1 Ch6 More about Normal Distribution

- Empirical Guidelines for Symmetric Bell-shaped Data Curve (p70)
  Approximately 68% of the data lie within \( \bar{x} \pm s \).
  Approximately 95% of the data lie within \( \bar{x} \pm 2s \).
  Approximately 99.7% of the data lie within \( \bar{x} \pm 3s \).
  Simply speaking, if your data is symmetric bell shaped, then your next observation will be very likely fall in \( \bar{x} \pm 3s \).
  These rules are actually coming from the shape of normal distribution. If data is symmetric bell shaped, then although it may not be from normal distribution, we use normal distribution shape to approximate it.

In other words, we know, for \( Z \sim N(0,1) \),

\[
P(0 - 1 < Z < 0 + 1) = 0.68 \\
P(0 - 2 < Z < 0 + 2) = 0.95 \\
P(0 - 3 < Z < 0 + 3) = 0.997
\] (1)

- Location and Shape related questions for Normal distribution.

2 Ch6.5 Transform \( N(\mu, \sigma) \) to \( N(0,1) \)

Why do we need this transformation? Because Appendix has only \( N(0,1) \) table. If you use software R, no need of this transformation. However, the concept of transformation from one random variable to another random variable is used frequently in the future. Transforming \( N(\mu, \sigma) \) to \( N(0,1) \) is a special case.

- Set manipulation

Let’s consider discrete case. This give you some intuition about transformation.

eg1) 5 cards in the box with number 5.1, 6.1, 7.1, 8.2, 9.
Pick up one card and let \( X = \) card number. We know

\[
P(X = 5.1) = P(X = 6.1) = P(X = 7.1)
\]

\[
P(X = 8.2) = P(X = 9) = 1/5.
\]

What is \( P[6 \leq X \leq 8] \) ?

ANS:
What is \( P[6 \leq X \leq 8] - 1 =? \)  
ANS:

What is \( P[6 - 1 \leq X - 1 \leq 8 - 1] =? \)  
ANS:

What is \( P[6 \leq 2X \leq 12] =? \)  
ANS:

What is \( 2P[6 \leq X \leq 12] =? \)  
ANS:

Now we are ready for our main dish.

- Transform \( N(\mu, \sigma) \) to \( N(0,1) \).

If \( X \sim N(\mu, \sigma) \) then \( Z \) becomes standard normal where,

\[
Z = \frac{X - \mu}{\sigma}.
\]  
Proof: Skip. (You have to use Moment Generating Function which you won’t learn in this course)

Originally random variable is \( X \). We create new random variable \( Z \) as above. At least, we can check \( Z \)'s mean and variance (See Lec 8 p14, 15)

\[
E[Z] = E\left(\frac{X - \mu}{\sigma}\right) =
\]

\[
Var[Z] = Var\left(\frac{X - \mu}{\sigma}\right) =
\]

eg1) Suppose \( X \sim N(\mu = 0, \sigma = 1) \), find \( P[|X| \leq 3] =? \)

eg2) Suppose \( X \sim N(\mu = 0, \sigma = 1) \), find \( P[|X| \geq 2] =? \)