1 Ch 9.4 The Relation Between C3 test and C.I.

This concept is work for both large sample case (Ch8) and small sample from normal distribution case (ch9).

- Ch8 and Ch9 had 3 types of hypothesis test:

<table>
<thead>
<tr>
<th>Case</th>
<th>nick name</th>
<th>$H_0$</th>
<th>$H_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>One sided hypothesis</td>
<td>$H_0 : \mu = \mu_0$</td>
<td>$H_1 : \mu &gt; \mu_0$</td>
</tr>
<tr>
<td>C2</td>
<td>One sided hypothesis</td>
<td>$H_0 : \mu = \mu_0$</td>
<td>$H_1 : \mu &lt; \mu_0$</td>
</tr>
<tr>
<td>C3</td>
<td>Two sided hypothesis</td>
<td>$H_0 : \mu = \mu_0$</td>
<td>$H_1 : \mu \neq \mu_0$</td>
</tr>
</tbody>
</table>

Summary of critical points and hypo. tests

<table>
<thead>
<tr>
<th>Case</th>
<th>Reject $H_0$ with significant level $\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ch8 C1</td>
<td>$&lt; Z_{obs}$</td>
</tr>
<tr>
<td>Ch8 C2</td>
<td>$&gt; Z_{obs}$</td>
</tr>
<tr>
<td>Ch8 C3</td>
<td>$&gt; Z_{obs}$ or $&lt; Z_{obs}$</td>
</tr>
<tr>
<td>Ch9 C1</td>
<td>$&lt; T_{obs}$</td>
</tr>
<tr>
<td>Ch9 C2</td>
<td>$&gt; T_{obs}$</td>
</tr>
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<td>Ch9 C3</td>
<td>$&gt; T_{obs}$ or $&lt; T_{obs}$</td>
</tr>
</tbody>
</table>
FACT: For Ch8, with

\[ Z_{obs} = \frac{\sqrt{n}(\bar{X} - \mu_0)}{S}, \]

\[ \mu' \text{ s } 100(1 - \alpha)\% C.I. = \frac{C3'\text{'s Rejection} (\mu_0 \leftrightarrow \mu)}{> Z_{obs} \cup \ < Z_{obs}} \]

\[ = \leq Z_{obs} \cap \geq Z_{obs} \]
FACT: For Ch9,

\[ T_{obs} = \frac{\sqrt{n} (\bar{X} - \mu_0)}{S}, \]

\( \mu' \)'s 100(1 - \alpha)\% C.I. = C3's Rejection(\mu_0 \leftarrow \mu) \]

\[ = \left\{ \begin{array}{c} > T_{obs} \cup \quad < T_{obs} \\ \leq T_{obs} \cap \quad \geq T_{obs} \end{array} \right. \]
In other words, (p382-p384)

• $\mu's$ 100(1 - $\alpha$)% C.I. does not contain $\mu_0$ for C3 hypothesis test
  $\Rightarrow$

• $\mu's$ 100(1 - $\alpha$)% C.I. does contain $\mu_0$ for C3 hypothesis test
  $\Rightarrow$

• ”Reject $H_0”$ for C3 hypothesis test,
  $\Rightarrow$ $\mu's$ 100(1 - $\alpha$)% C.I.

• ”Not Reject $H_0”$ for C3 hypothesis test,
  $\Rightarrow$ $\mu's$ 100(1 - $\alpha$)% C.I.
Try: p383: Example 5. Relation between 95% confidence interval and $\alpha = 0.05\%$ test.

Given $n=9$, from a normal population, $\bar{x} = 8.3$, $s=1.2$.

(a) Find 95% confidence interval for $\mu$.
(Hint: $t_{df=8,\alpha=0.025} = 2.306$).

(b) What is the conclusion of $H_0 : \mu = 9.0$, $H_1 : \mu \neq 9.0$ with $\alpha = 0.05\%$?

(c) What is the conclusion of $H_0 : \mu = 9.0$, $H_1 : \mu \neq 9.0$ with $\alpha = 0.01\%$?
2 Review

Try: p317 [Ch7.4][12]. n=100, σ = 20

(a) $P[-2 \leq \bar{X} - \mu \leq 2] = ?$

(b) $P[-k \leq \bar{X} - \mu \leq k] = 0.90$. What is $k$?

(c) What is the probability that $\bar{X}$ will differ from $\mu$ by more than 4 units?

(d) Can you calculate

$$\frac{-P[-2 \leq \bar{X} - \mu \leq 2]}{P[Z < 0]} + 1 = ?$$

(1)
Try: p379 [Ch9.3] [7]. n=12. We know 95% C.I. for $\mu$ is [18.6, 26.2].

(a) What were $\bar{x}$ and $s$ for that sample?
• Why do we need to study Statistics?

• R-related questions are:

• Here are some questions you will see in the mid 2:
  (Really!):

• Sleep well before the exam.