Generalized SVM Optimization Formulation

In general, an SVM optimization problem can be viewed as instances of the following problem:

\[
\min_w \{ f(\langle w, \psi(x_1) \rangle, \langle w, \psi(x_2) \rangle, \ldots, \langle w, \psi(x_m) \rangle) + R(\|w\|) \}
\]

That is, \( f \) is a function with the mapping

\[
f : \mathbb{R}^m \rightarrow \mathbb{R}
\]

and \( R \) is a monotonically non-decreasing function with the mapping

\[
R : [0, \infty) \rightarrow \mathbb{R}
\]

Hard-SVM

Recall the hard-SVM formulation:

\[
\text{minimize} \quad \|w\|^2 \\
\text{subject to} \quad \forall i \ y_i \langle w, x \rangle \geq 1
\]

We see here that \( f \) is the function

\[
f(v) = \begin{cases} 
\infty & \text{if } \exists i \ y_i v_i < 1 \\
0 & \text{otherwise}
\end{cases}
\]

and \( R \) is the function

\[
R(a) = a^2
\]

Soft-SVM

Recall the soft-SVM formulation:

\[
\text{minimize} \quad \|w\|^2 + \lambda L^\text{hinge}_S(w)
\]

where

\[
L^\text{hinge}_S(w) = \frac{1}{m} \sum_{i=1}^{m} \max\{0, 1 - y_i \langle w, \psi(x_i) \rangle\}
\]

Here, \( f \) is the function

\[
f(v) = \lambda L^\text{hinge}_S(w)
\]

and \( R \) is the function

\[
R(a) = a^2
\]