CS730 Open-Book Final Exam

May 10, 2001
7:15-9:15 a.m., 1221 Computer Sciences & Statistics

Answer all questions: 1, 2 & 3. If any question is missing from your sheets, please inform the instructor.

Answers to all three questions can be printed on a single page.

Last Name (Print):____________________

First Name: ____________________

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Grades

1. **Question 1:**____________

2. **Question 2:**____________

3. **Question 3:**____________

4. **Total:**_______________

1. (10 points) Let $(x^{i+1},\, u^{i+1})$ be a KKT point of the IQP iteration:

$$\min_x \{\nabla f(x^i)(x - x^i) + \frac{1}{2}(x - x^i)G^i(x - x^i) \;\Big|\; g(x^i) + \nabla g(x^i)(x - x^i) \leq 0\},$$

for the nonlinear program

$$\min_x \;\{f(x) \mid g(x) \leq 0\}, \tag{$*$}$$

where $f\colon R^n \to R$ and $g\colon R^n \to R^m$ are differentiable and convex on R^n and G^i is a symmetric $n \times n$ positive definite matrix. Suppose that $(x^{i+1} - x^i)G^i(x^{i+1} - x^i) = 0$. Is x^{i+1} a solution of (*)? Justify your claim.

Answer: Yes. . . . . .No. . . . . .

Justification:

Problem 1 Sheet

2. (10 points) Consider the quadratic program

$$\min_{x_1,x_2} -x_1^2 + x_1x_2 - \frac{1}{2}x_2^2 - x_1$$

$$\text{s.t.} - 1 \leq x_1 \leq 1, \quad -1 \leq x_2 \leq 1$$

Start with $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Compute x^1 using the Gradient Projection Algorithm:

$$x^1 = P(x - \lambda \nabla f(x)), \quad \lambda \geq 0.$$

Is x^1 a solution of the quadratic program? Justify.

Answer: Yes. . . . . .No. . . . . .

Justification:

Problem 2 Sheet

3. (10 points) Let $f\colon R^n \to R$ have a Lipschitz continuous gradient with Lipschitz constant $K > 0$. If $\nabla f(x^i) \neq 0$ for some x^i in R^n, give an **upper bound** to $\min_x f(x)$, in terms of x^i and K, that is strictly **less** than $f(x^i)$.

Hint: Use the following inequality, a consequence of the Quadratic Bound Lemma:

$$\min_x f(x) \ \leq \ \min_x (f(x^i) + \nabla f(x^i)(x - x^i) + \frac{K}{2}\|x - x^i\|^2)$$

Answer:

$$\min_x f(x) \ \leq \ f(x^i) - \ldots$$

Justification:

Problem 3 Sheet

Extra Sheet