Computer Science 540
Introduction to Artificial Intelligence
Homework assignment 3
due: October 27th at the beginning of class

1. Create the following in propositional logic:
   (a) If possible construct a valid sentence that uses only the symbols P, Q, and R and the implication connective. Symbols and connectives may be used more than once and all of the symbols must be used at least once. If it is possible then provide the sentence with its associated truth table. If it is not then justify your answer.
   (b) If possible construct a satisfiable sentence that is not valid using the same symbols and connectives as in (a) above. If it is possible then provide the sentence with its associated truth table. If it is not then justify your answer.
   (c) If possible construct an unsatisfiable sentence with the same limitations on symbols and connectives as in (a) above? If it is possible then provide the sentence with its associated truth table. If it is not then justify your answer.
2. For each of the following construct a proof using natural deduction. For each step of the proof identify which previous sentences are being used and what inference mechanism is being used. You may only use the following inference mechanisms: Modus Ponens, And-Elimination, And-Introduction and the equivalences found in figure 7.11 of the textbook.
   (a) $\neg C \Rightarrow (A \vee B)$
   $\neg D \Rightarrow (C \vee \neg B)$
   $\neg (C \vee D)$

   $(A \vee B) \wedge (C \vee \neg B)$

   (b) $\neg (N \Rightarrow I)$
   $\neg I \Rightarrow C$

   C

   (c) $G \Leftrightarrow \neg H$

   $\neg G \Leftrightarrow H$

3. Below is listed a knowledge base in first order logic:

   $\forall x\ Horse(x) \vee Cow(x) \vee Pig(x) \Rightarrow Mammal(x)$

   $\forall x,y\ Offspring(x,y) \wedge Horse(y) \Rightarrow Horse(x)$

   $Horse(Bluebeard)$

   $Parent(Bluebeard,Charlie)$

   $\forall x,y\ Offspring(x,y) \Leftrightarrow Parent(y,x)$

   $\forall x\ Mammal(x) \Rightarrow \exists y\ Parent(y,x)$

   (a) Convert the FOL sentences into Conjunctive Normal Form.
   (b) Using the CNF sentences that you made for part (b) and resolution by refutation, construct the resolution tree that proves ***Horse(Charlie)***.

4. For each pair of atomic sentences, give the most general unifier if it exists:
   (a) P(A,B,B), P(x,y,z).
   (b) Q(y,G(A,B)), Q(G(x,x),y).
   (c) Older(Father(y),y), Older(Father(x),John).
   (d) Knows(Father(y),y), Knows(x,x).
5. Convert the following English sentences into first order logic. Use only the predicates and constants listed below:

   Predicates:

   barber(x) – x is a barber
   man(x) – x is a man
   person(x) – x is a person
   shaves(x,y) – x shaves y
   time(t) – t is a time
   fool(x,y,t) – x fools y at time t
   happy(x) – x is happy
   x=y – x is the same object as y

   constants:

   You

   English sentences:

   (a) Any person who shaves himself is happy.
   (b) There are exactly two people who are happy.
   (c) Either you shave yourself or you are happy, but not both.
   (d) There is a barber who shaves all men who do not shave themselves.
   (e) You can fool some of the people all of the time, and you can fool all of the people some of the time, but you can't fool all of the people all of the time.