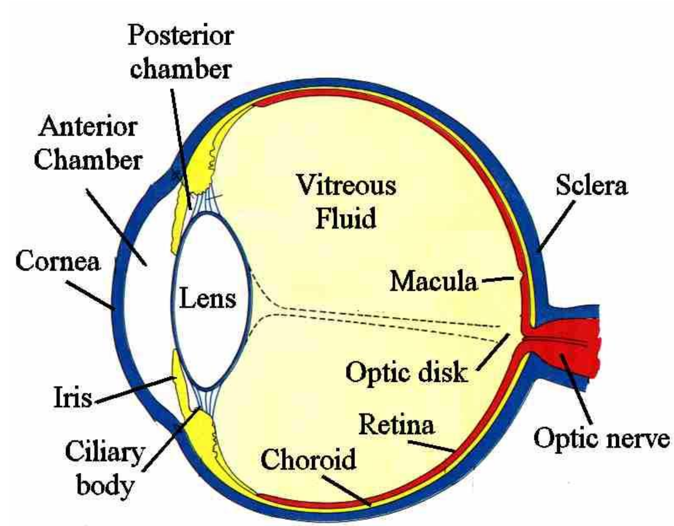
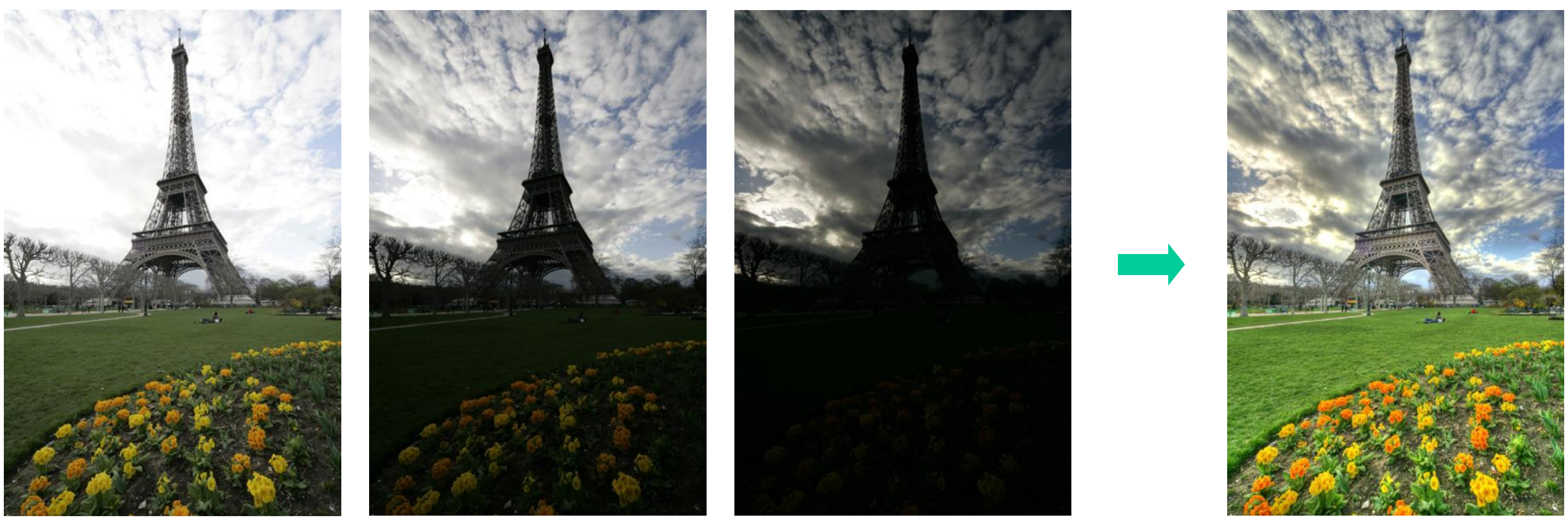
Last Lecture



Kimber, D.C.; C.E. Gray, and C.E. Stackpole. (1966). *Anatomy and Physiology*. MacMillan Co., NY. pg.335.



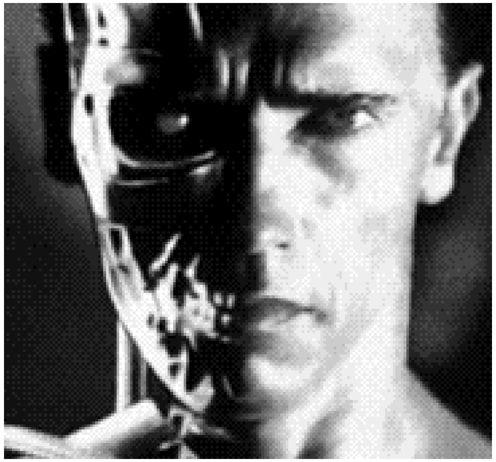
photomatix.com

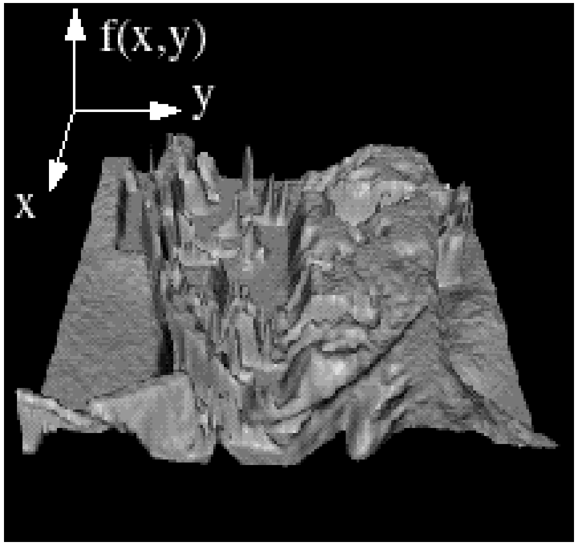
Today

Image Processing: from basic concepts to latest techniques

- Filtering
- Edge detection
- Re-sampling and aliasing
- Image Pyramids (Gaussian and Laplacian)
- Removing handshake blur from a single image

Image as a discreet function





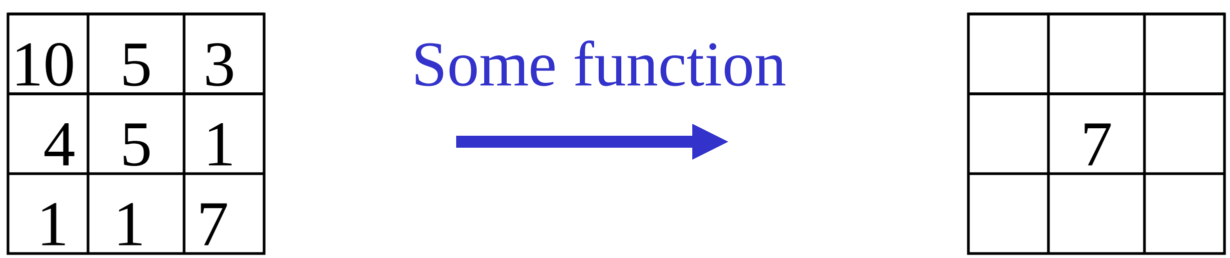
Represented by a matrix

$j \rightarrow$, $i \downarrow$

62	79	23	119	120	105	4	0
10	10	9	62	12	78	34	0
10	58	197	46	46	0	0	48
176	135	5	188	191	68	0	49
2	1	1	29	26	37	0	77
0	89	144	147	187	102	62	208
255	252	0	166	123	62	0	31
166	63	127	17	1	0	99	30

What is image filtering?

- Modify the pixels in an image based on some function of a local neighborhood of the pixels.



Local image data

Modified image data

Linear functions

- Simplest: linear filtering.
  - Replace each pixel by a linear combination of its neighbors.
- The prescription for the linear combination is called the “convolution kernel”.

10	5	3
4	5	1
1	1	7

Local image data

0	0	0
0	0.5	0
0	1	0.5

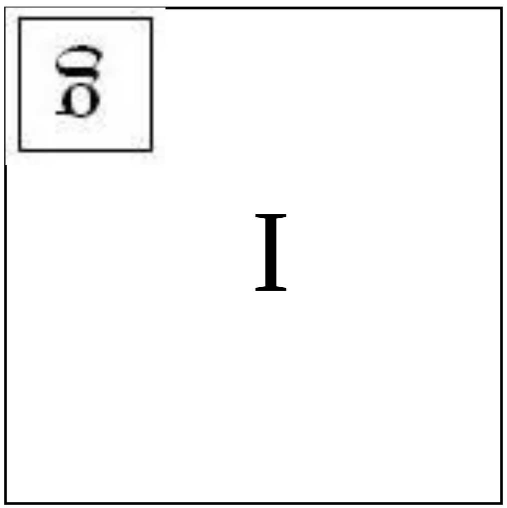
kernel

	7	

Modified image data

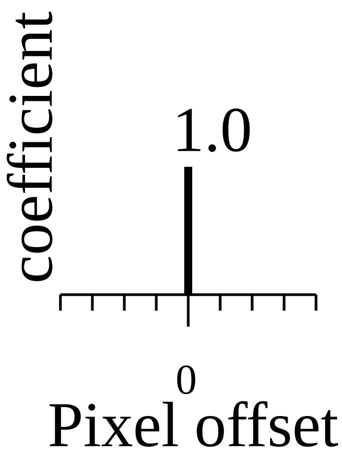
Convolution

$$f[m,n] = I \otimes g = \sum_{k,l} I[m-k, n-l]\, g[k,l]$$



Linear filtering (warm-up slide)

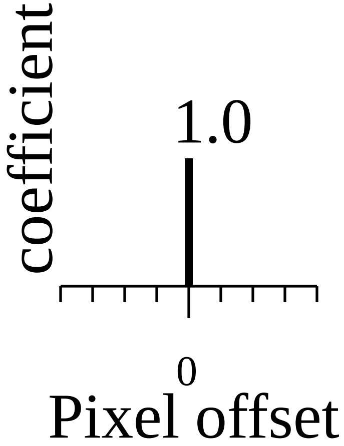
original



?

Linear filtering (warm-up slide)

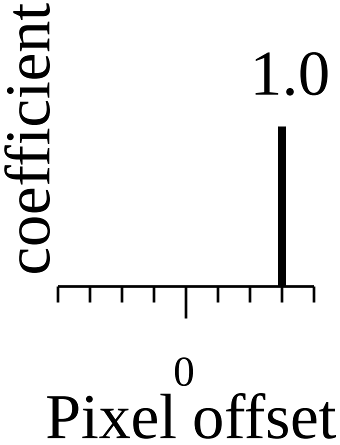
original



Filtered
(no change)

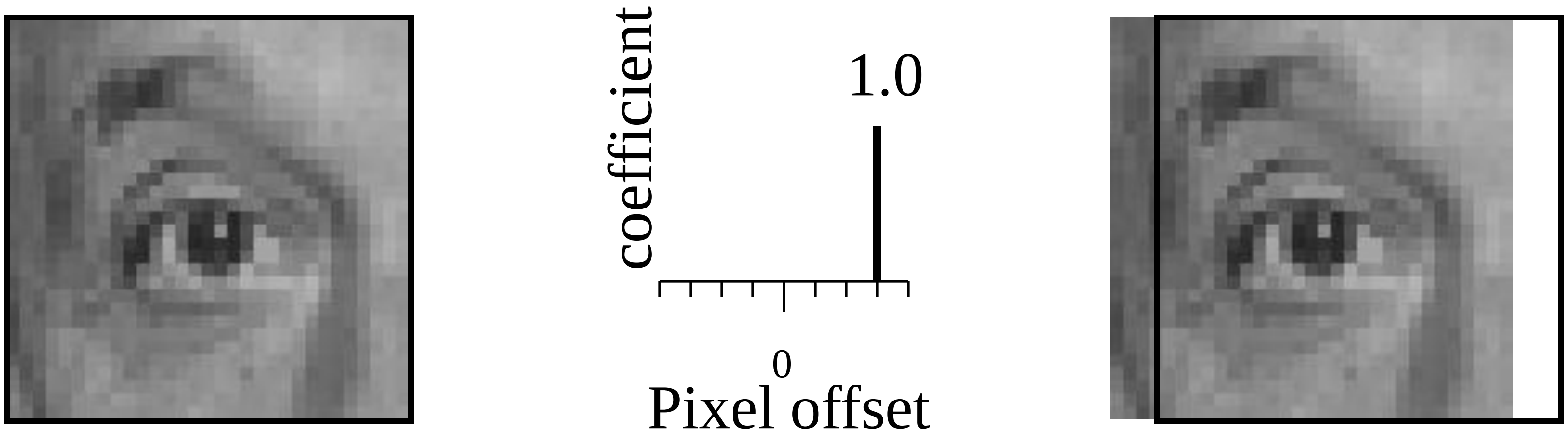
Linear filtering

original



?

shift

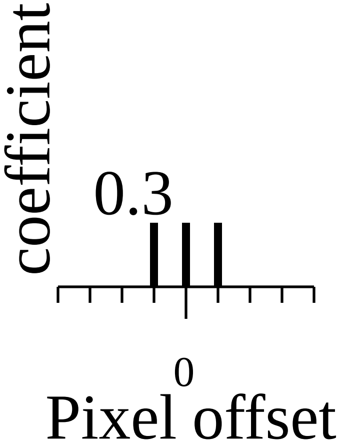


original

shifted

Linear filtering

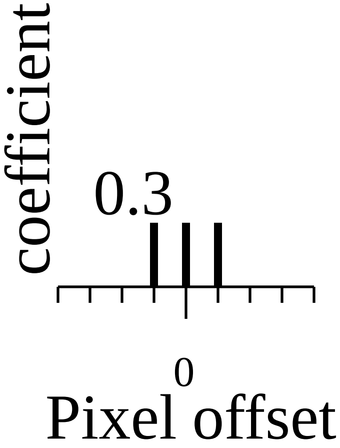
original



?

Blurring

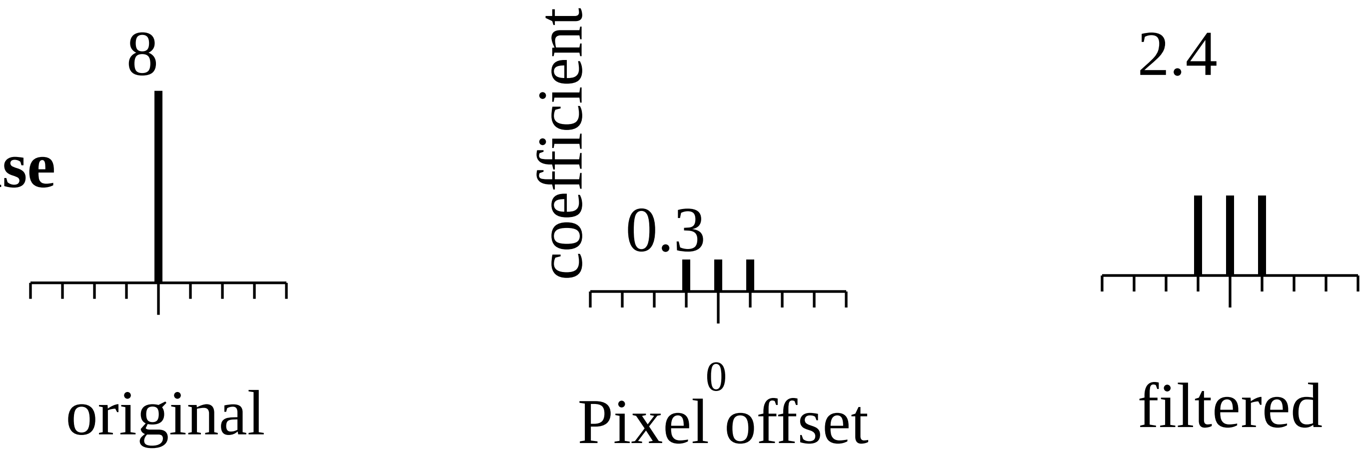
original



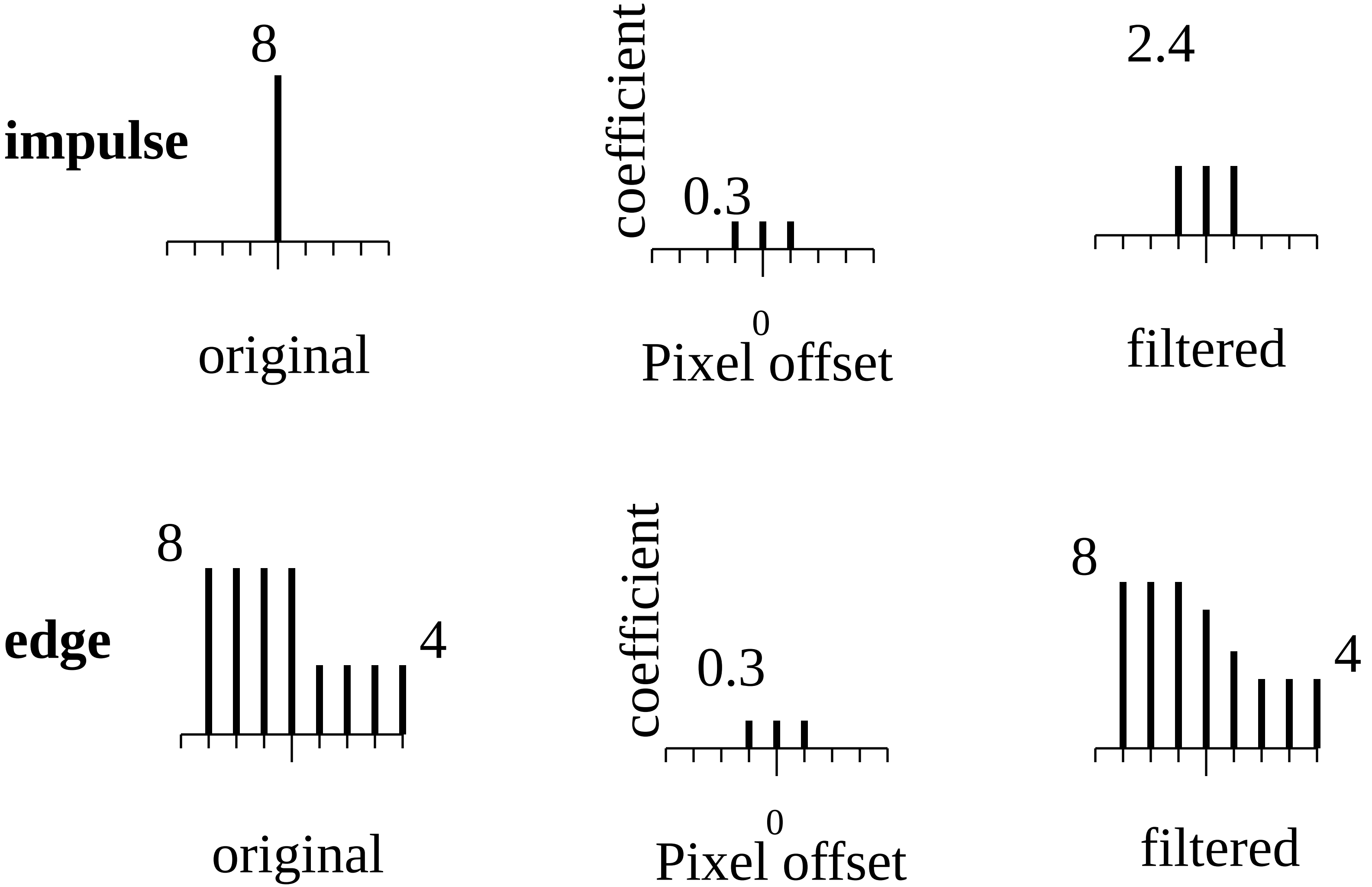
Blurred (filter applied in both dimensions).

Blur Examples

impulse

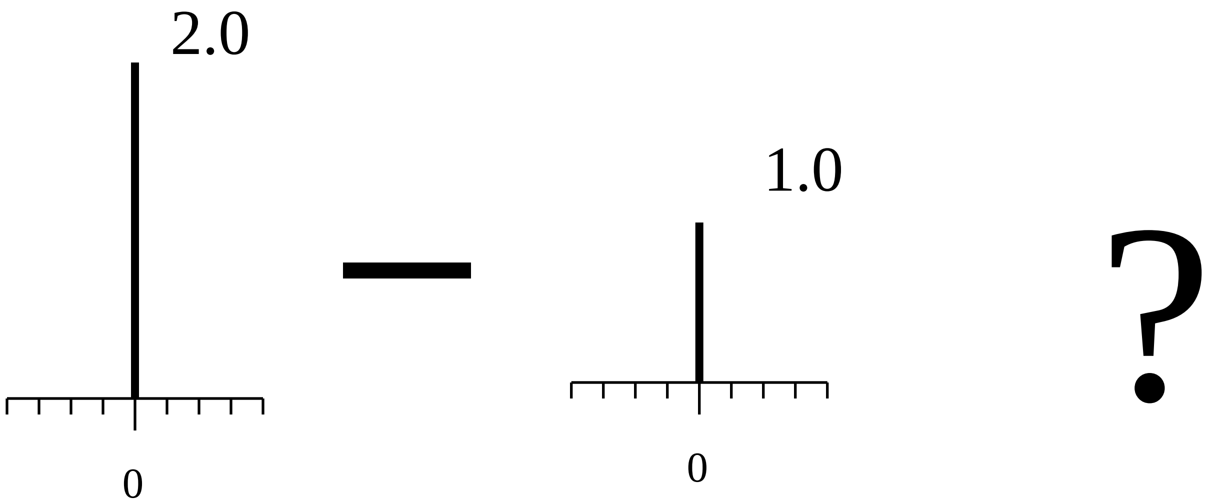


Blur Examples



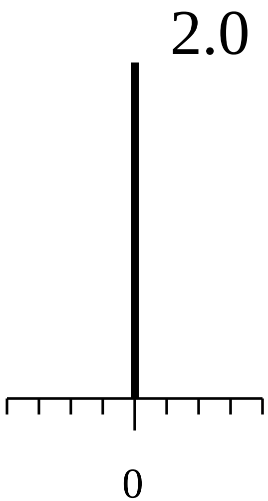
Linear filtering (warm-up slide)

original

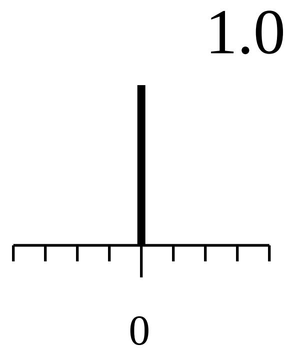


Linear Filtering (no change)

original



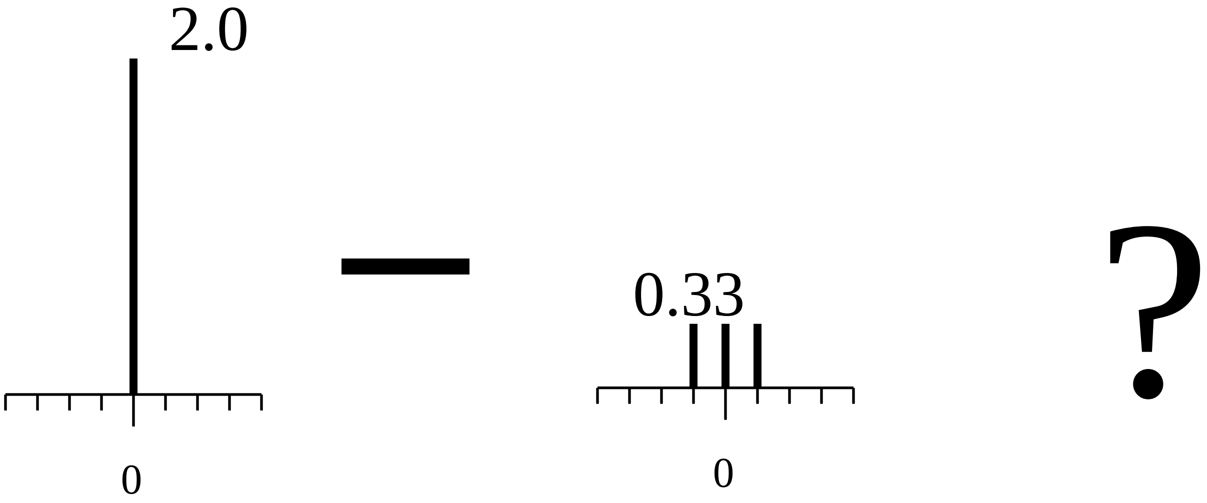
−



Filtered
(no change)

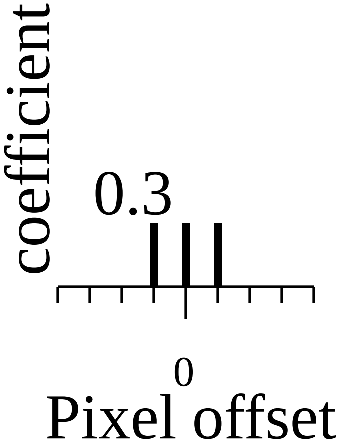
Linear Filtering

original



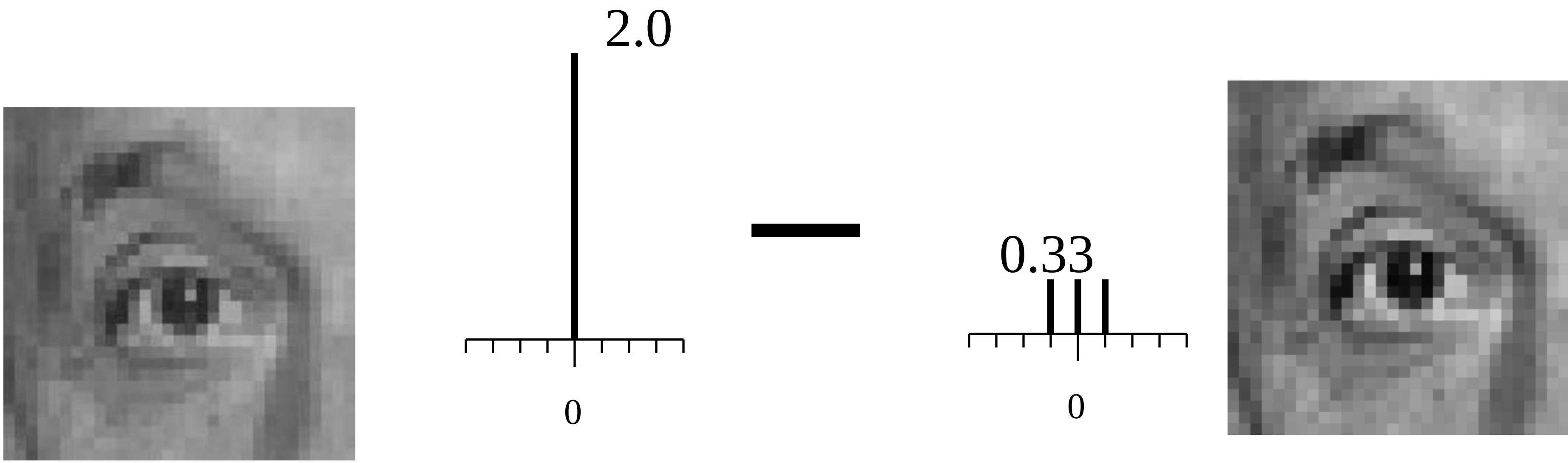
(remember blurring)

original



Blurred (filter applied in both dimensions).

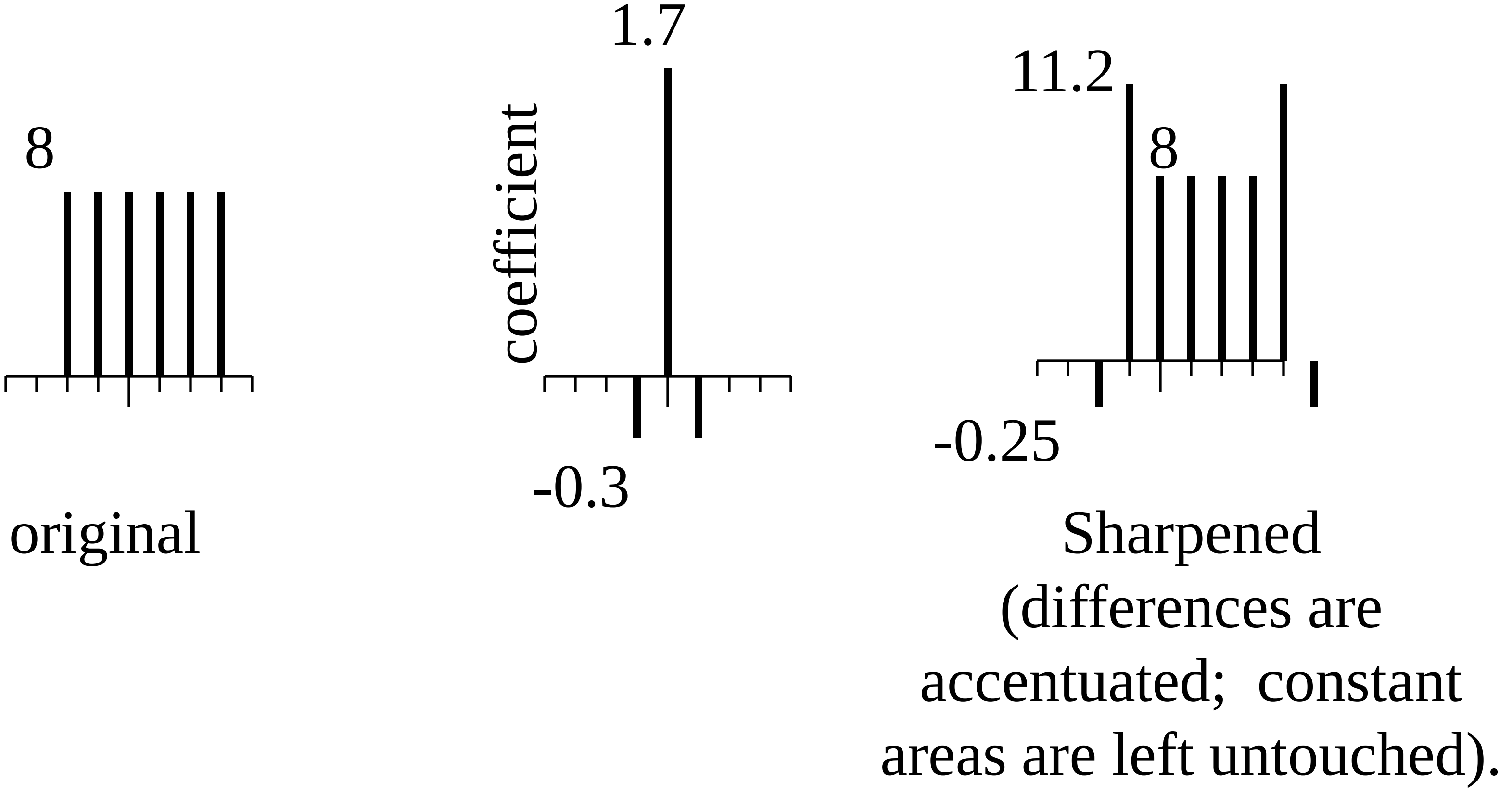
Sharpening



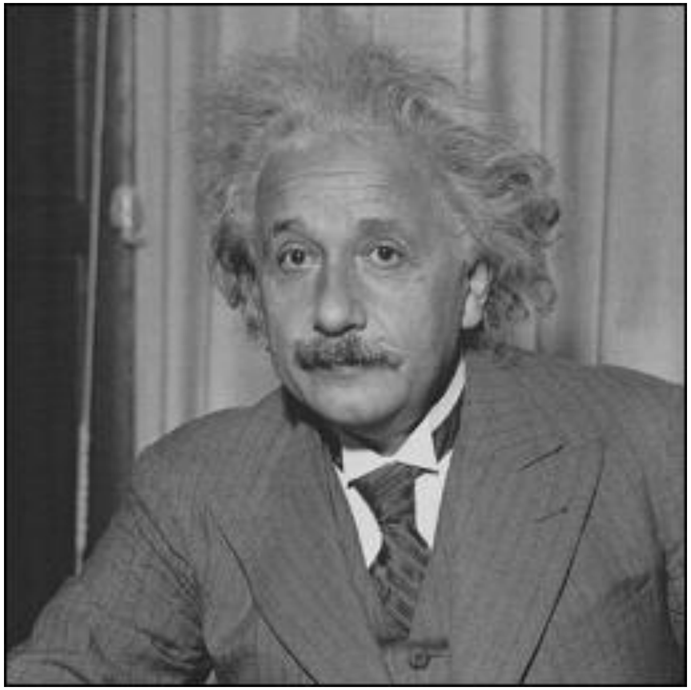
original

Sharpened original

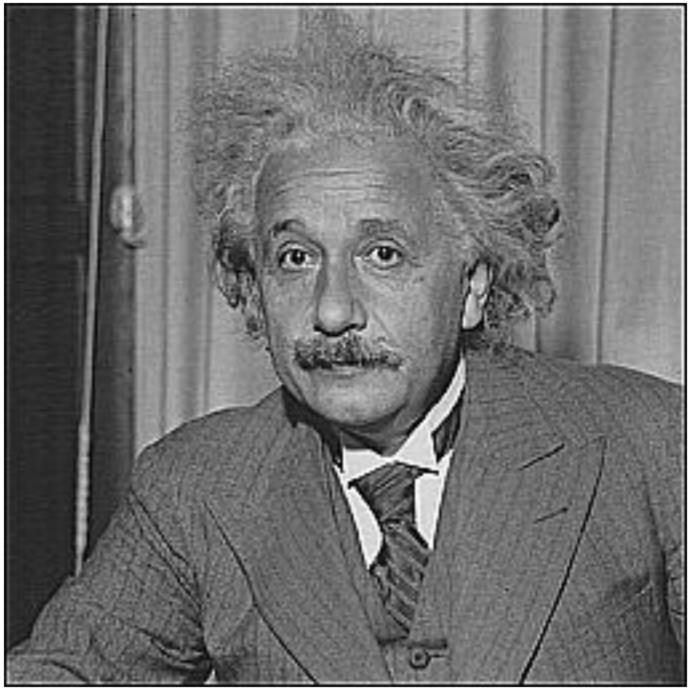
Sharpening example



Sharpening



before



after

Spatial resolution and color

original

R

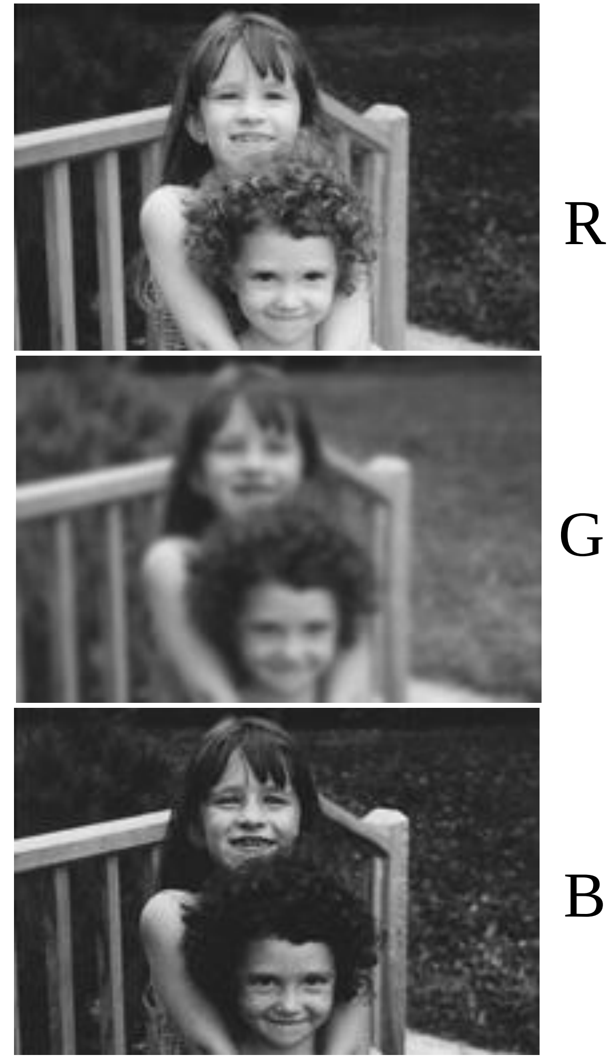
G

B

Blurring the G component

original

processed



Blurring the R component

original

processed

R

G

B

Blurring the B component

original

processed

R

G

B

Lab Color Component

L

a

b

A rotation of the color coordinates into directions that are more perceptually meaningful:
L: luminance,
a: red-green,
b: blue-yellow

Bluring L

original

processed

L

a

b

Bluring a

original

processed

L

a

b

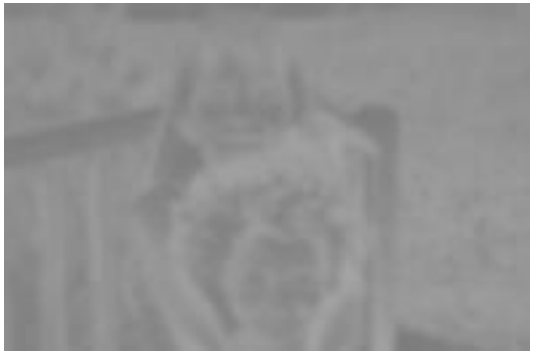
Bluring b

original

processed

L

a

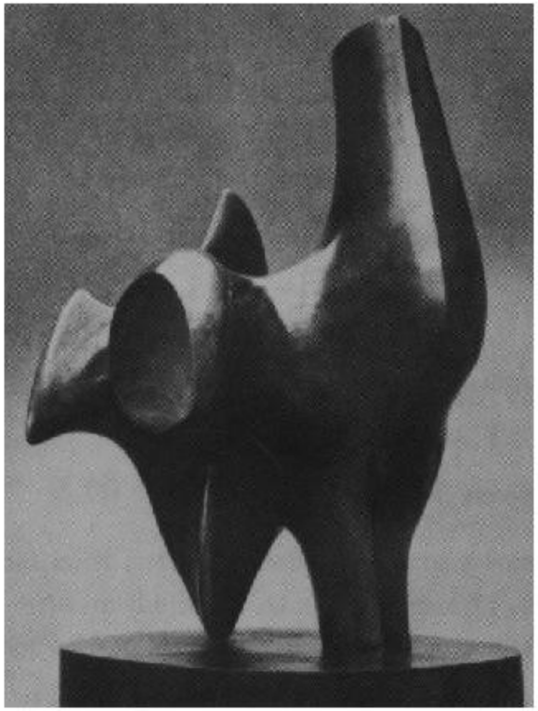


b

Application to image compression

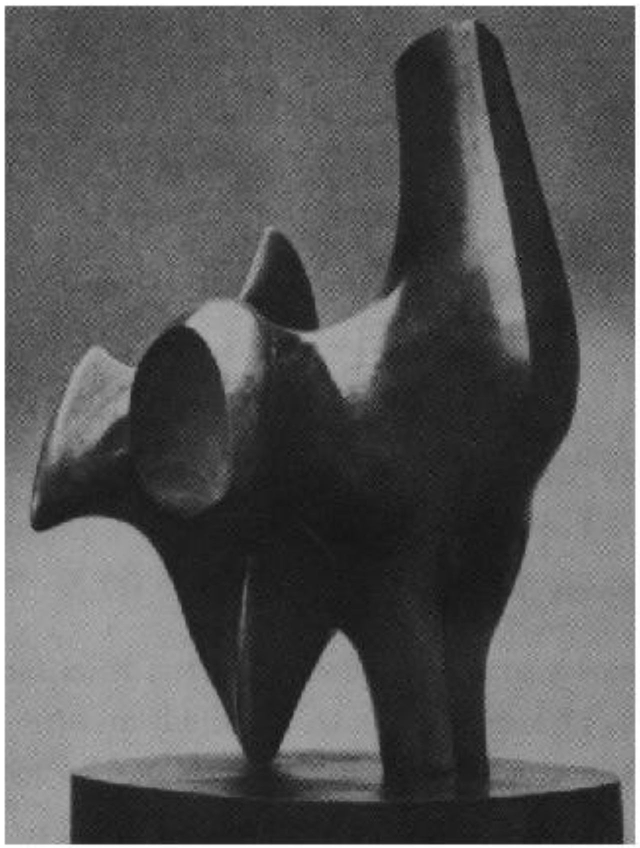
- (compression is about hiding differences from the true image where you can’t see them).

Edge Detection



- Convert a 2D image into a set of curves
  - Extracts salient features of the scene
  - More compact than pixels

How can you tell that a pixel is on an edge?



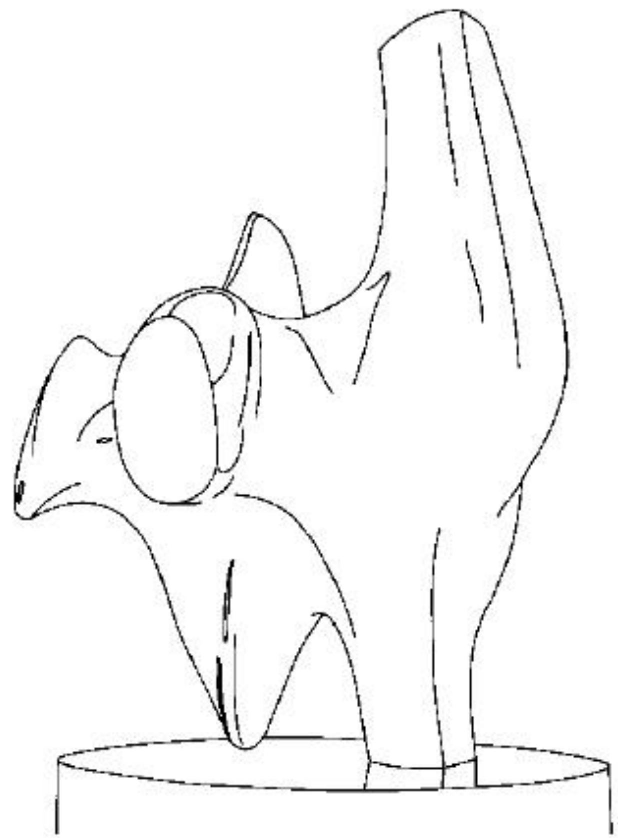
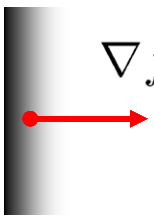


Image gradient

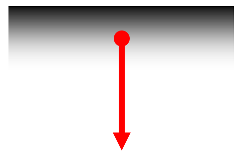
- The gradient of an image:

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

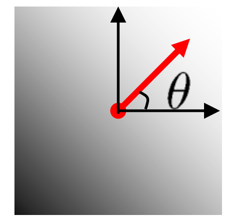
- The gradient points in the direction of most rapid change in intensity



$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$



$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

- The gradient direction is given by:

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

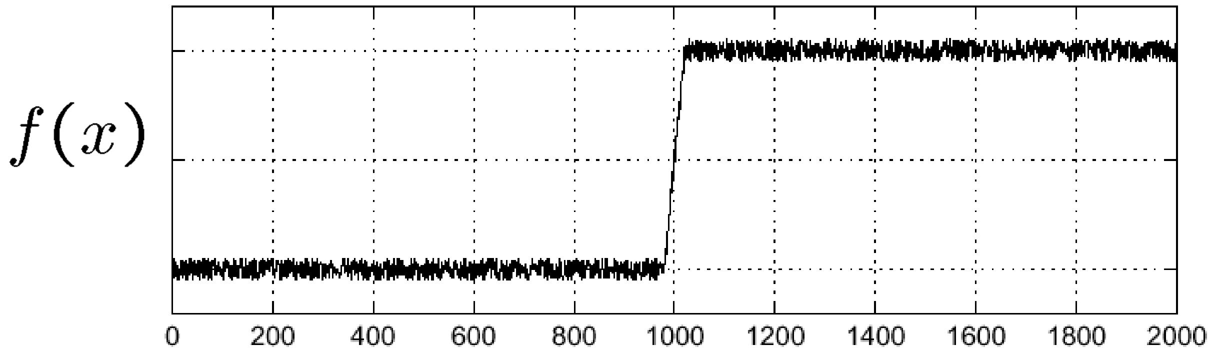
  – how does the gradient relate to the direction of the edge?

- The *edge strength* is given by the gradient magnitude

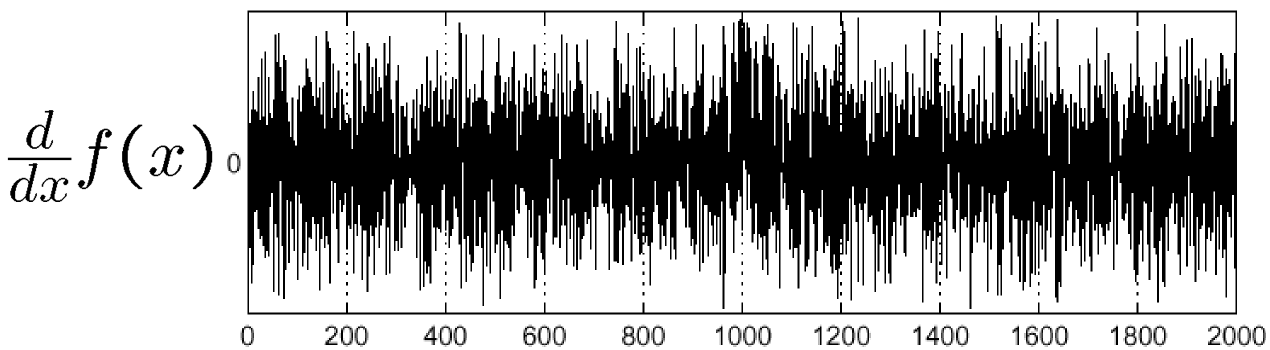
$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Effects of noise

- Consider a single row or column of the image
  - Plotting intensity as a function of position gives a signal

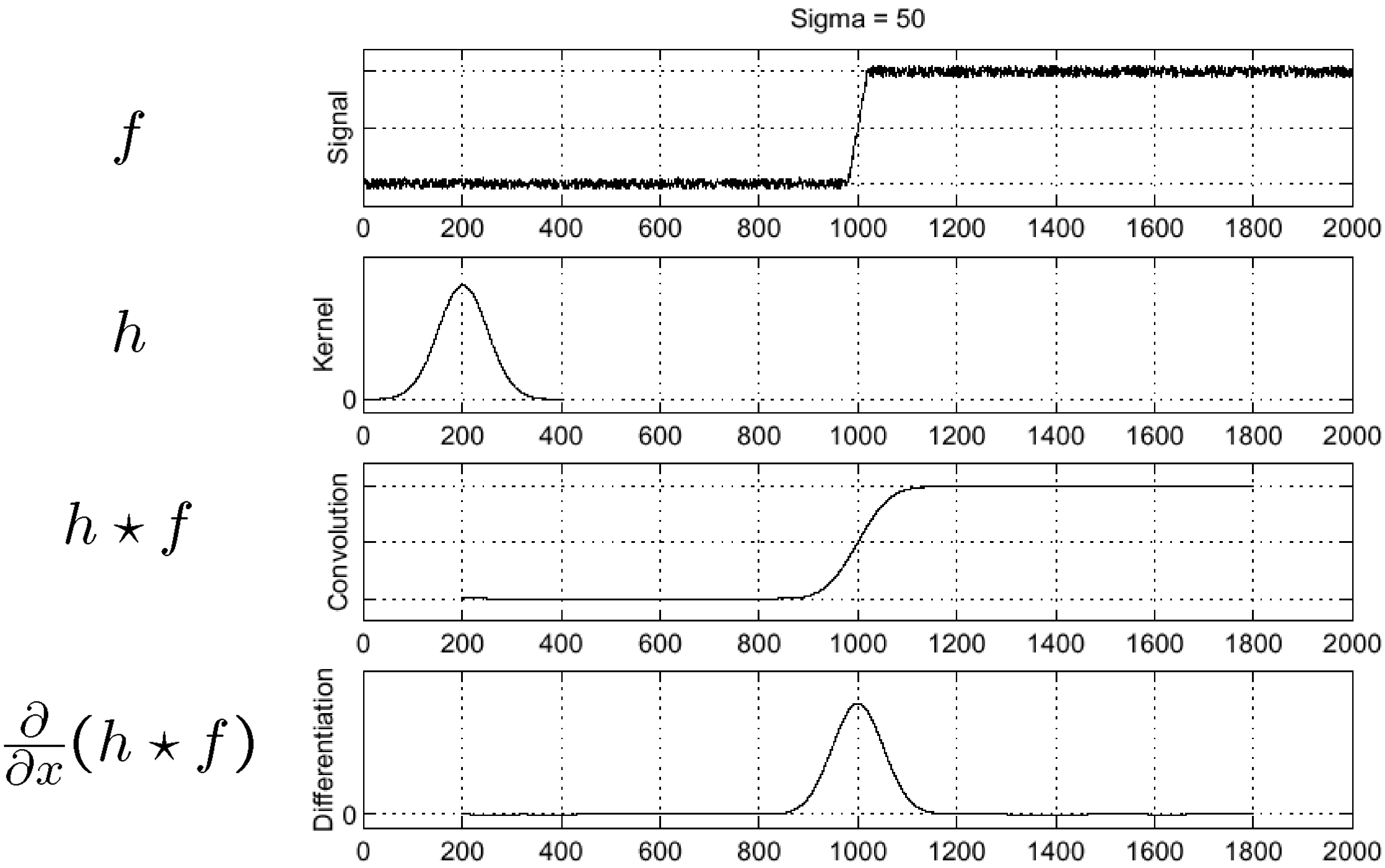


How to compute a derivative?



- Where is the edge?

Solution: smooth first



- Where is the edge?
- Look for peaks in $\frac{\partial}{\partial x}(h \star f)$

Derivative theorem of convolution

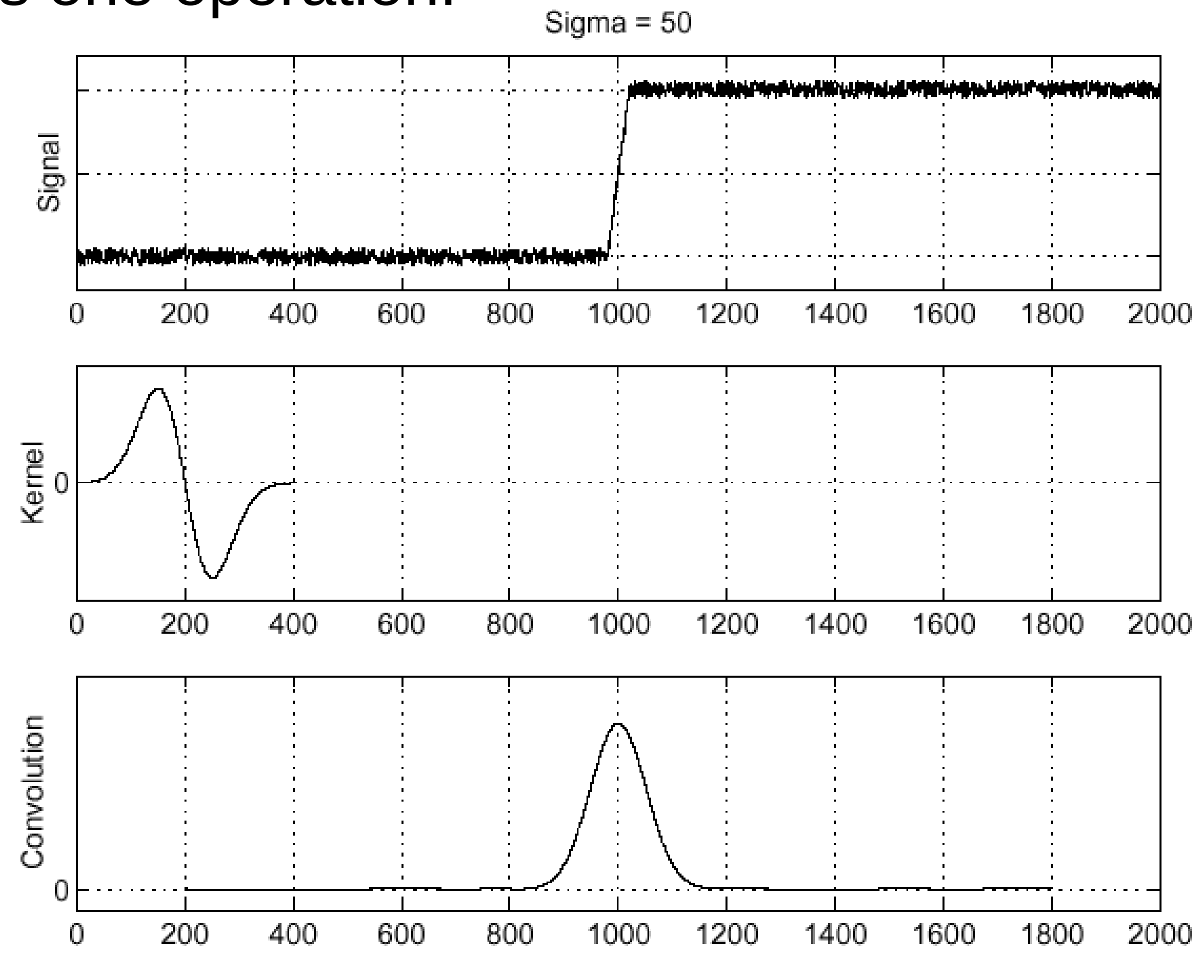
$$\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$$

- This saves us one operation:

f

$\frac{\partial}{\partial x}h$

$(\frac{\partial}{\partial x}h) \star f$



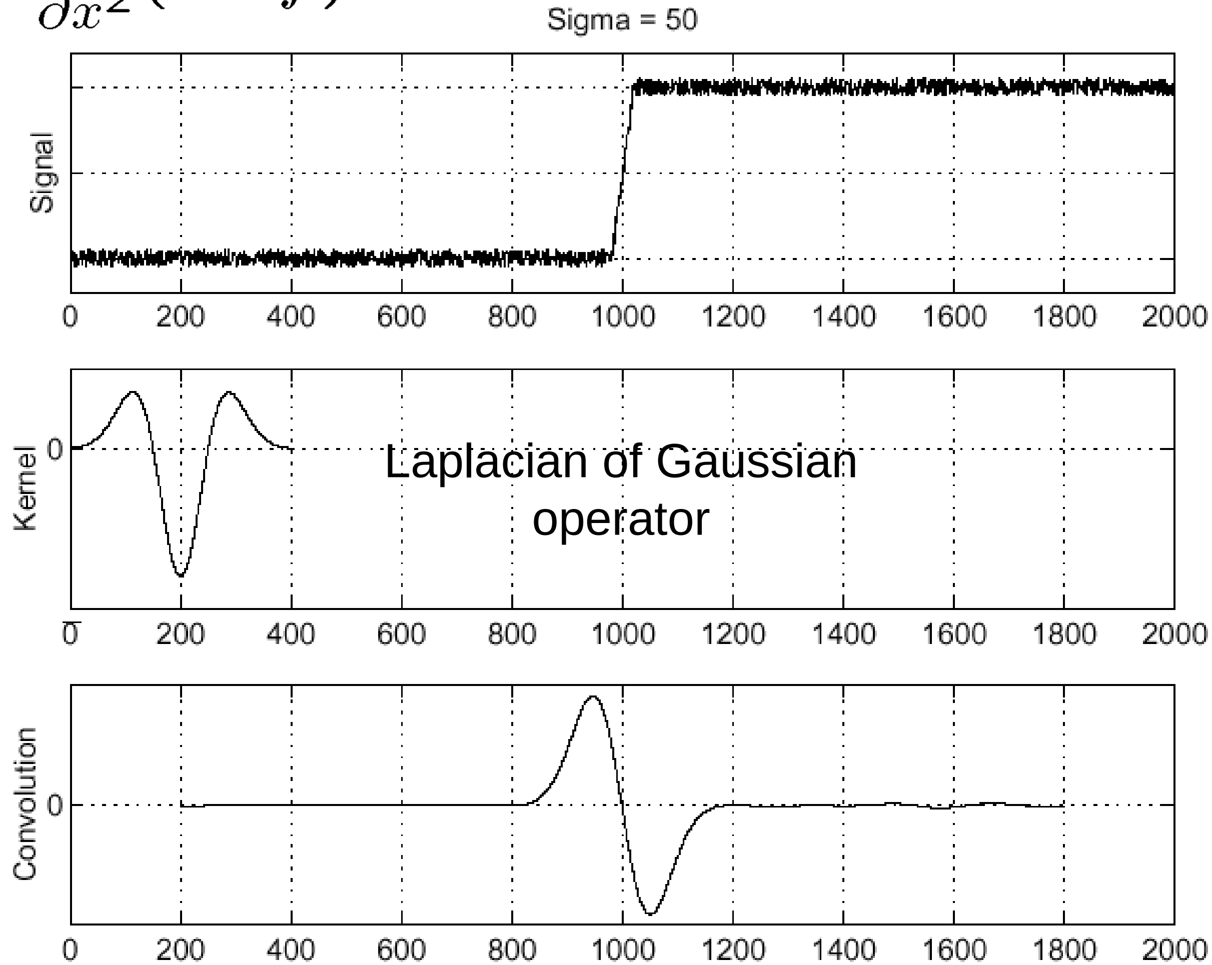
Laplacian of Gaussian

- Consider $\frac{\partial^2}{\partial x^2}(h \star f)$

$$f$$

$$\frac{\partial^2}{\partial x^2}h$$

$$(\frac{\partial^2}{\partial x^2}h) \star f$$

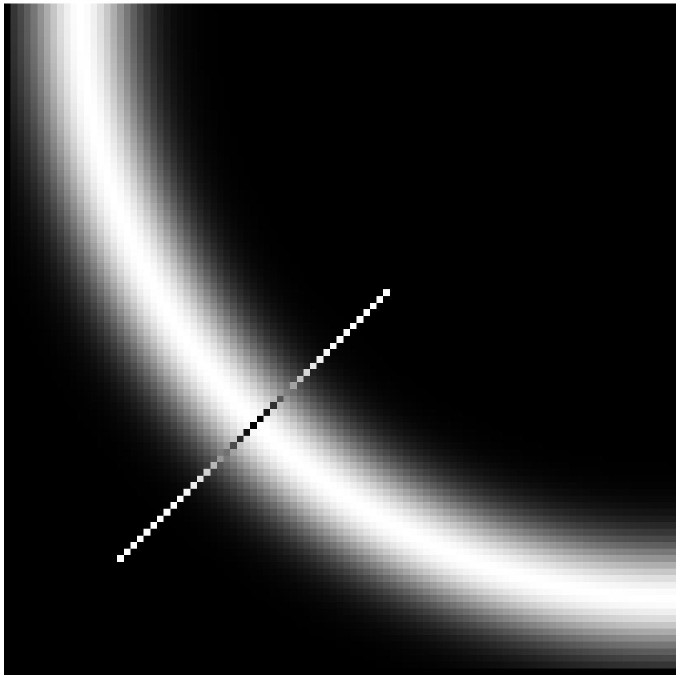


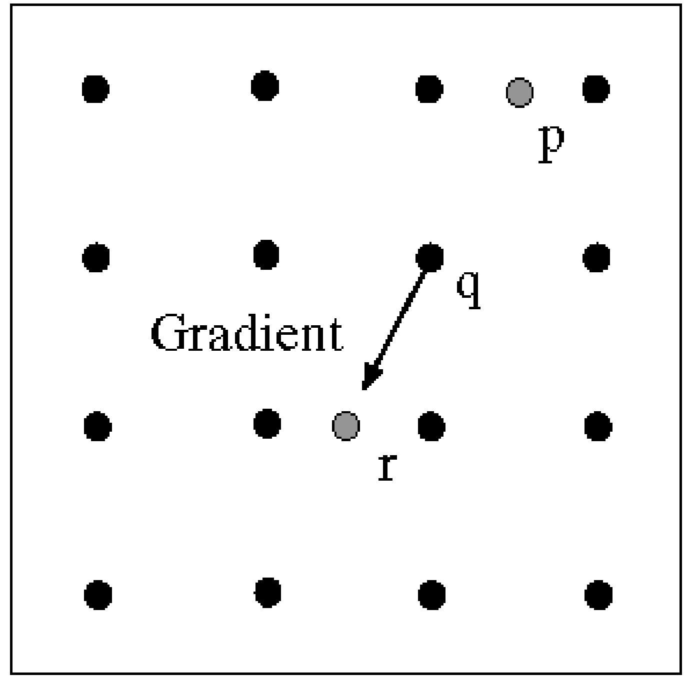
- Where is the edge?
- Zero-crossings of bottom graph

Canny Edge Detector

- Smooth image *I* with 2D Gaussian: $G * I$
- Find local edge normal directions for each pixel $\theta = \arctan \frac{I_y}{I_x}$
- Along this direction, compute image gradient $\left| \nabla (G * I) \right|$
- Locate edges by finding max gradient magnitude (**Non-maximum suppression**)

Non-maximum Suppression





- Check if pixel is local maximum along gradient direction
  - requires checking interpolated pixels p and r

The Canny Edge Detector

original image (Lena)

The Canny Edge Detector

magnitude of the gradient

The Canny Edge Detector

After non-maximum suppression

Canny Edge Detector

original

Canny with $\sigma = 1$

Canny with $\sigma = 2$

- The choice of σ depends on desired behavior
  - large σ detects large scale edges
  - small σ detects fine features

Image Scaling

This image is too big to fit on the screen. How can we reduce it?

How to generate a half-sized version?

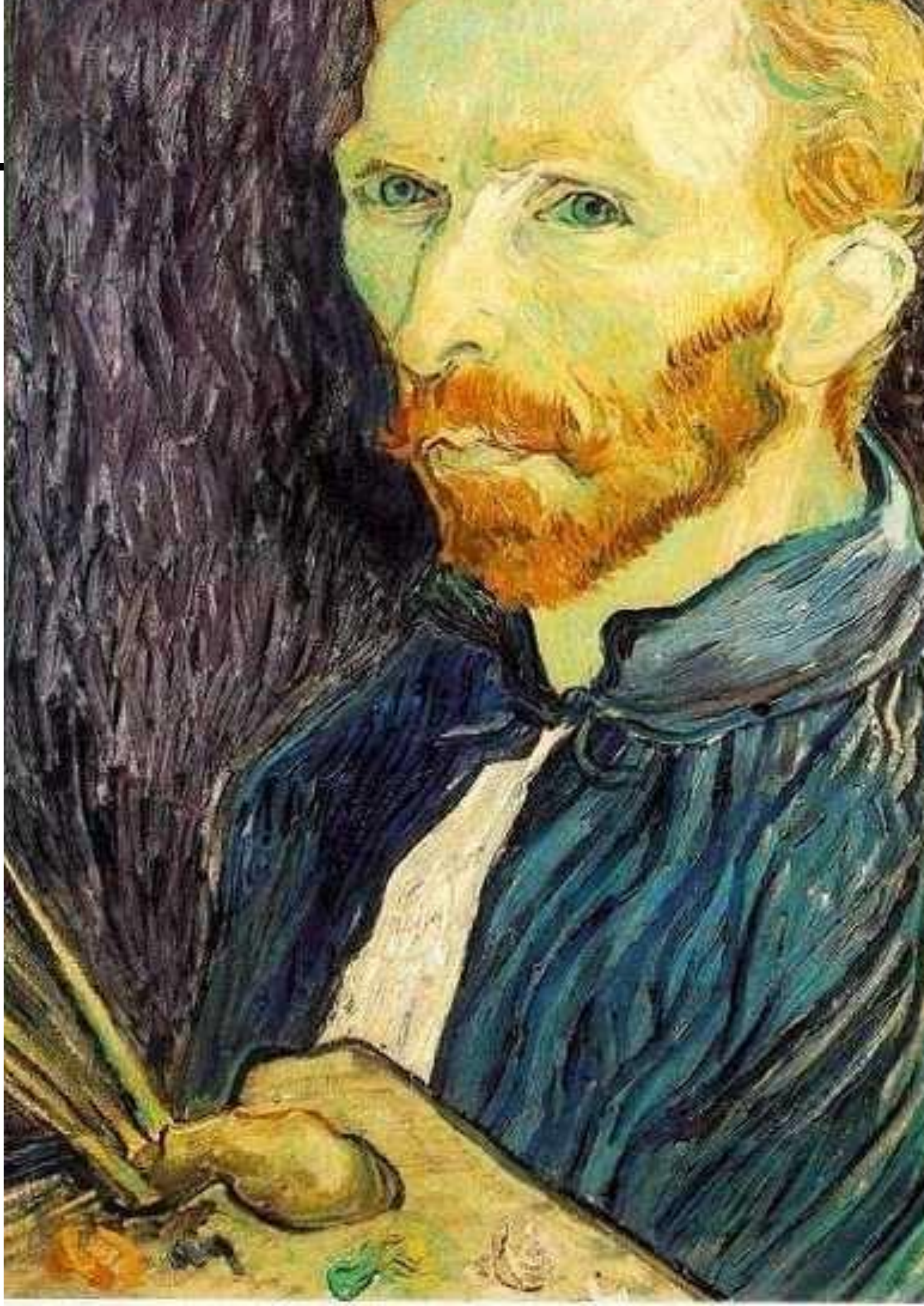
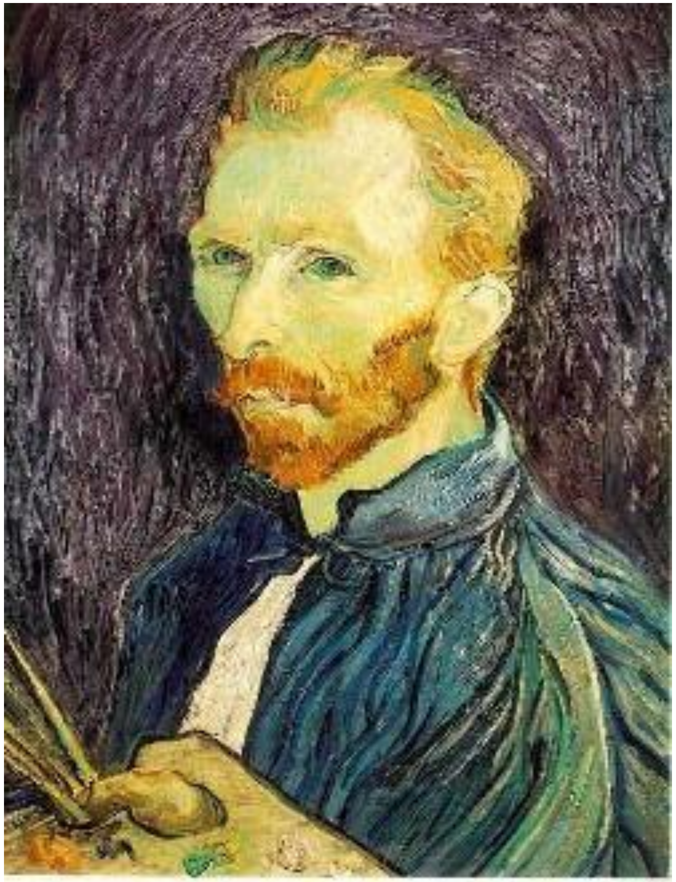


Image sub-sampling

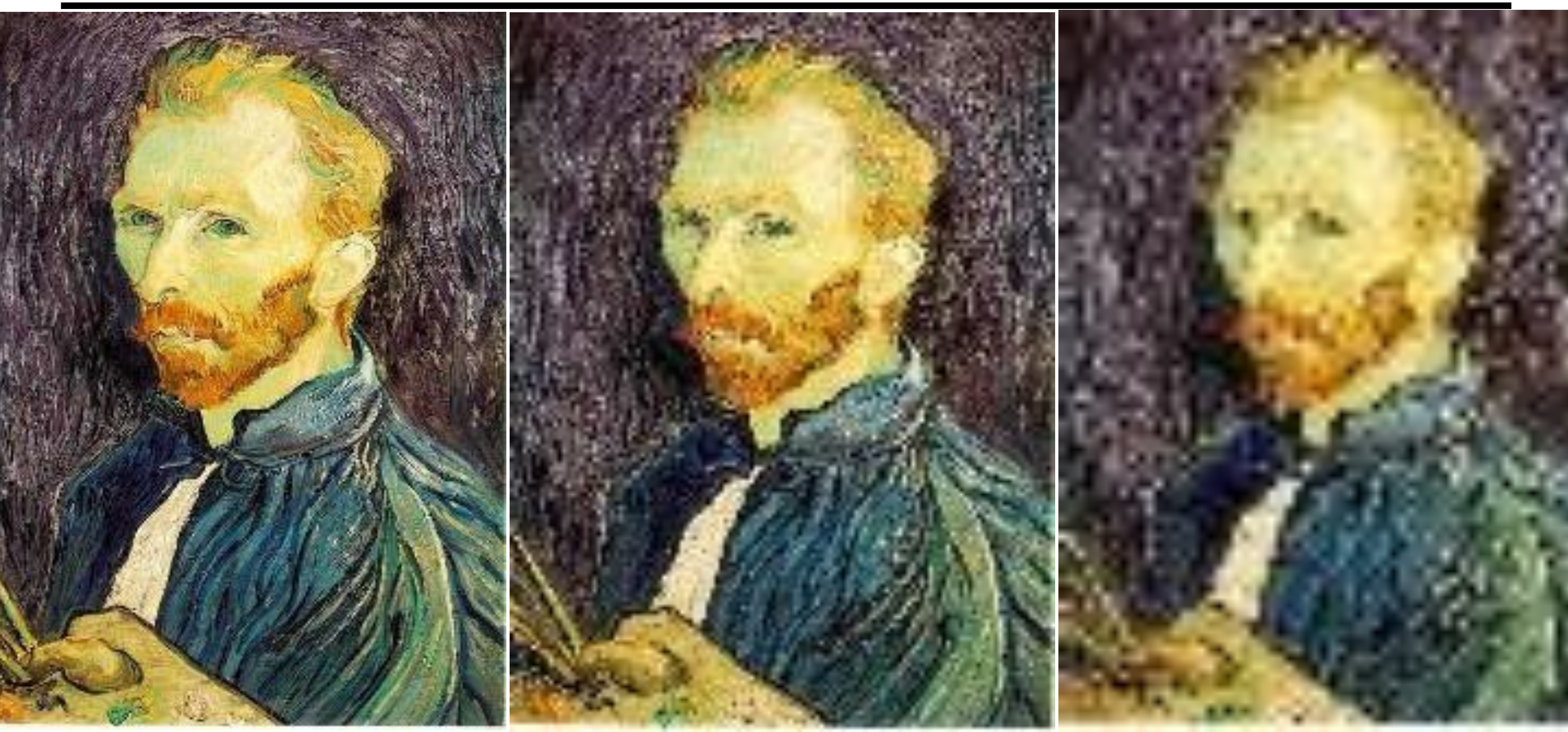


1/4

1/8

Throw away every other row and column to create a 1/2 size image
- called *image sub-sampling*

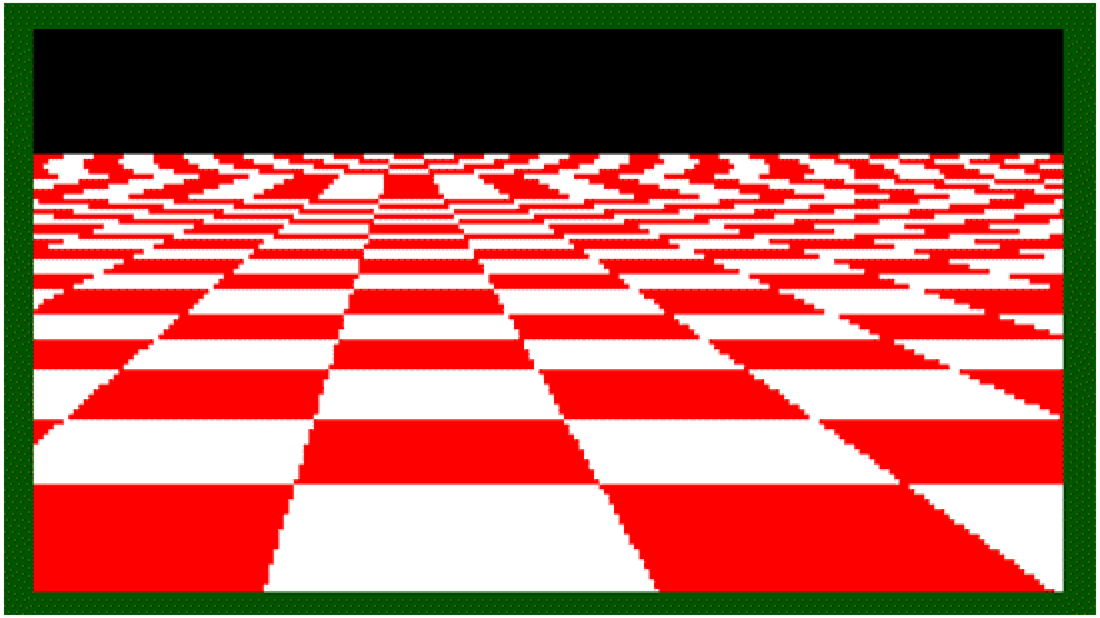
Image sub-sampling



1/2 1/4 (2x zoom) 1/8 (4x zoom)

Why does this look so crufty?

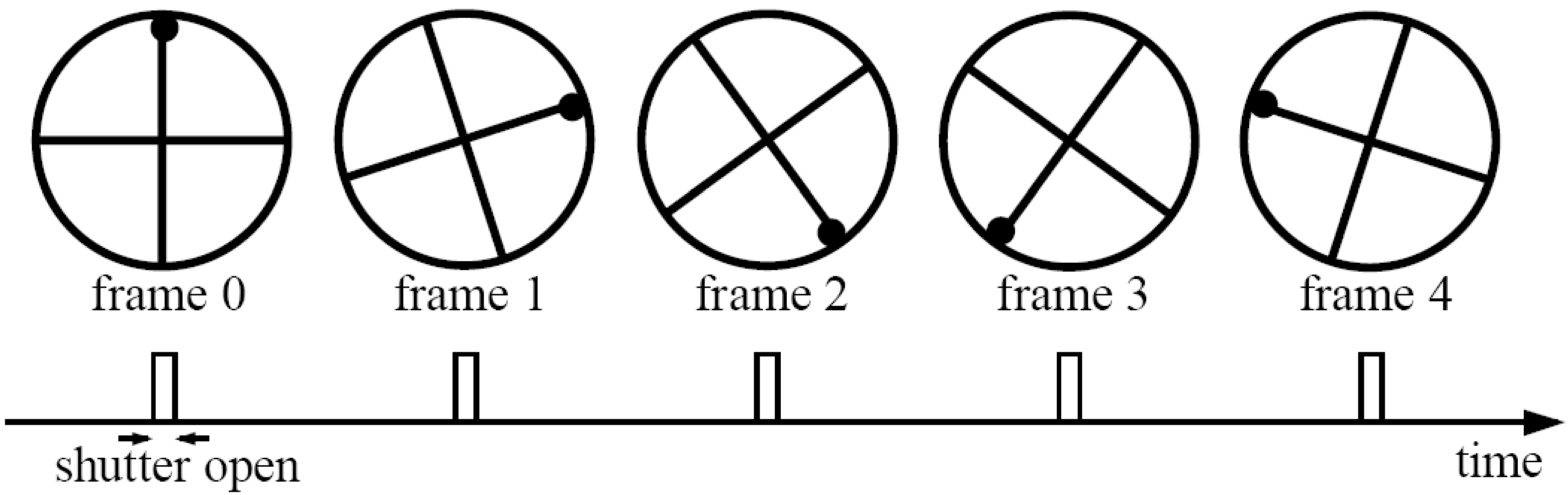
Even worse for synthetic images



Really bad in video

Imagine a spoked wheel moving to the right (rotating clockwise). Mark wheel with dot so we can see what's happening.

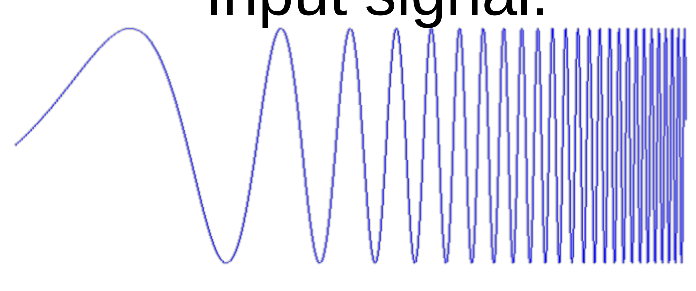
If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):



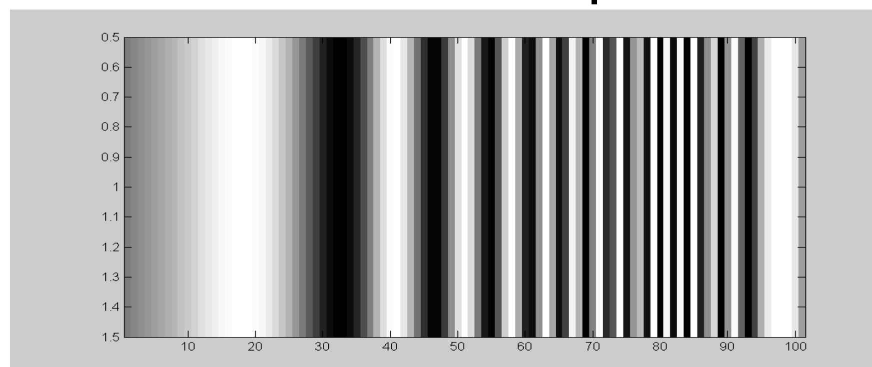
Without dot, wheel appears to be rotating slowly backwards! (counterclockwise)

Alias: n., an assumed name

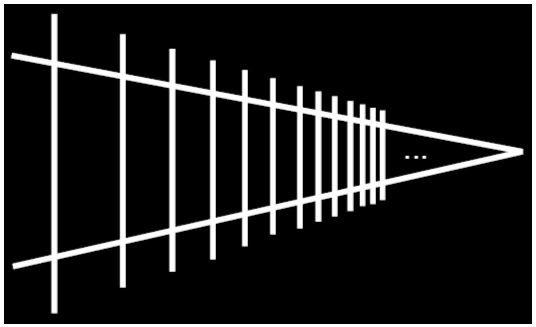
Input signal:



Matlab output:

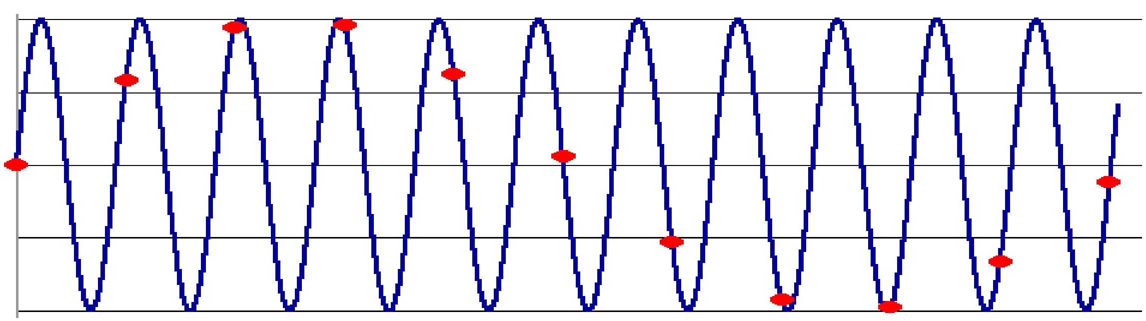


x = 0:.05:5; imagesc(sin((2.^x).*x))



Picket fence receding
Into the distance will
produce aliasing…

WHY?



Aj-aj-aj:
Alias!

Not enough samples

Aliasing

- occurs when your sampling rate is not high enough to capture the amount of detail in your image
- Can give you the wrong signal/image—an *alias*

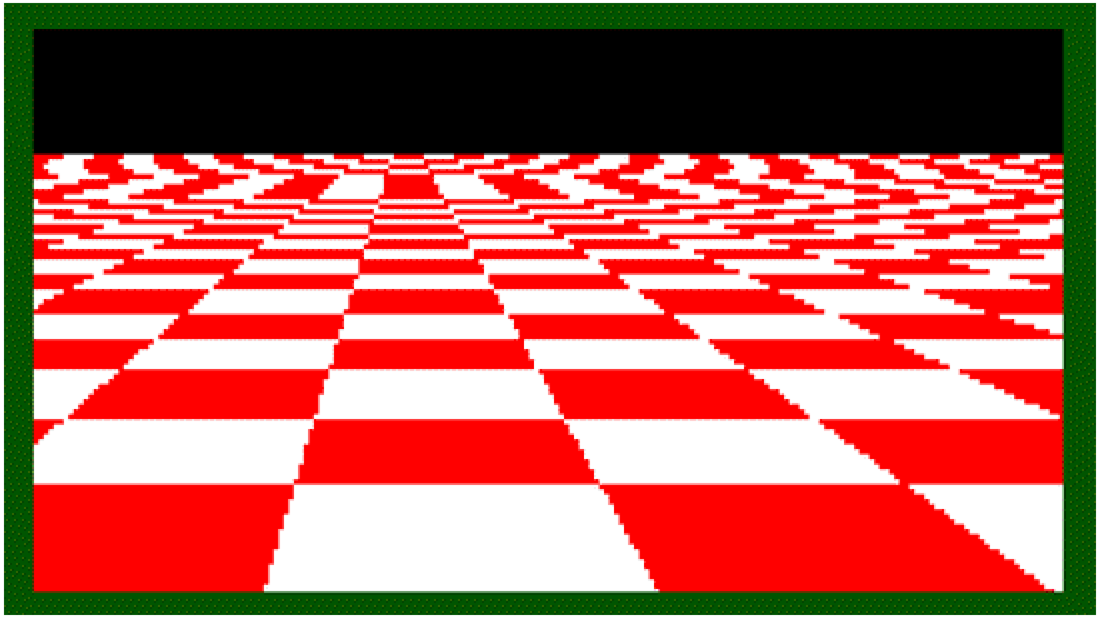
- Where can it happen in images?
- During image synthesis:
  - **sampling** continous singal into discrete signal
  - e.g. ray tracing, line drawing, function plotting, etc.
- During image processing:
  - **resampling** discrete signal at a different rate
  - e.g. Image warping, zooming in, zooming out, etc.

- To do sampling right, need to understand the structure of your signal/image
- Enter Monsieur Fourier…

Antialiasing

- What can be done?

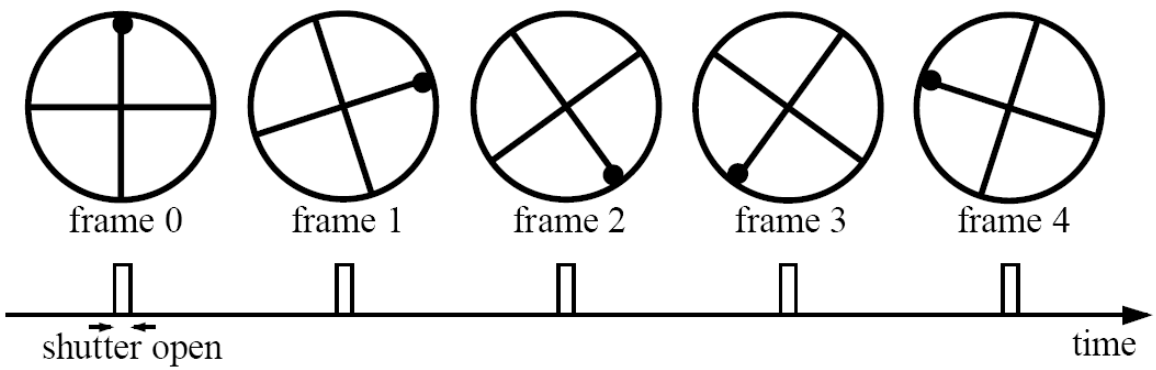
1. Raise sampling rate by *oversampling*
   - *Sample at k times the resolution*



Antialiasing

- What can be done?

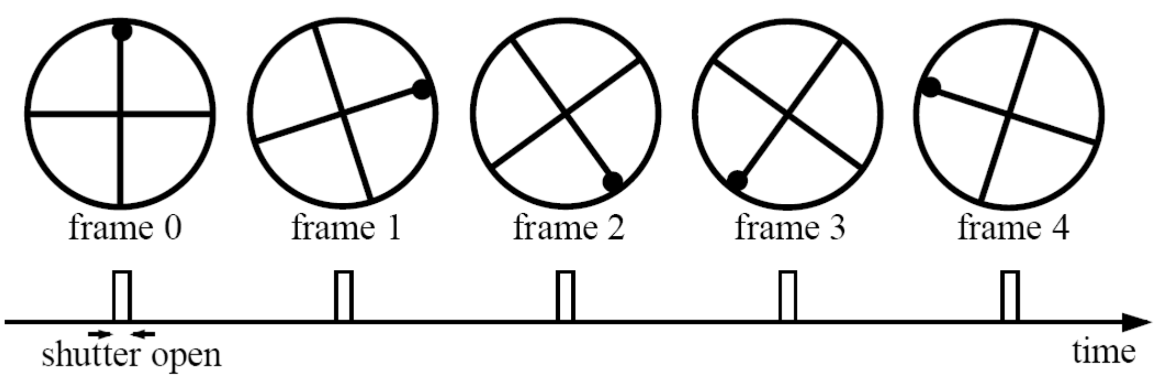
1. Raise sampling rate by *oversampling*
   - *Sample at k times the resolution*



Antialiasing

- What can be done?

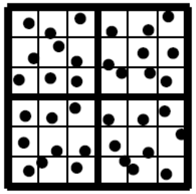
1. Raise sampling rate by *oversampling*
   - *Sample at k times the resolution*
2. Lower the max frequency by *prefiltering*
   - Smooth the signal enough
   - Works on discrete signals



Antialiasing

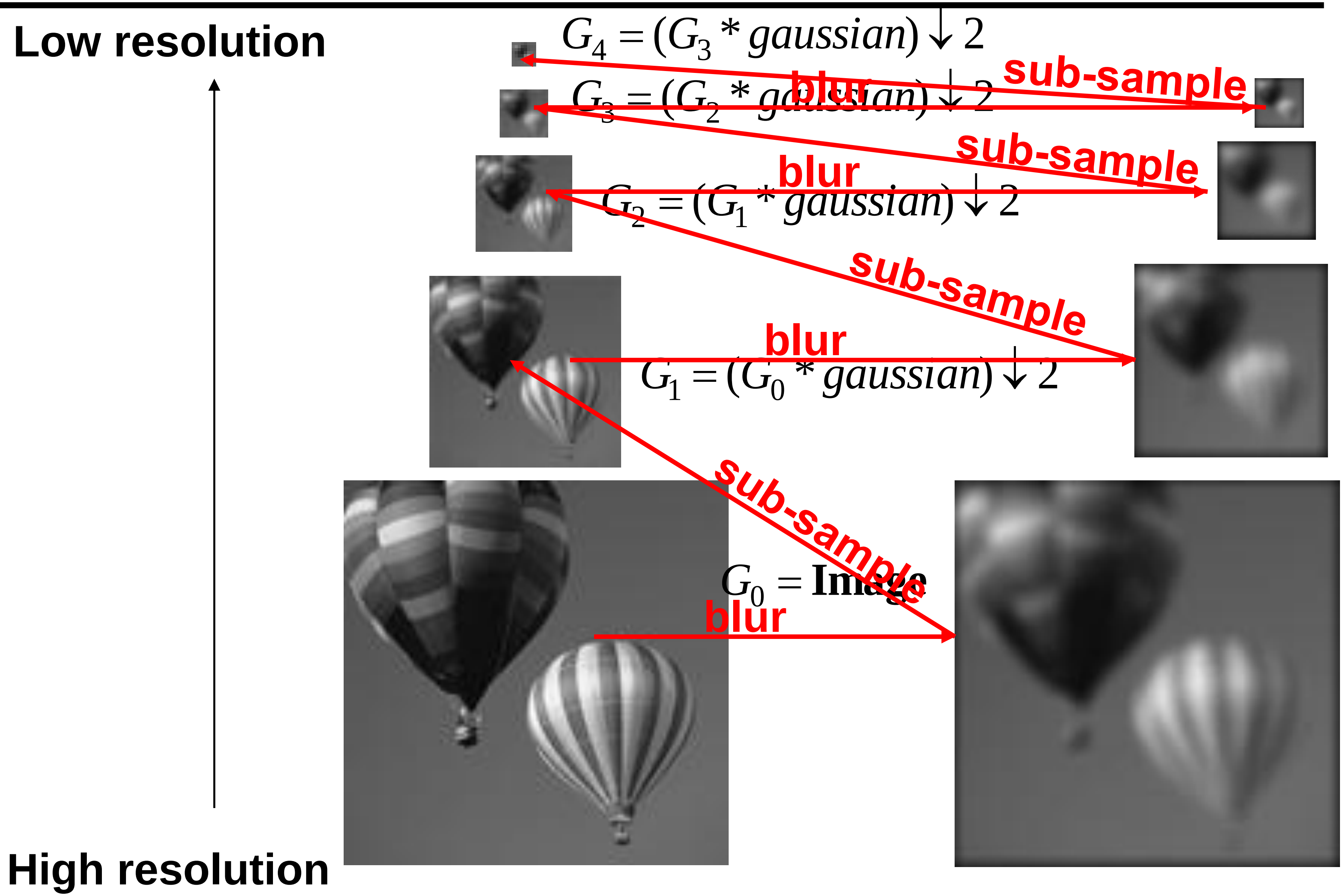
- What can be done?

1. Raise sampling rate by *oversampling*
   - *Sample at k times the resolution*
2. Lower the max frequency by *prefiltering*
   - Smooth the signal enough
   - Works on discrete signals
3. Improve sampling quality with better sampling (CS559)

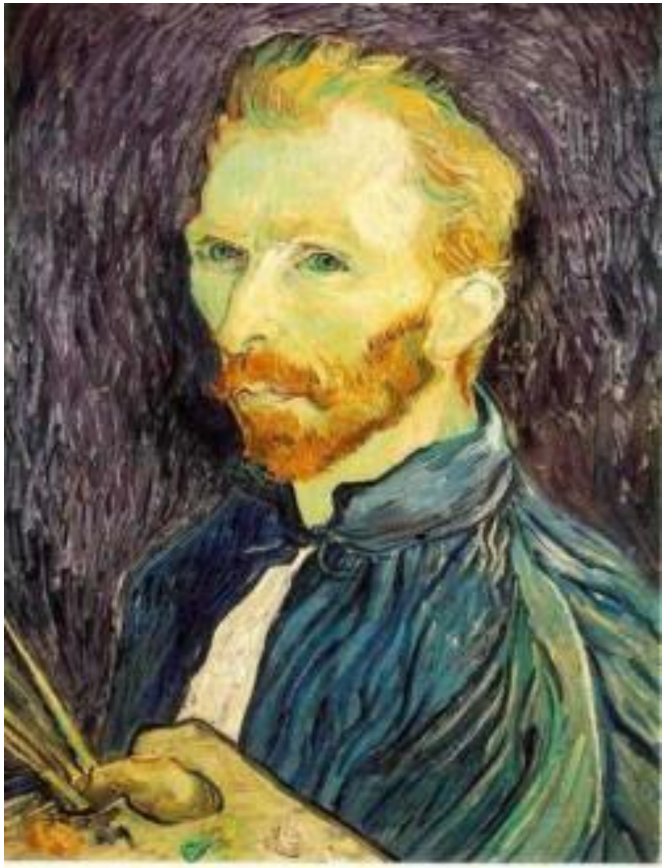


jittered,
9 samples per pixel

The Gaussian Pyramid



Gaussian pre-filtering



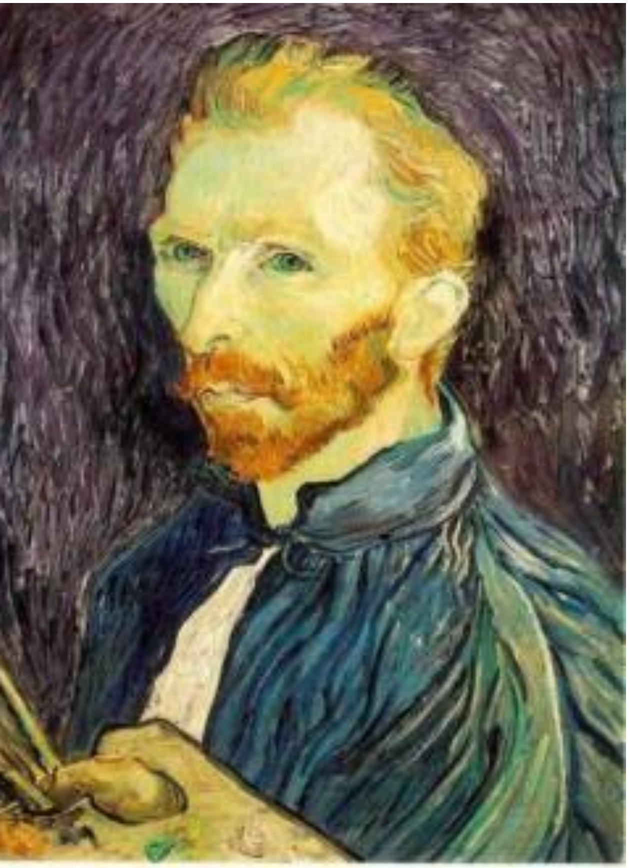
Gaussian 1/2

G 1/4

G 1/8

- Solution: filter the image, *then* subsample

Subsampling with Gaussian pre-filtering



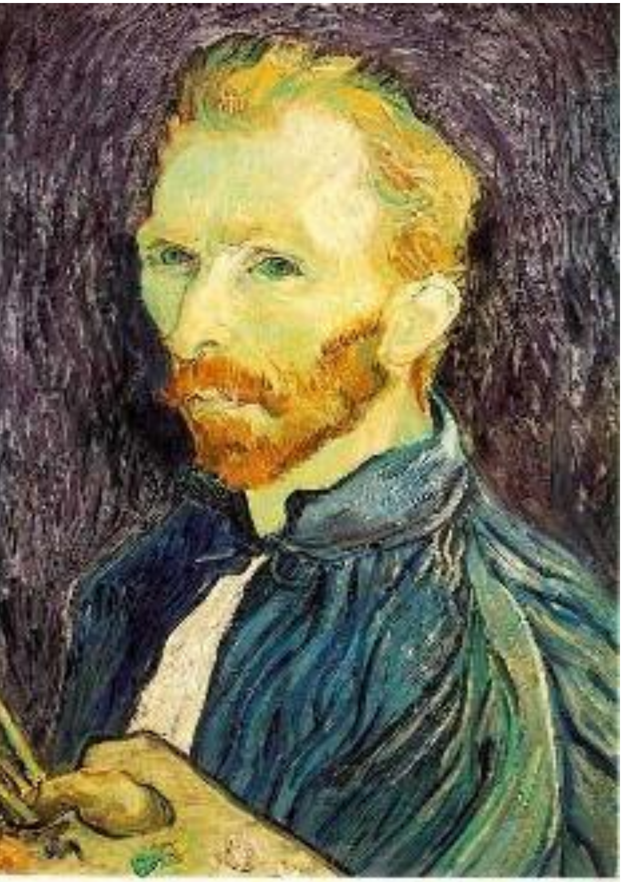
Gaussian 1/2

G 1/4

G 1/8

- Solution: filter the image, *then* subsample

Compare with...

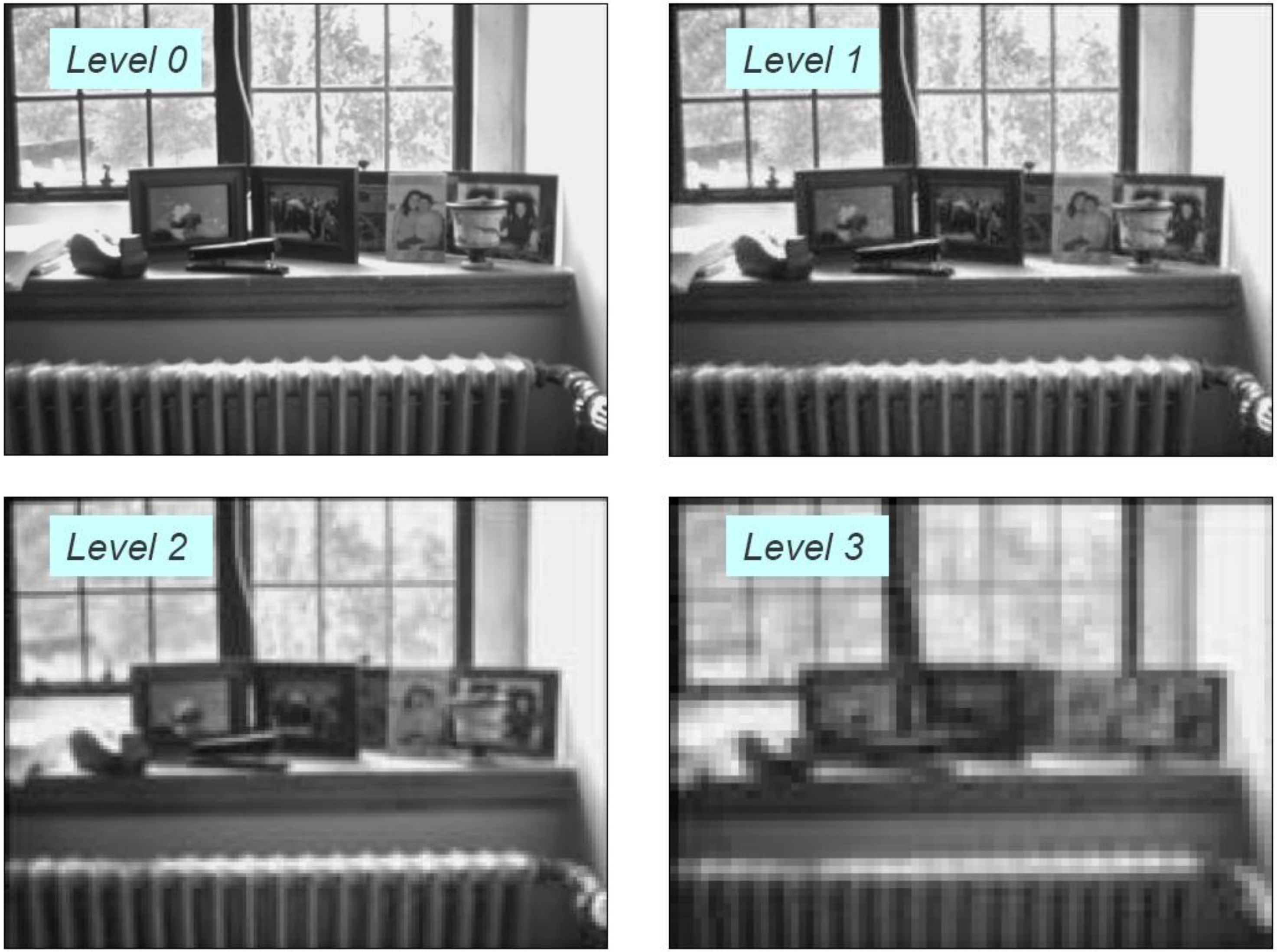


1/2

1/4 (2x zoom)

1/8 (4x zoom)

Pyramids at Same Resolution



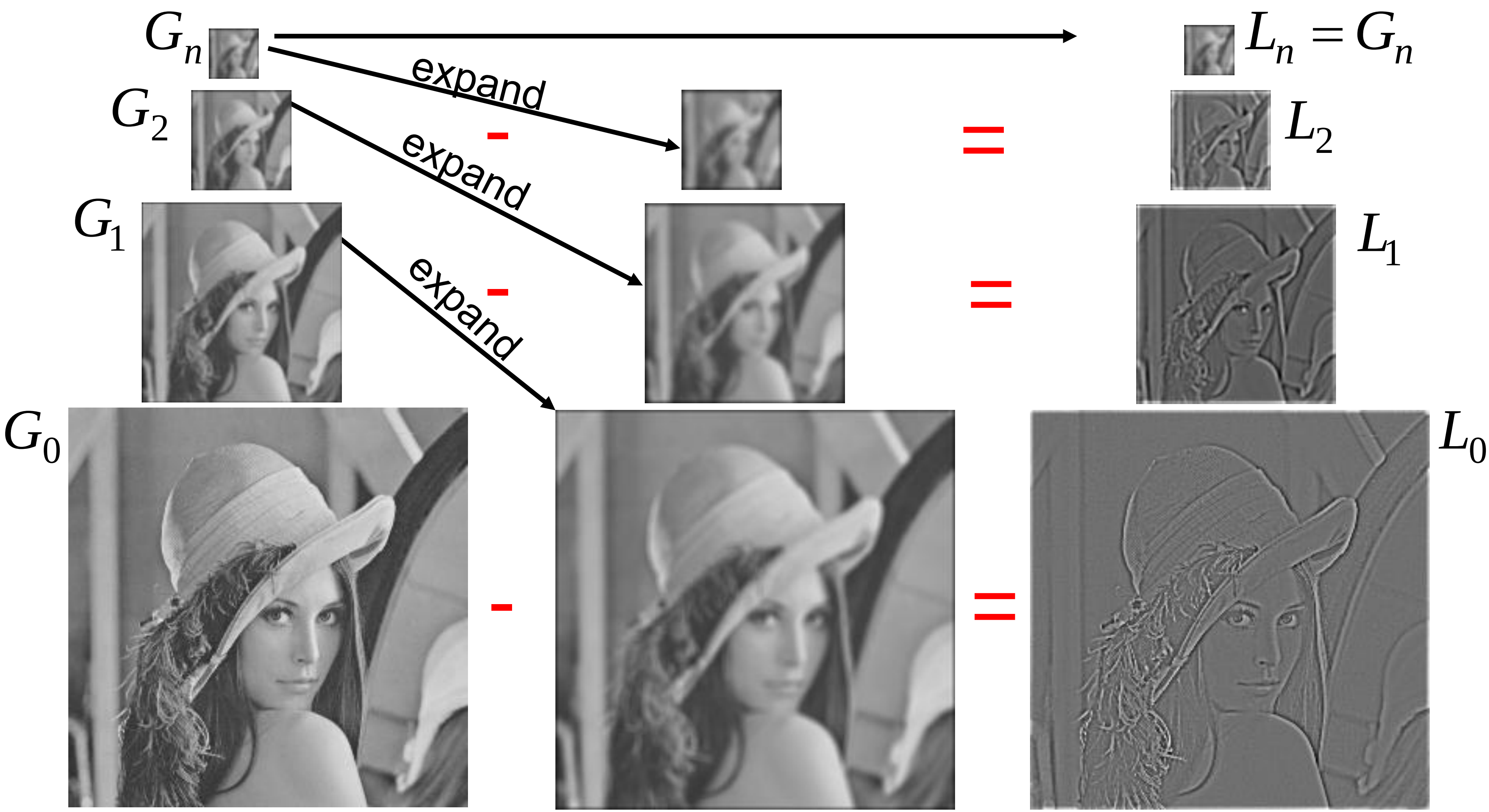
The Laplacian Pyramid

$$L_i = G_i - \text{expand}(G_{i+1})$$

$$G_i = L_i + \text{expand}(G_{i+1})$$

Gaussian Pyramid

Laplacian Pyramid



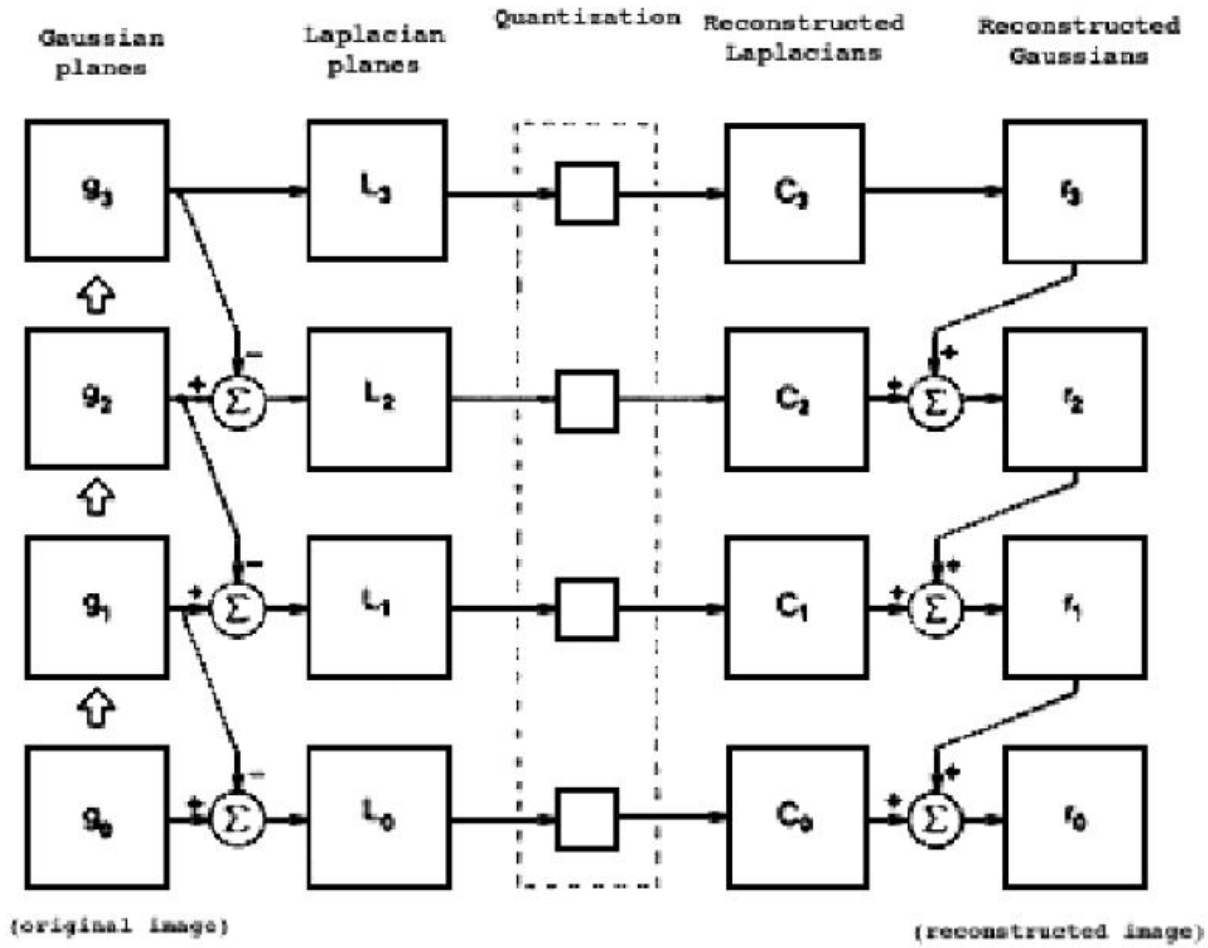


Fig. 10. A summary of the steps in Laplacian pyramid coding and decoding. First, the original image g_0 (lower left) is used to generate Gaussian pyramid levels g_1, g_2, … through repeated local averaging. Levels of the Laplacian pyramid L_0, L_1, … are then computed as the differences between adjacent Gaussian levels. Laplacian pyramid elements are quantized to yield the Laplacian pyramid code C_0, C_1, C_2, …. Finally, a reconstructed image r_0 is generated by summing levels of the code pyramid.

Recap

Image Processing: from basic concepts to latest techniques

- Filtering
- Edge detection
- Re-sampling and aliasing
- Image Pyramids (Gaussian and Laplacian)
- Next …