Generalized SVM Optimization Formulation

In general, an SVM optimization problem can be viewed as instances of the following problem:

$$\min_{\mathbf{w}} \{ \, f(\langle \mathbf{w}, \psi(\mathbf{x}_1) \rangle, \langle \mathbf{w}, \psi(\mathbf{x}_2) \rangle, \ldots, \langle \mathbf{w}, \psi(\mathbf{x}_m) \rangle) + R(\|\mathbf{w}\|) \, \}$$

That is, f is a function with the mapping

$$f : \mathbb{R}^m \to \mathbb{R}$$

and R is a monotonically non-decreasing function with the mapping

$$R : [0, \infty) \to \mathbb{R}$$

Hard-SVM

Recall the hard-SVM formulation:

$$\begin{array}{cc} \text{minimize} & \|\mathbf{w}\|^2 \\ \text{subject to} & \forall i \; y_i \langle \mathbf{w}, \mathbf{x} \rangle \geq 1 \end{array}$$

We see here that f is the function

$$f(\mathbf{v}) = \begin{cases} \infty & \exists i \; y_i v_i < 1 \\ 0 & \text{otherwise} \end{cases}$$

and R is the function

$$R(a) = a^2$$

Soft-SVM

Recall the soft-SVM formulation:

$$\begin{array}{cc} \text{minimize} & \|\mathbf{w}\|^2 + \lambda L_S^{\text{hinge}}(\mathbf{w}) \end{array}$$

where

$$L_S^{\text{hinge}}(\mathbf{w}) = \frac{1}{m} \sum_{i=1}^{m} \max\{0, 1 - y \langle \mathbf{w}, \psi(\mathbf{x}_i) \rangle\}$$

Here, f is the function

$$f(\mathbf{v}) = \lambda L_S^{\text{hinge}}(\mathbf{w})$$

and R is the function

$$R(a) = a^2$$