CS 730 May 6, 1996 Solutions

1. Yes, the point $x^0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is optimal because:

   (a) For $\lambda > 0$:

   $$\left(x^0 - \lambda \nabla f(x^0)\right)_+ = \left(\begin{pmatrix} 0 \\ 1 \end{pmatrix} - \lambda \begin{pmatrix} 4\cdot 0 - 2\cdot 1 + 4 \\ -2\cdot 0 + 2\cdot 1 - 2 \end{pmatrix} \right)_+ = \begin{pmatrix} -2\lambda \\ 1 \end{pmatrix}_+ = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = x^0$$

   and

   (b) The objective function $f(x)$ is convex, since

   $$\nabla f(x) = \begin{bmatrix} 4x_1 - 2x_2 + 4 \\ -2x_1 + 2x_2 - 2 \end{bmatrix}, \ \nabla^2 f(x) = \begin{bmatrix} 4 & -2 \\ -2 & 2 \end{bmatrix},$$

   and $\nabla^2 f(x)$ is positive definite.

2. Since $\nabla f\big(x(r)\big) + r \sum_{j=1}^{m} \frac{1}{-g_j(x(r))} \nabla g_j\big(x(r)\big) = 0$ and $g\big(x(r)\big) < 0$, it follows that the point $\left(x(r),\ u_i = \frac{r}{-g_i(x(r))},\ i = 1, \ldots, m\right)$ is dual feasible and:

   $$f(x(r)) \geq \inf \left\{ f(x) \middle| g(x) \leq 0 \right\} \geq f(x(r)) - rm$$

   (a) The best estimate is $f\big(x(r)\big) - \frac{rm}{2}$.

   (b) A guaranteed error bound is $\frac{rm}{2}$.

3. (a) x^i and the multiplier u^i of the IQP are also KKT points of $\min\ f(x) \quad \text{s.t.}\ g(x) \leq 0$.

   (b) x^i solves $\min\ f(x) \quad \text{s.t.}\ g(x) \leq 0$.