CS 730 Nonlinear Programming Algorithms
Open-Book Final Exam
Thursday May 11, 1995
11:00 a.m. - 12:55 p.m., Room 1263 C.S.

1. Is the origin $(0,0) \in R^2$ an optimal solution for $\displaystyle\min_{(x_1,x_2)\geq 0} \frac{1}{2}x_1^2 - x_1 + x_1x_2$? Justify your answer.

2.

   (a) Derive an explicit expression for $x_1(\alpha)$ and $x_2(\alpha)$ where

   $$x(\alpha) \in \arg\min_{x\in R^2} f(x) + \frac{\alpha}{2}\|g(x)_+\|_2^2, \ \alpha > 0,$$

   for the linear program

   $$\min\ x_1 + x_2 \qquad \text{s.t.} \ -x_1 + x_2 + 1 \leq 0, \ -x_2 \leq 0.$$

   (b) Obtain a solution to the linear program from $x(\alpha)$.

3. Let $f : R^n \to R$, $g : R^n \to R^m$, be differentiable and convex on R^n. Suppose that for some $u^i \in R^m$ and $\alpha > 0$:

   $$x^i \in \arg\min_{x\in R^n} f(x) + \frac{1}{2\alpha}\left\|\left(\alpha g(x) + u^i\right)_+\right\|_2^2,$$

   $$\left(\alpha g(x^i) + u^i\right)_+ g(x^i) \geq 0, \ g(x^i) \leq 0$$

   What can you say about x^i relative to the problem:

   $$\min f(x) \qquad \text{s.t.} \ g(x) \leq 0?$$

   Justify your claim.