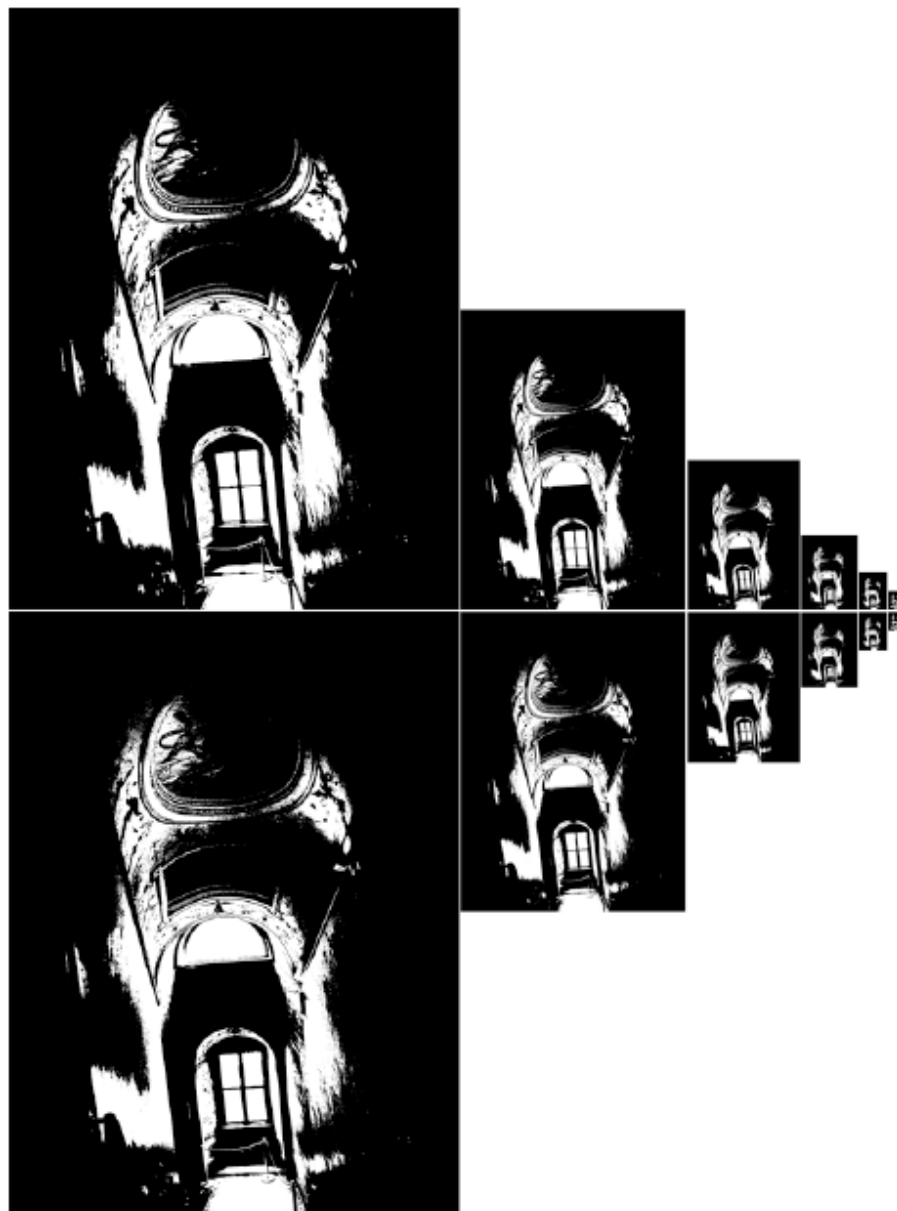
---

- Consider only integral translations. It is enough empirically.
- The inputs are N grayscale images. (You can either use the green channel or convert into grayscale by  $Y=(54R+183G+19B)/256$ )
- MTB is a binary image formed by thresholding the input image using the median of intensities.

# Search for the optimal offset

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- Try all possible offsets.
- Gradient descent
- Multiscale technique
- $\log(\text{max\_offset})$  levels
- Try 9 possibilities for the top level
- Scale by 2 when passing down; try its 9 neighbors

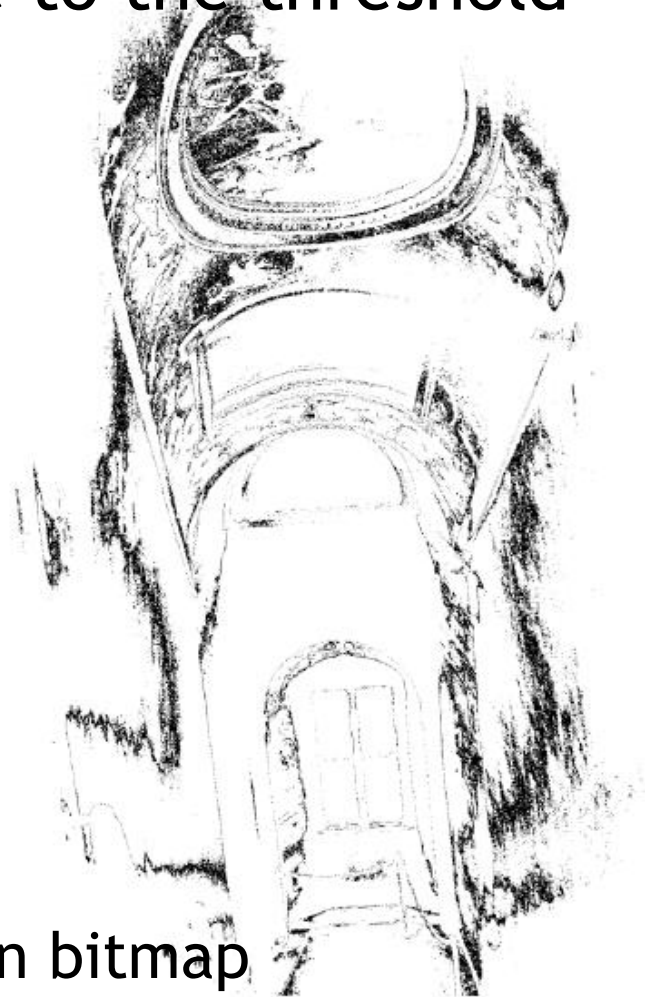


# Threshold noise

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ignore pixels that are close to the threshold



exclusion bitmap



# Results

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Success rate = 84%. 10% failure due to rotation.  
3% for excessive motion and 3% for too much  
high-frequency content.





# Equipment

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We provide 3 sets:



Contact TA for checkout.

# HDR Video

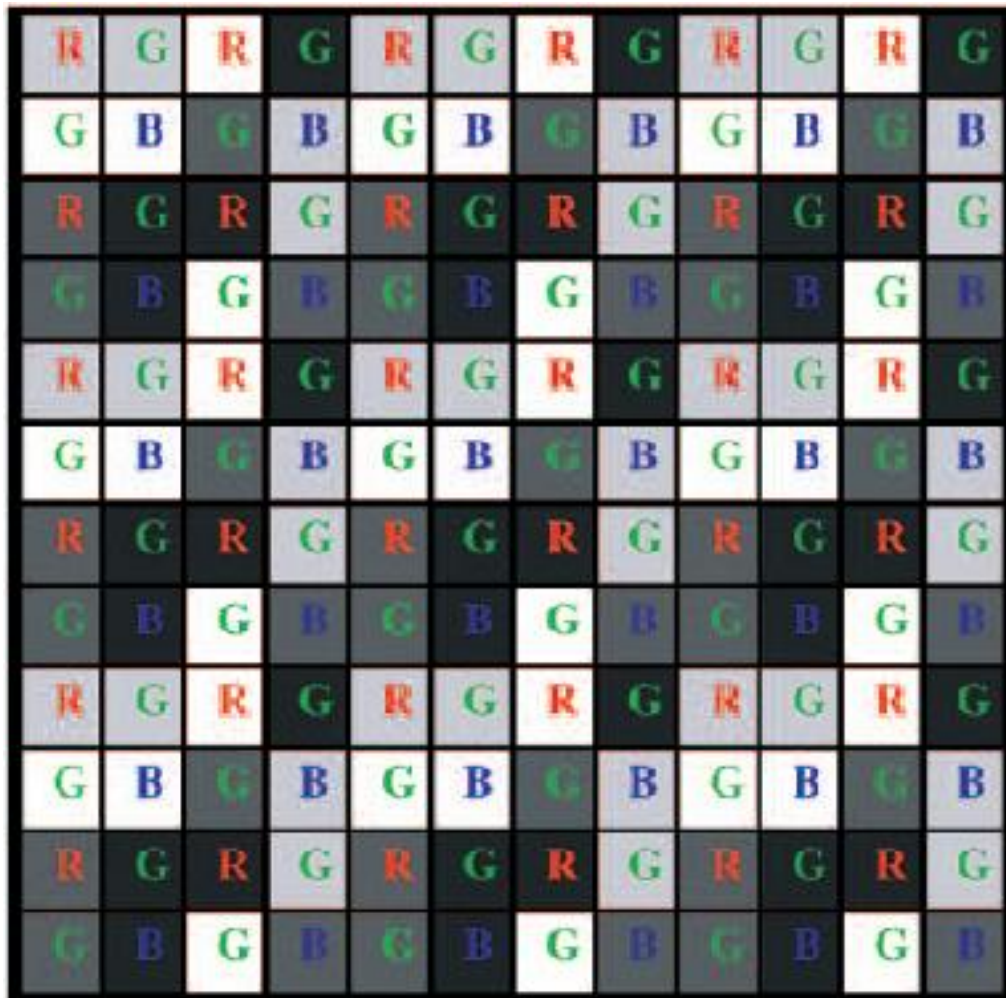
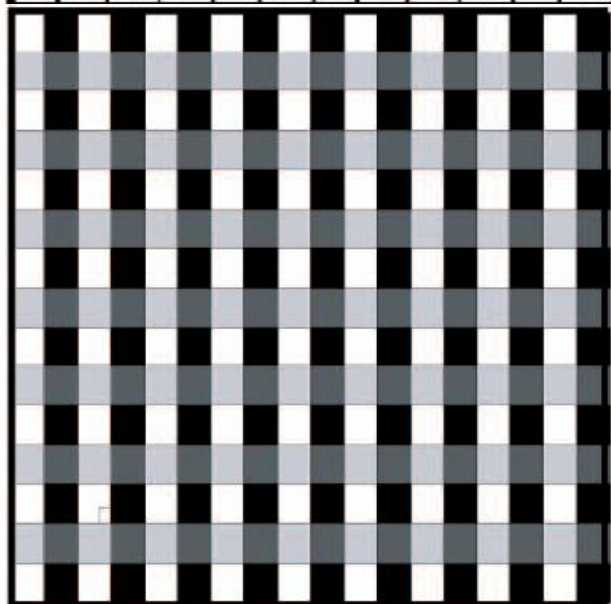
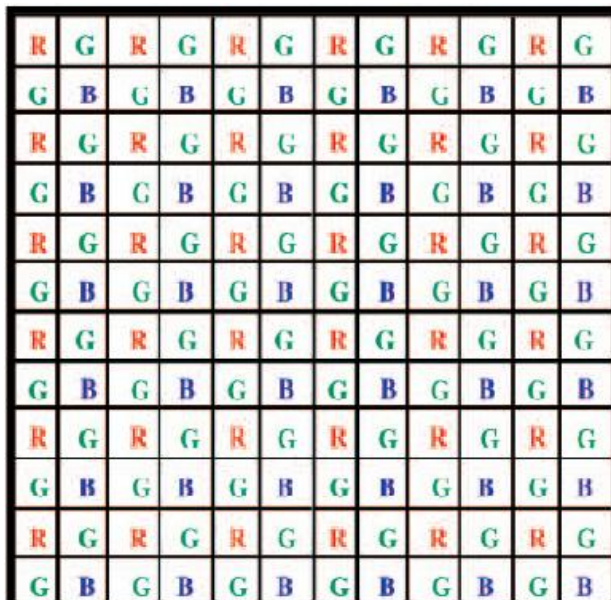
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## **High Dynamic Range Video**

Submitted to SIGGRAPH 2003

Paper #125

# Assorted pixel (Single Exposure HDR)



# Assorted pixel

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# Assorted pixel

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Normal Camera

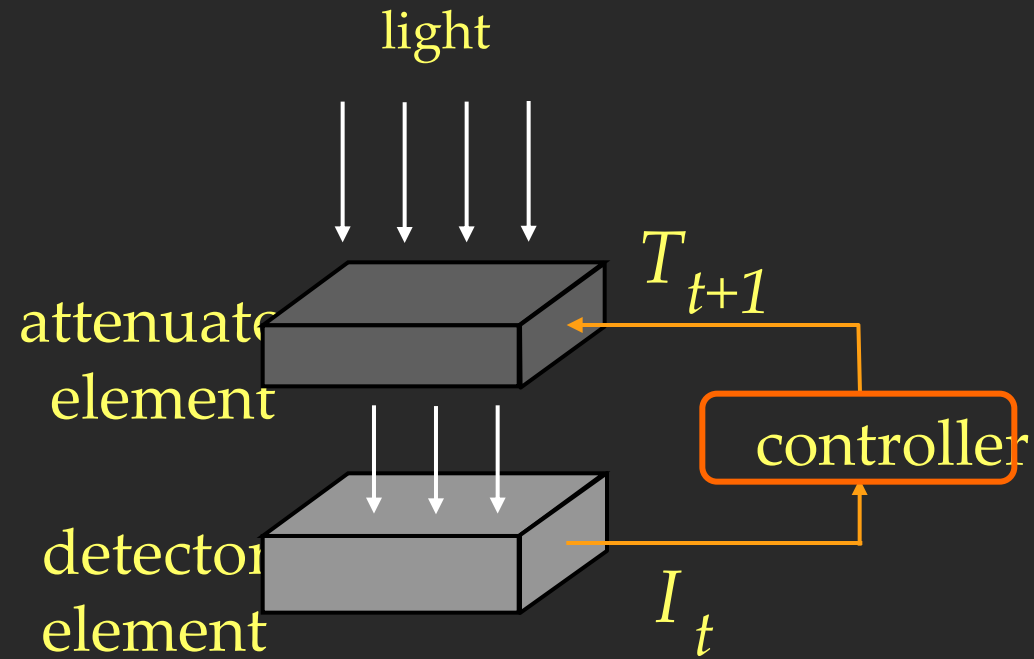


Assorted Pixel Camera



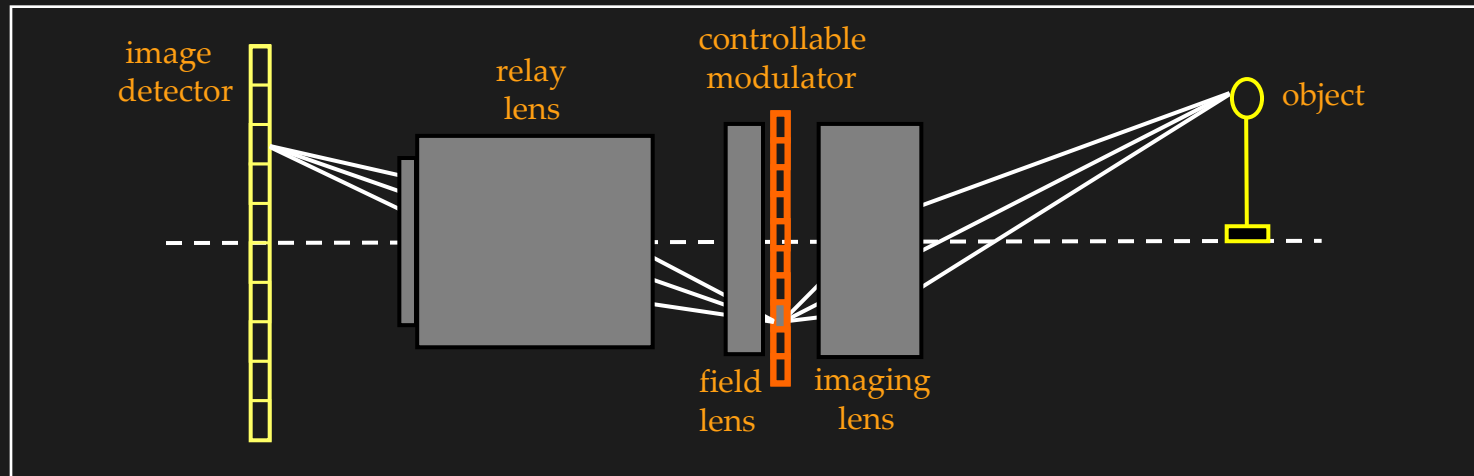
# Pixel with Adaptive Exposure Control

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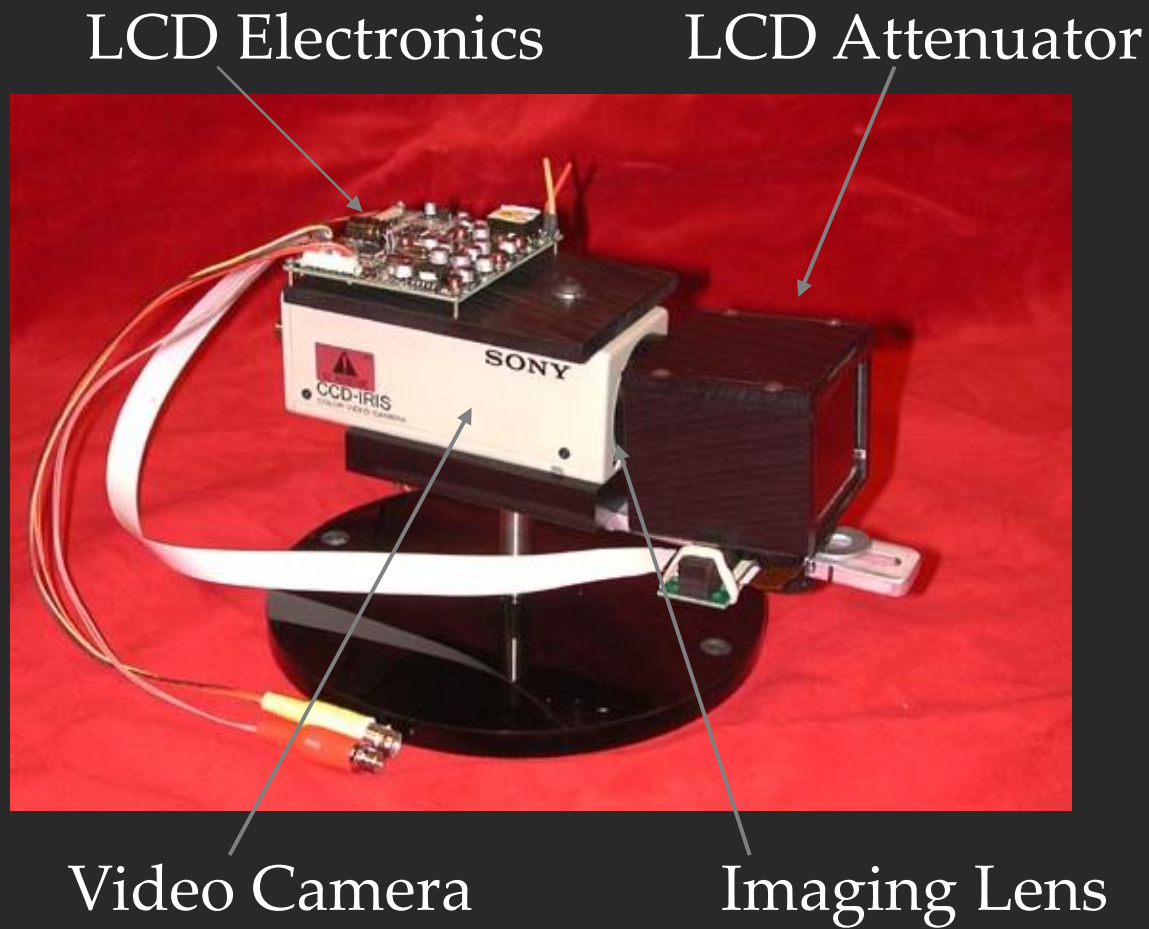


# ADR Imaging with Spatial Light Modulator

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# ADR Camera with LCD Attenuator





# ADAPTIVE DYNAMIC RANGE IMAGING

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OPTICAL DYNAMIC ATTENUATION