1 Small samples from two NORMAL populations and UNEQUAL variance

- $X_1, X_2, \ldots, X_{n_1}$ is a random sample of size $n_1$ from population 1 with $N(\mu_1, \sigma_1^2)$.

- $Y_1, Y_2, \ldots, Y_{n_2}$ is a random sample of size $n_2$ from population 2 with $N(\mu_2, \sigma_2^2)$.

- The samples are independent. In other words, the response measurements under one treatment are unrelated to the response measurements under the other treatment.

- Small sample case.

- "$s_1/s_2 < 1/2$" or "$2 < s_1/s_2$"
  (i.e. we can assume $\sigma_1 \neq \sigma_2$).

Estimator of $\mu_1 - \mu_2$: $\overline{X} - \overline{Y}$ (same as the previous section)

$100(1 - \alpha)\%$ CI of $\mu_1 - \mu_2$ is
\[
\overline{X} - \overline{Y} \pm t_{df = \min(n_1 - 1, n_2 - 1), \alpha/2} \cdot \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} \quad (1)
\]
Note: What is $df = \min(n_1 - 1, n_2 - 1)$?.

Suppose $n_1 = 10$, $n_2 = 6$, then

$$\min(n_1 - 1, n_2 - 1) = \min(10 - 1, 6 - 1) = \min(9, 5) = 5.$$ 

It just takes a smaller of $n_1 - 1$ and $n_2 - 1$.

eg) If $n_1 = 5$, $n_2 = 5$, then

$$\min(n_1 - 1, n_2 - 1) = \min(5 - 1, 5 - 1) = \min(4, 4) = 4.$$
Hypothesis test with $\alpha$ level (C3):

$H_0 : \mu_1 - \mu_2 = \delta_0$

$H_1 : \mu_1 - \mu_2 \neq \delta_0$

Step 1) Get

$$T_{obs} = \frac{(\bar{X} - \bar{Y}) - \delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \quad (2)$$

If $T_{obs}$ is in rejection region, ”Reject $H_0$.” In other word, if $|T_{obs}| > t_{df=\min(n_1-1,n_2-1)}, \alpha/2$, then ”Reject $H_0$.”

Suppose you do $H_1 : \mu_1 - \mu_2 > \delta_0$ (C1) test.

If $T_{obs} > t_{df=\min(n_1-1,n_2-1)}, \alpha$, then ”Reject $H_0$.”

Suppose you do $H_1 : \mu_1 - \mu_2 < \delta_0$ (C2) test.

If $T_{obs} < -t_{df=\min(n_1-1,n_2-1)}, \alpha$, then ”Reject $H_0$.”

• p-value for C1, C2, C3 (Figures:)
EG1) Independent random samples from two Normal populations have provided the summary statistics as follows:

Sample 1: \( n_1 = 4, \bar{x} = 5, s_1^2 = 4 \)
Sample 2: \( n_2 = 3, \bar{y} = 7, s_2^2 = 9 \)

Q1) Which method do you use:
(a) large sample
(b) small normal distribution samples with equal variance
(c) small normal distribution samples with unequal variance

Q2) Give estimates of \( \sigma_1 \) and \( \sigma_2 \). (p413)

Q3) Do hypothesis test \( H_0 : \mu_1 - \mu_2 = -1, H_0 : \mu_1 - \mu_2 < -1 \), and find p-value.
Independent random samples from two Normal populations have provided the summary statistics as follows:

Sample 1: \( n_1 = 4, \bar{x} = 5, s_1^2 = 1 \)
Sample 2: \( n_2 = 3, \bar{y} = 7, s_2^2 = 9 \)

Q1) Which method do you use:
(a) large sample
(b) small normal distribution samples with equal variance
(c) small normal distribution samples with unequal variance

Q2) Give estimates of \( \sigma_1 \) and \( \sigma_2 \). (p413)

Q3) Do hypothesis test \( H_0 : \mu_1 - \mu_2 = -1, H_0 : \mu_1 - \mu_2 < -1 \) and find p-value.