CS 730 Nonlinear Programming Algorithms
Open-Book Final Exam
Monday, May 6, 1996
11:00 a.m. - 12:55 p.m., Room 1325 C.S.

1. Consider the problem:
$$\min_{(x_1,x_2)\geq 0} \quad 2x_1^2 - 2x_1x_2 + x_2^2 + 4x_1 - 2x_2$$
Take one gradient projection step from $x^0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Is x^0 optimal? Why?

2. Let
$$x(r) \in \arg\min\left\{f(x) - r\sum_{i=1}^{m} \log\Big(-g_j(x)\Big)\Big|g(x) < 0\right\}$$
where $f\colon R^n \to R$, $g\colon R^n \to R^m$ are differentiable, convex functions on R^n and $r > 0$.
   (a) Give the <u>best</u> estimate $s(r)$ of
   $$\inf_x \left\{f(x)\Big|g(x) \leq 0\right\},$$
   in terms of $x(r)$ and other <u>known</u> quantities. (Not the best is not good enough!)

   (b) Give a bound on the error in your estimate of the infimum, that is on
   $$\left|s(r) - \inf_x \left\{f(x)\Big|g(x) \leq 0\right\}\right|.$$

3. Let $f\colon R^n \to R$, $g\colon R^n \to R^m$ be differentiable on R^n, let $H^i \in R^{n\times n}$ and let
$$x^i \in \arg\min_x \left\{\nabla f(x^i)(x - x^i) + \frac{1}{2}(x - x^i)H^i(x - x^i)\Big|g(x^i) + \nabla g(x^i)(x - x^i) \leq 0\right\}$$
   (a) What is the most you can say about x^i relative to
   $$\min f(x) \qquad \text{s.t. } g(x) \leq 0?$$

   (b) What more can you say if f and g are convex on R^n?