Panoramic Image Stitching

Feature Detection and Matching
Today

More on Mosaic
Projective Geometry
Single View Modeling

Vermeer's *Music Lesson*

Reconstructions by Criminisi et al.
Image Alignment

- Cylinder: Translation 2 DoF
- Plane: Homography 8 DoF
Plane perspective mosaics

- 8-parameter generalization of affine motion
  - works for pure rotation or planar surfaces
- Limitations:
  - local minima
  - slow convergence
Revisit Homography

\[
\begin{bmatrix}
    x_1 \\
    y_1 \\
    1
\end{bmatrix}
\sim
\begin{bmatrix}
    f & 0 & x_c \\
    0 & f & y_c \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    X \\
    Y \\
    Z
\end{bmatrix}
\]

\[
\begin{bmatrix}
    x_2 \\
    y_2 \\
    1
\end{bmatrix}
\sim
\begin{bmatrix}
    f & 0 & x_c \\
    0 & f & y_c \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    X \\
    Y \\
    Z
\end{bmatrix}
\]

\[KRK^{-1}x_1 \sim x_2\]
Estimate $f$ from $H$?

\[
\begin{align*}
&\begin{pmatrix} x_1 - x_c \\ y_1 - y_c \\ 1 \end{pmatrix} \sim \begin{bmatrix} f_1 & 0 & 0 \\ 0 & f_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \\
&\begin{pmatrix} x_2 - x_c \\ y_2 - y_c \\ 1 \end{pmatrix} \sim \begin{bmatrix} f_2 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}
\end{align*}
\]

\[
(K_2 R K_1^{-1}) x_1 \sim x_2
\]

\[
H
\]

\[
\begin{align*}
R &\sim K_2^{-1} H K_1 \\
&= \begin{bmatrix}
 a & b & c / f_1 \\
 d & e & g / f_1 \\
 h * f_2 & i * f_2 & j * f_2 / f_1 \\
\end{bmatrix}
\end{align*}
\]

$f_1 = \text{?}, \ f_2 = \text{?}$
The drifting problem

- Error accumulation
  - small errors accumulate over time
Bundle Adjustment

Associate each image $i$ with $K_i, R_i$

Each image $i$ has features $p_{ij}$

Trying to minimize total matching residuals

$$E(\text{all } f_i \text{ and } R_i) = \sum_{(i,m)} \sum_j \left\| p_{ij} \sim K_i R_i R_m^{-1} K_m^{-1} p_{mj} \right\|^2$$
Rotations

• How do we represent rotation matrices?

1. Axis / angle \((n, \theta)\)
   \[
   R = I + \sin \theta \, [n]_\times + (1 - \cos \theta) \, [n]_\times^2
   \]
   (Rodriguez Formula), with \([n]_\times\) be the cross product matrix.

\[
\mathbf{a} \times \mathbf{b} = [\mathbf{a}]_\times \mathbf{b} = \begin{bmatrix}
0 & -a_3 & a_2 \\
 a_3 & 0 & -a_1 \\
-a_2 & a_1 & 0
\end{bmatrix} \begin{bmatrix}
b_1 \\
b_2 \\
b_3
\end{bmatrix}
\]
Incremental rotation update

1. Small angle approximation
\[ \Delta R = I + \sin\theta \ [n]_\times + (1 - \cos\theta) \ [n]_\times^2 \]
\[ \approx I + \theta \ [n]_\times = I + [\omega]_\times \]
*linear in* \( \omega = \theta n \)

2. Update original \( R \) matrix
\[ R \leftarrow R \Delta R \]
Recognizing Panoramas

[Brown & Lowe, ICCV’03]
Finding the panoramas
Finding the panoramas
Algorithm overview

Algorithm: Panoramic Recognition

Input: \( n \) unordered images
Algorithm overview

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Input: $n$ unordered images

I. Extract SIFT features from all $n$ images
Algorithm overview

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II. Find $k$ nearest-neighbours for each feature using a k-d tree
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IV. Find connected components of image matches
Finding the panoramas
Finding the panoramas
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V. For each connected component:
   (i) Perform bundle adjustment to solve for the rotation $\theta_1, \theta_2, \theta_3$ and focal length $f$ of all cameras
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V. For each connected component:
   (i) Perform bundle adjustment to solve for the rotation \( \theta_1, \theta_2, \theta_3 \) and focal length \( f \) of all cameras

   (ii) Render panorama using multi-band blending

**Output:** Panoramic image(s)
[Brown & Lowe, ICCV 2003] 
[Brown, Szeliski, Winder, CVPR’05]
How well does this work?

Test on 100s of examples…
How well does this work?

Test on 100s of examples…

…still too many failures (5-10%) for consumer application
Matching Mistakes: False Positive
Matching Mistakes: False Positive
Matching Mistakes: False Negative

- Moving objects: large areas of disagreement
Matching Mistakes

• Accidental alignment
  – repeated / similar regions

• Failed alignments
  – moving objects / parallax
  – low overlap
  – “feature-less” regions

• No 100% reliable algorithm?
How can we fix these?

- Tune the feature detector
- Tune the feature matcher (cost metric)
- Tune the RANSAC stage (motion model)
- Tune the verification stage
- Use “higher-level” knowledge
  - e.g., typical camera motions

→ Sounds like a big “learning” problem
  - Need a large training/test data set (panoramas)
on to 3D…

Enough of images!

We want more from the image

We want real 3D scene walk-throughs:

Camera rotation
Camera translation
So, what can we do here?

- Model the scene as a set of planes!
The projective plane

- Why do we need homogeneous coordinates?
  - represent points at infinity, homographies, perspective projection, multi-view relationships
- What is the geometric intuition?
  - a point in the image is a ray in projective space

- Each point \((x, y)\) on the plane is represented by a ray \((sx, sy, s)\)
  - all points on the ray are equivalent: \((x, y, 1) \cong (sx, sy, s)\)
Projective lines

• What does a line in the image correspond to in projective space?

• A line is a plane of rays through origin
  – all rays \((x,y,z)\) satisfying: \(ax + by + cz = 0\)

  \[
  \text{in vector notation: } 0 = \begin{bmatrix} b & c \\ \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \end{bmatrix}
  \]

• A line is also represented as a homogeneous 3-vector \(l\)
Point and line duality

- A line \( l \) is a homogeneous 3-vector
- It is \( \perp \) to every point (ray) \( p \) on the line: \( l \cdot p = 0 \)

What is the line \( l \) spanned by rays \( p_1 \) and \( p_2 \)?
- \( l \) is \( \perp \) to \( p_1 \) and \( p_2 \) \( \Rightarrow \) \( l = p_1 \times p_2 \)
- \( l \) is the plane normal

What is the intersection of two lines \( l_1 \) and \( l_2 \)?
- \( p \) is \( \perp \) to \( l_1 \) and \( l_2 \) \( \Rightarrow \) \( p = l_1 \times l_2 \)

Points and lines are dual in projective space
- can switch the meanings of points and lines to get another formula
Ideal points and lines

- **Ideal point ("point at infinity")**
  - $p \approx (x, y, 0)$ – parallel to image plane
  - It has infinite image coordinates

**Ideal line**
- $l \approx (a, b, 0)$ – parallel to image plane
- Corresponds to a line in the image (finite coordinates)
Homographies of points and lines

• Computed by 3x3 matrix multiplication
  – To transform a point:  \( p' = Hp \)
  – To transform a line:  \( lp=0 \rightarrow l'p'=0 \)
    – 0 = lp = lH⁻¹Hp = lH⁻¹p’ \( \Rightarrow \)  l’ = lH⁻¹
  – lines are transformed by postmultiplication of H⁻¹
3D projective geometry

- These concepts generalize naturally to 3D
  - Homogeneous coordinates
    - Projective 3D points have four coords: \( P = (X,Y,Z,W) \)
  - Duality
    - A plane \( N \) is also represented by a 4-vector
    - Points and planes are dual in 4D: \( N \cdot P = 0 \)
  - Projective transformations
    - Represented by 4x4 matrices \( T \): \( P' = TP, \quad N' = N \cdot T^{-1} \)
3D to 2D: “perspective” projection

• Matrix Projection: \( \mathbf{p} = \begin{bmatrix} wx \\ wy \\ w \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \Pi \mathbf{p} \)

What is \textit{not} preserved under perspective projection?

What IS preserved?
Vanishing points

- Vanishing point
  - projection of a point at infinity
Vanishing points (2D)
Vanishing points

- Properties
  - Any two parallel lines have the same vanishing point $v$
  - The ray from $C$ through $v$ is parallel to the lines
  - An image may have more than one vanishing point
    - in fact every pixel is a potential vanishing point
Vanishing lines

- Multiple Vanishing Points
  - Any set of parallel lines on the plane define a vanishing point
  - The union of all of these vanishing points is the *horizon line*
    - also called *vanishing line*
  - Note that different planes define different vanishing lines
Vanishing lines

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Computing vanishing points

- Properties
  - $\mathbf{v} = \Pi \mathbf{P}_\infty$
  - $\mathbf{P}_\infty$ is a point at infinity, $\mathbf{v}$ is its projection
  - They depend only on line direction
  - Parallel lines $\mathbf{P}_0 + t\mathbf{D}$, $\mathbf{P}_1 + t\mathbf{D}$ intersect at $\mathbf{P}_\infty$

\[
\mathbf{P}_t = \begin{bmatrix}
P_x + tD_x \\
P_y + tD_y \\
P_z + tD_z \\
1
\end{bmatrix} \equiv \begin{bmatrix}
P_x / t + D_x \\
P_y / t + D_y \\
P_z / t + D_z \\
1 / t
\end{bmatrix} \quad t \to \infty \quad \mathbf{P}_\infty \equiv \begin{bmatrix}
D_x \\
D_y \\
D_z \\
0
\end{bmatrix}
\]
Computing vanishing lines

• Properties
  – \( I \) is intersection of horizontal plane through \( C \) with image plane
  – Compute \( I \) from two sets of parallel lines on ground plane
  – All points at same height as \( C \) project to \( I \)
    • points higher than \( C \) project above \( I \)
  – Provides way of comparing height of objects in the scene
Fun with vanishing points
Perspective cues
Perspective cues
Perspective cues
Comparing heights

Vanishing Point
Measuring height
Computing vanishing points (from lines)

• Intersect $p_1 q_1$ with $p_2 q_2$

\[ v = (p_1 \times q_1) \times (p_2 \times q_2) \]

Least squares version

• Better to use more than two lines and compute the “closest” point of intersection

• See notes by Bob Collins for one good way of doing this:
Measuring height without a ruler

Compute $Z$ from image measurements

- Need more than vanishing points to do this
The cross ratio

- A Projective Invariant
  - Something that does not change under projective transformations (including perspective projection)

The cross-ratio of 4 collinear points

\[ \frac{\|P_3 - P_1\| \|P_4 - P_2\|}{\|P_3 - P_2\| \|P_4 - P_1\|} \]

Can permute the point ordering

- \(4! = 24\) different orders (but only 6 distinct values)

This is the fundamental invariant of projective geometry
Measuring height

$$\frac{\|T - B\|}{\|R - B\|} \frac{\|\infty - R\|}{\|\infty - T\|} = \frac{H}{R}$$
scene cross ratio

$$\frac{\|t - b\|}{\|r - b\|} \frac{\|v_z - r\|}{\|v_z - t\|} = \frac{H}{R}$$
image cross ratio

scene points represented as $P = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$
image points as $p = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$
Measuring height

vanishing line (horizon)

\[ v \approx (b \times b_0) \times (v_x \times v_y) \]

\[ t \approx (v \times t_0) \times (r \times b) \]

image cross ratio

\[
\frac{\|t - b\| \|v_Z - r\|}{\|r - b\| \|v_Z - t\|} = \frac{H}{R}
\]
What if the point on the ground plane $b_0$ is not known?

- Here the guy is standing on the box, height of box is known
- Use one side of the box to help find $b_0$ as shown above
Computing \((X, Y, Z)\) coordinates

- Okay, we know how to compute height (\(Z\) coords)
  - how can we compute \(X, Y\)?
Camera calibration

• Goal: estimate the camera parameters
  – Version 1: solve for projection matrix
    \[
    \mathbf{x} = \begin{bmatrix}
    wx \\
    wy \\
    w
    \end{bmatrix}
    = \begin{bmatrix}
    * & * & * & * \\
    * & * & * & * \\
    * & * & * & *
    \end{bmatrix}
    \begin{bmatrix}
    X \\
    Y \\
    Z \\
    1
    \end{bmatrix}
    = \Pi \mathbf{x}
    \]

• Version 2: solve for camera parameters separately
  – intrinsics (focal length, principle point, pixel size)
  – extrinsics (rotation angles, translation)
  – radial distortion
Vanishing points and projection matrix

\[
\Pi = \begin{bmatrix}
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
\pi_1 & \pi_2 & \pi_3 & \pi_4
\end{bmatrix} = \begin{bmatrix}
1 & \pi_2 & \pi_3 & \pi_4
\end{bmatrix}
\]

- \(\pi_1 = \Pi \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T = \mathbf{v}_x\) (X vanishing point)
- similarly, \(\pi_2 = \mathbf{v}_y, \pi_3 = \mathbf{v}_z\)
- \(\pi_4 = \Pi \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T = \) projection of world origin

\[
\Pi = \begin{bmatrix}
\mathbf{v}_X & \mathbf{v}_Y & \mathbf{v}_Z & \mathbf{o}
\end{bmatrix}
\]

Not So Fast! We only know \(\mathbf{v}\)'s up to a scale factor

\[
\Pi = \begin{bmatrix}
\mathbf{v}_X & b\mathbf{v}_Y & c\mathbf{v}_Z & \mathbf{o}
\end{bmatrix}
\]

- Can fully specify by providing 3 reference points
3D Modeling from a photograph

The Virtual Museum
A. Criminisi @ Microsoft, 2002

https://research.microsoft.com/vision/cambridge/3d/3dart.htm