CS730 Assignment #4

Consider the conjugate directions algorithm presented in class for solving the quadratic minimization problem:

$$\min_{x \in R^n} \frac{1}{2} xCx + ax$$

where C is a symmetric positive definite $n \times n$ matrix and $a \in R^n$, with direction choice:

$$p^{i+1} = -\nabla f(x^{i+1}) + \alpha^{i+1} p^i,$$

and min-along-direction stepsize, that is:

$$\nabla f(x^{i+1}) p^i = 0.$$

Using the results established in class, how are the following three choices for α^{i+1} related?

1. "Pure" Conjugate Directions Coefficient:

$$\beta^{i+1} = \frac{p^i C \nabla f(x^{i+1})}{p^i C p^i}$$

2. Polyak-Polak-Ribière Coefficient:

$$\gamma^{i+1} = \frac{(\nabla f(x^{i+1}) - \nabla f(x^i)) \nabla f(x^{i+1})}{\|\nabla f(x^i)\|^2}$$

3. Fletcher-Reeves Coefficient:

$$\delta^{i+1} = \frac{\|\nabla f(x^{i+1})\|^2}{\|\nabla f(x^i)\|^2}$$