CS730 Assignment #5

Consider the following nonlinear program:

$$\min_{x \in R^n} \quad f(x) \;\text{ s.t. }\; g(x) \le 0 \qquad\qquad (1)$$

where $f : R^n \to R^1$ and $g : R^n \to R^m$, are differentiable convex funtions on R^n. Let α be a fixed positive number and let u be a fixed arbitrary vector in R^m. Let x be a solution of the following unconstrained minimization problem:

$$\min_{x \in R^n} \quad L(x, u, \alpha) = f(x) + \frac{1}{2\alpha}[\| \, (\alpha g(x) + u)_+ \, \|^2 - \|u\|^2],$$

where $\| \cdot \|$ denotes the 2-norm and $(\zeta)_+ :=$max $\{\zeta, 0\}$.

1. Give a lower bound to

   $$\inf_{x \in R^n} \quad f(x) \;\text{ s.t. }\; g(x) \le 0,$$

   in terms of x, u and α.

2. Give a single (vector) condition on x, u and α that is sufficient for x to solve (1).

Hint $\quad a = (a - b)_+ \iff 0 \le a, \; ab = 0, \; b \ge 0,$
where a, b are vectors in R^k and ab denotes the scalar product and use the sufficiency of the KKT conditions.

Note $L(x, u, \alpha)$ is *not* the ordinary Lagrangian, but in fact it is the augmented Lagrangian.