1 Small samples from two NORMAL populations and UNEQUAL variance

- \(X_1, X_2, \cdots, X_{n_1}\) is a random sample of size \(n_1\) from population 1 with \(N(\mu_1, \sigma_1^2)\).

- \(Y_1, Y_2, \cdots, Y_{n_2}\) is a random sample of size \(n_2\) from population 2 with \(N(\mu_2, \sigma_2^2)\).

- The samples are independent. In other words, the response measurements under one treatment are unrelated to the response measurements under the other treatment.

- Small sample case.

- “\(s_1/s_2 < 1/2\)” or “\(2 < s_1/s_2\)” (i.e. we can assume \(\sigma_1 \neq \sigma_2\)).

Estimator of \(\mu_1 - \mu_2\): \(\bar{X} - \bar{Y}\) (same as the previous section)

- \(100(1 - \alpha)%\) CI of \(\mu_1 - \mu_2\) is

\[
\bar{X} - \bar{Y} \pm \frac{t_{df=\min(n_1-1,n_2-1),\alpha/2}}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \tag{1}
\]

Note: What is \(df = \min(n_1 - 1, n_2 - 1)\)?

Suppose \(n_1 = 10, n_2 = 6\), then

\(\min(n_1-1, n_2-1) = \min(10-1, 6-1) = \min(9, 5) = 5\).

It just takes a smaller of \(n_1 - 1\) and \(n_2 - 1\).

eg) If \(n_1 = 5, n_2 = 5\), then

\(\min(n_1-1, n_2-1) = \min(5-1, 5-1) = \min(4, 4) = 4\).

Hypothesis test with \(\alpha\) level (C3):

- \(H_0: \mu_1 - \mu_2 = \delta_0\)
- \(H_1: \mu_1 - \mu_2 \neq \delta_0\)

Step1) Get

\[T_{obs} = \frac{(\bar{X} - \bar{Y}) - \delta_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \tag{2}\]

If \(T_{obs}\) is in rejection region, “Reject \(H_0\)” In other word, if \(|T_{obs}| > t_{df=\min(n_1-1,n_2-1), \alpha/2}\), then “Reject \(H_0\)”.

Suppose you do \(H_1: \mu_1 - \mu_2 > \delta_0\) (C1) test.

If \(T_{obs} > t_{df=\min(n_1-1,n_2-1), \alpha}\), then “Reject \(H_0\)”.

Suppose you do \(H_1: \mu_1 - \mu_2 < \delta_0\) (C2) test.

If \(T_{obs} < -t_{df=\min(n_1-1,n_2-1), \alpha}\), then “Reject \(H_0\)”.

- p-value for C1, C2, C3 (Figures: ...)
EG1) Independent random samples from two normal populations have provided the summary statistics as follows:

Sample 1: $n_1 = 4$, $\bar{x} = 5$, $s_1^2 = 4$
Sample 2: $n_2 = 3$, $\bar{y} = 7$, $s_2^2 = 9$

Q1) Which method do you use:
(a) large sample
(b) small normal distribution samples with equal variance
(c) small normal distribution samples with unequal variance

Q2) Give estimates of $\sigma_1$ and $\sigma_2$. (p413)

Q3) Do hypothesis test $H_0 : \mu_1 - \mu_2 = -1, H_0 : \mu_1 - \mu_2 < -1$, and find p-value.

EG2) Independent random samples from two normal populations have provided the summary statistics as follows:

Sample 1: $n_1 = 4$, $\bar{x} = 5$, $s_1^2 = 1$
Sample 2: $n_2 = 3$, $\bar{y} = 7$, $s_2^2 = 9$

Q1) Which method do you use:
(a) large sample
(b) small normal distribution samples with equal variance
(c) small normal distribution samples with unequal variance

Q2) Give estimates of $\sigma_1$ and $\sigma_2$. (p413)

Q3) Do hypothesis test $H_0 : \mu_1 - \mu_2 = -1, H_0 : \mu_1 - \mu_2 < -1$, and find p-value.