

The sum of n uniform sums over the range $[0, 1]$ is the Irwin-Hall distribution. ^(IHD) The cumulative distribution function (CDF) of the IHD is:

$$\frac{1}{n!} \sum_{k=0}^{\lfloor x \rfloor} (-1)^k \binom{n}{k} (x-k)^{n-1}$$

We want to figure out the likelihood of n draws being less than x . So, x is 1. This simplifies as below:

$$\frac{1}{n!} \sum_{k=0}^1 (-1)^k \binom{n}{k} (1-k)^{n-1}$$

$$= \frac{1}{n!} \left((-1)^0 \binom{n}{0} (1-0)^{n-1} + (-1)^1 \binom{n}{1} (1-1)^{n-1} \right)$$

$$= \frac{1}{n!} (1 \cdot 1 \cdot 1 + 0)$$

$$= \frac{1}{n!}$$