1 Reminder

Mid1 (Oct 7(Thur) in class 75 min) = Ch1, 2, 4, 5, 6 (up to Oct 5 lecture).

• Did you receive seating seat?

What are parameters?

Parameters determine its distribution function uniquely in the model.

eg1) Suppose your model is Binomial distribution. If you specify n=1, p=0.5 (actually this is Bernoulli distribution), then the distribution is like:

eg2) Suppose your model is Binomial distribution. If you specify n=1, p=0.1 (actually this is Bernoulli distribution), then the distribution is like:

eg3) Suppose your model is Binomial distribution. If you specify n=2, p=0.3, then the distribution is like:

eg4) Suppose your model is Binomial distribution. If you specify n=1, then the distribution is like: like what?

eg5) Suppose your model is N(mean=0, sd=1). (standard normal) Then, its distribution is like:

eg6) $W_1 \sim N(mean = -2, sd = 1)$, $W_2 \sim N(mean = -2, sd = 2)$, $W_3 \sim N(mean = -2, sd = 0.5)$, $W_4 \sim N(mean =$

eg7) Suppose $Y$ is from normal distribution with $EY = 0, Var(y) = 1$. Then the distribution is like:

eg8) Suppose $Y$ is from normal distribution with $EY = 1, Var(y) = 0.5$. Then the distribution is like:

eg9) Suppose $T$ is from Binomial distribution with $ET = 1, Var(T) = 0.5$. Then the distribution is like:
2 Ch6 More on Normal distribution

• (general result) Difference of median and mean (p258): (A1) Symmetric density, (A2) left skewed (A3) right skewed. Note: Normal distribution is always (A1) type.

• Relation between location and area (p257):
  Assume general continuous distribution. It can be Normal distribution, Uniform distribution etc. But the concept of area, location is same. Let’s take a look.

  eg) Suppose a, b are given values with the relation $a < 0 < b$ and $X \sim N(\mu = 0, \sigma = 1)$. Show me those areas:
  (A1) $P[a < X < b] = ?$
  (A2) $P[a < X] = ?$
  (A3) $P[X < a] = ?$
  (A4) $P[(a < X) \cup (X < a)] = ?$
  (A5) $P[(a < X) \cap (X < a)] = ?$

Typically 2 kind of questions in this section.
Type 1) Given a location, find the area:
eg1) (small) $z=3$ is given, $P(Z \leq z) = P(Z \leq 3) = ?$ Use R.

eg2) Suppose I got IQ test = 100. How many people (percentile) are below me?

Hint: Let $W \sim N(mean = 100, sd = 15)$ (Wikipedia said this is the IQ distribution). Use R.

Type 2) Given an area, find the location:
eg1) 0.95 (95%) is given, also define $P(Z \leq z) = 0.95$. what is (small) $z$? Use R.
eg2) To be (95%) percentile, what IQ score you should take? Let \( W \sim N(100, sd = 15) \), and solve small \( w \);
\[ P(W \leq w) = 0.95? \]
Answer: Using R program:
\[
> \text{qnorm}(0.95, \text{mean}=100, \text{sd}=15) \\
124.6728
\]

• Find (A1) \( P[Z \leq 1.37] = ? \) (A2) \( P[Z \geq 1.37] = ? \). Use R.

• Find (A3) \( P[Z \leq -1.37] = ? \) (A4) \( P[-1.37 \leq Z \leq 1.37] = ? \)

• \( P[-0.15 \leq Z \leq 1.60] = ? \) (p265. Eg2)

3 Things you need to review

• Review basic definitions you learned: sample mean, population mean.
sample variance, population variance.
Sample median (= \( Q_2 \)), \( Q_1 \), \( Q_3 \).
Population median (= \( Q_2 \)), population \( Q_1 \), population \( Q_3 \), interquartile.

Population \( Q_1 \): Cut the distribution so that the area below the location is 25%. Figure:

Population median: Cut the distribution half so that the area below the location is 50%. Figure:

Is a sample mean fixed value or random?

• Normal distribution \( N(\mu, \sigma) \): Can you convert area (probability) and location?
Is it possible to have \( N(\mu = -2, \sigma = -2) \)?

• Binomial distribution: What are parameters for the Binomial dist.? When do you use this distribution?

mean=?
Variance=?

• Basic R commands.
• What does \( P(A|B) = P(A) \) mean?
• What does \( P(A \cup B) = P(A) + P(B) \) mean?

• What does \( P(AB) = 0 \) mean?

• When does \( P(A \cup B \cup C) = P(A) + P(B) + P(C) \) hold?

• No question like "Draw histograms by hand."

• Can you use "N choose r"?

• When you place 7 different cards from left to right, how many combinations?

• Transformation formula for Expectation and variance. eg1) Suppose \( E[-X] = 3 \) and \( Var(-X) = 2 \). Find: \( E[-3X + 5] \)?

\[ Var([-2]^3X + 10^2) = ? \]

eg2) \( Y = 2X - 1 \). Find: \( E[-3Y + 5] \)?

\[ Var([-2]^3Y + 10^2) = ? \]

• What is the relation between \( P(Z < -b) \) and \( P(b < Z) \) for \( Z \sim N(0, 1) \)? Why?