1 Review

- **Alternative Hypothesis** $H_1$ (p340): the claim we wish to establish.
  eg. "Aki is guilty"

- **Null Hypothesis** $H_0$ (p340): negation of the claim.
  (eg. "Aki is NOT guilty")

- **Type 1 Error** (p345): Rejection of $H_0$ when $H_0$ is true.
  $\equiv$ Level of significance
  $\equiv \alpha$
  eg) An innocent person gets guilty.

- **Type 2 Error** (p345): Not reject $H_0$ when $H_1$ is true.
  $\equiv \beta$
  eg) A criminal gets no charge.

- **Decision under the hypothesis** (p342): We have only one of two conclusions:
  Conclusion 1: Reject $H_0$
  Conclusion 2: Not Reject $H_0$ (\(=\) Retain $H_0$)
Let’s consider 3 kinds of hypothesis test for population mean. (p353) Remember, \( \mu_0 \) is some number.

<table>
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<th>nick name</th>
<th>( H_0 )</th>
<th>( H_1 )</th>
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<tr>
<td>C3</td>
<td>Two sided hypothesis</td>
<td>( H_0 : \mu = \mu_0 )</td>
<td>( H_1 : \mu \neq \mu_0 )</td>
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Eg) Do hypothesis test. What is your conclusion? \( H_0 : \mu = 270 \) \( H_1 : \mu > 270 \), and \( \alpha = 0.1 \)
2   Ch8.4 Hypotheses test and p-value

**p-value**: Evidence of How likely $H_0$ is.

Consider the following conversation. There are only two options, yes or no. However, there is a gray zone in reality.

You: ”Shall we marry?”

P1: ”Yes, because I love your fortune.”

P2: ”Yes, because I am polygamy.”

P3: ”No, I am busy today.”

P4: ”No, we don’t need it.”

Q) Which person loves you?

Lesson:
The method below is for $C1$. For $C2, C3$, all we need is modify Step1 and Step5.

- Hypothesis test for $C1$

  Step 1) Decide $H_1$.

  Researchers (you) want to establish $\mu > \mu_0$. We set

  $H_1 : \mu > \mu_0$

  where $\mu_0$ is a number researcher decide (100, 150, -3.2, etc). (usually given in a problem.)

  Step 2) Decide $H_0$.

  Then automatically, we have

  $H_0 : \mu = \mu_0$

  Step 3) Decide level of significance $\alpha \in (0, 1)$ (= Type 1 error). We control $\alpha$ (Type I Error) not $\beta$ (Type II Error). Hence we (Researcher) decide $\alpha$, not $\beta$.

  We usually use $\alpha = 0.05$

  Step 4) Get $Z_{obs}$ (TEST STATISTIC for $\mu$).

  $$Z_{obs} = \frac{\sqrt{n}(\bar{X} - \mu_0)}{S}$$

  (1)
where S = sample sd. i.e.,

\[ S = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}}. \]  

(2)

Step 5-a) Find **critical point = c**

c = location of N(0,1) which has right area = \( \alpha \). Use Z-table. ( = \( Z_\alpha \). See Fig324 Fig)

Step 5-b) Find **rejection region**

NEW

Step 5-c) Find **p-value \( \in (0, 1) \)**

DEF: **p-value** = Area right side of \( Z_{obs} \)
Picture of p-value for C1:
Step6) Conclude your hypothesis test (Only two options !)

Option 1: If $Z_{obs} \in \text{Rejection Region}$, 
(or if p-value $\leq \alpha$) then

Conclusion: ”Reject $H_0$.”

Option 2: Otherwise,

Conclusion: ”NOT reject $H_0$.”

Note: In step6, to get a conclusion, all you have to do is just heck one of them:

Check criteria 1: $Z_{obs} \in \text{Rejection Region}$
Check criteria 2: p-value $\leq \alpha$

You should get the same conclusion. I mean,

”Check criteria 1” = Yes $\Leftrightarrow$ ”Check criteria 2” =Yes $\Leftrightarrow$ ”Reject $H_0$.”.

”Check criteria 1” = No $\Leftrightarrow$ ”Check criteria 2” =No $\Leftrightarrow$ ”NOT Reject $H_0$.”.

Last time we did not use p-value but still get a conclusion. So why do we need p-value?
The reason is that p-value give you strength of evidence against $H_0$. (p348)

Small p-value $\Rightarrow H_0$ is strongly rejected. Highly statistically significant. This is a good thing for researchers because ”Reject $H_0$” (in favor of $H_1$) is what researchers want.

Note that:

- **p-value** depends on data, i.e., random. You know this value after collection the data.

- **Level of significance $=\alpha$** is constant. This is usually given before collecting the data.
• Hypothesis test for C2

Step 1) \( H_1 : \mu < \mu_0 \)

Step 5-a) Find critical point = c

c = location of N(0,1) which has left area = \( \alpha \). Use Z-table. \( ( = -Z_\alpha \). See p324 Fig. )

Step 5-b) Find rejection region

NEW

Step 5-c) Find p-value \( \in (0, 1) \)

DEF: p-value = Area left side of \( Z_{obs} \)

Picture of p-value for C2:
• Hypothesis test for C3
  Step 1) \( H_1 : \mu \neq \mu_0 \)
  Step 5-a) Find critical point = c
  \( c = \) location of \( N(0,1) \) which has right area = \( \alpha/2 \). Use Z-table.
  \( ( = -Z_{\alpha/2}. \text{ See p324 Fig.} ) \)

Step 5-b) Find rejection region
NEW
Step 5-c) Find p-value \( \in (0, 1) \)
DEF: p-value = \( 2 \times (\text{Area right side of } |Z_{\text{obs}}|) \)

Picture of p-value for C3:
Eg1) For Bee example, do C1. with $H_0 : \mu = 270$, and $\alpha = 0.05$

Eg2) For Bee example, do C1. with $H_0 : \mu = 270$, and $\alpha = 0.10$

Eg3) For Bee example, do C2. with $H_0 : \mu = 270$, and $\alpha = 0.05$

Eg4) For Bee example, do C3. with $H_0 : \mu = 270$, and $\alpha = 0.05$

Eg5) For Bee example, do C1. with $H_0 : \mu = 230$, and $\alpha = 0.05$
Eg: A sample of 42 measurements was taken. The sample mean and standard deviation were found to be 8.79 and 1.27, respectively. Perform the hypothesis test.

Q1) The purpose is to test whether population mean is less than 8.5. Use 0.10 as the level of significance.
Write the hypothesis.
What is $Z_{obs}$?
What is p-value?
What is your conclusion?

Q2) The purpose is to test whether population mean is bigger than 8.5. Use 0.10 as the level of significance.
Write the hypothesis.
What is $Z_{obs}$?
What is p-value?
What is your conclusion?

Q3) The purpose is to test whether population mean is not equal to 8.5. Use 0.20 as the level of significance.
Write the hypothesis.
What is $Z_{obs}$?
What is p-value?
What is your conclusion?
Q4) The purpose is to test whether population mean is not equal to 8.5. Use 0.05 as the level of significance.
Write the hypothesis.
What is $Z_{obs}$?
What is p-value?
What is your conclusion?

Q5) (Tricky!) The purpose is to test whether population mean is less than 8.0. Use 0.10 as the level of significance.
Write the hypothesis.
Which is bigger, Q5’s p-value or Q1’s p-value?
Which is bigger, Q5’s $Z_{obs}$ or Q1’s $Z_{obs}$?
What is your conclusion?

Q6) (Tricky!) The purpose is to test whether population mean is bigger than 8.0. Use 0.10 as the level of significance.
Write the hypothesis.
Which is bigger, Q5’s p-value or Q2’s p-value?
Which is bigger, Q5’s $Z_{obs}$ or Q2’s $Z_{obs}$?
What is your conclusion?