ST 301 (AKI)

PLEASE DO NOT OPEN YET!
TURN OFF YOUR CELL PHONE!

FIRST NAME: ANSWER

THIS IS VERSION 1 (seating number = odd). For those seating number = even, see the scramble sheet.

Answer and problems by: AKI = [1]-[14], JENNY = [15]-[27], CHANHAN = [28]-[35].
Choose one answer for each question.

1. You have sorted 20 data set:

\[202 \ 204 \ 206 \ 207 \ 208 \ 209 \ 210 \ 211 \ 211 \ 212 \ 212 \ 214 \ 217 \ 218 \ 219 \ 219 \ 219 \ 219 \ 222 \ 225.\]

The sample mean is 213.2. You delete the smallest, and add two more data, 100 and 300. Then what is the sample mean for this new data? In other words, calculate the sample mean of the following data set.

\[100 \ 204 \ 206 \ 207 \ 208 \ 209 \ 210 \ 211 \ 211 \ 212 \ 212 \ 214 \ 217 \ 218 \ 219 \ 219 \ 219 \ 219 \ 222 \ 225 \ 300.\]

(a) \((-300 - 100 + 213.2 \times 19 + 202)/21\)

(b) \((300 + 100 + 213.2 \times 21 - 202)/19\)

(c) \((300 + 100 + 213.2 \times 20 - 202)/21\)

(d) \((300 + 100 + 213.2 \times 20 - 202)/20\)

(e) None of them

Note: This is from LEC3, p9. Answer(C)

2. eg) You have data: 39, 34, 35, 40, 42, 40, 70.

Then stem-and-leaf display is

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<table>
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<th>3</th>
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<th>4</th>
<th>5</th>
</tr>
</thead>
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<td>(a)</td>
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<tr>
<td>7</td>
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</tr>
</tbody>
</table>
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(e) None of them

Note: This is from LEC2, p9. Answer=C. Need to sort the data.

3. You have sorted 20 data set:

\[202 \ 204 \ 206 \ 207 \ 208 \ 209 \ 210 \ 211 \ 211 \ 212 \ 212 \ 214 \ 217 \ 218 \ 219 \ 219 \ 219 \ 219 \ 222 \ 225.\]

The sample mean is 213.2. You delete the smallest, and add two more data, 100 and 300. Then what is the maximum value for the new data:

\[100 \ 204 \ 206 \ 207 \ 208 \ 209 \ 210 \ 211 \ 211 \ 212 \ 212 \ 214 \ 217 \ 218 \ 219 \ 219 \ 219 \ 219 \ 222 \ 225 \ 300.\]

(a) 300

(b) 225

(c) 100

(d) \((225 + 300)/2\)

(e) None of them

Note: This is from LEC3, p9. Answer=A

4. You observed: \(x_1 = 0, x_2 = 0, x_3 = 0.\) What is the sample variance?

(a) -1

(b) 0

(c) 1

(d) 2

(e) None of them
5. Suppose $X$ is a random variable. Which expression is mathematically correct?
(a) Banana + Monkey = Smile  
(b) $P((X = 0.1) + (X = 0.2)) = 0.3$
(c) $P[2 < X] \cup P[X < 3] = P[2 < X < 3]$  
(d) $P[X = 0.1] \cap P[X = 0.2] = \phi$
(e) None of them

Note: This is from LEC9, p5. Answer=E. (B) is wrong because set + set. (C) is wrong because number \cup number. (D) is wrong because left hand side is a number, while right hand side is a set ($\phi$).

6. Simplify the expression $A \cap \overline{\overline{B} \cap A} = ?$
(a) $A \cap B$  
(b) $A \cap \overline{B}$  
(c) $A$  
(d) $\phi$  
(e) None of them

Note This is from LEC4, p13-p15. Answer=D. $[\overline{B} \cap A] = B \cap A$. Hence $A \cap (B \cap A) = (A \cap A) \cap B = \phi \cap B = \phi$.

7. Use an unfair 6-face dice, with the letter A, B, C, D, E, F. You throw it 5 times. Let $X_i$ the i-th trial and $i = 1, 2, \cdots, 5$, $X_i = \{1, 2, 3, 4, 5, 6\}$.

Then, $P(\text{you observe D exactly 3 times}) = ?$
(a) $\left(\binom{5}{3}\right)0.4^30.6^2$  
(b) $0.4^3 + 0.6^2$  
(c) $0.4^3 \cdot 0.6^2$  
(d) $\left(\binom{5}{3}\right)0.4^30.05^2$  
(e) None of them

Note This is from LEC9, p12. Answer=A. Use Binomial distribution with $p = 0.4, n = 5$ and number of success =3. Note $\binom{5}{3} = \binom{5}{2}$.

8. To create boxplot in R, which command do you use?
(a) histbox()  
(b) boxplot()  
(c) plotbox()  
(d) boxR()  
(e) None of them

Note This is R related question. LEC6, p6. Answer=B.

9. Suppose $Y \sim \text{Binomial} (n=30, p=0.4)$. Which R command do you use to calculate $P(Y \geq 10)$?
(a) $1 - \text{pbinom}(9, \text{size}=30, \text{prob}=0.6)$  
(b) $1 - \text{pbinom}(10, \text{size}=30, \text{prob}=0.6)$  
(c) $\text{pbinom}(10, \text{size}=30, \text{prob}=0.4)$  
(d) $1 - \text{pbinom}(9, \text{size}=30, \text{prob}=0.4)$  
(e) None of them

Note This is R related question, LEC9, p16. Answer=D. $P(Y \geq 10) = 1 - P(Y \leq 9)$. 

Note: This is from LEC3, p13. Answer=B. $n = 3, \bar{x} = 0, ((0 - 0)^2 + (0 - 0)^2 + (0 - 0)^2)/(3 - 1) = 0/2 = 0$. 

5. Suppose $X$ is a random variable. Which expression is mathematically correct?
(a) Banana + Monkey = Smile  
(b) $P((X = 0.1) + (X = 0.2)) = 0.3$
(c) $P[2 < X] \cup P[X < 3] = P[2 < X < 3]$  
(d) $P[X = 0.1] \cap P[X = 0.2] = \phi$
(e) None of them

Note: This is from LEC9, p5. Answer=E. (B) is wrong because set + set. (C) is wrong because number \cup number. (D) is wrong because left hand side is a number, while right hand side is a set ($\phi$).
10. You use hist() in R to create histogram. If you want to vertical line at x axis= 4, which command and option do you use?
   (a) addline(v=4)   (b) lineadd(v=4)   (c) vertline(v=4)   (d) abline(v=4)   (e) None of them
   
   Note This is R related question IntroRc.txt last line. Answer=D.

11. Use R to tile the output screen, by 1 × 4 (total 4 figures, 1 row and 4 column.) Which command do you use?
   (a) par(mfrow=c(2,2,1))   (b) par(mfrow=c(1,4))   (c) par(mfrow=c(4,1))   (d) par(mfrow=c(2,1,2))   (e) None of them
   
   Note This is R related question, LEC6, p7. Answer=B

12. You have 2 data sets in R, height and time.
   > height ← c(120,155, 180, 200, 512, 80)
   > time ← c(10, 12, 13, 14, 15, 21)

   Use plot() function to create a x-y plot with x-axis=time, y-axis=height. Which command is correct?
   (a) plot(height ~ time)   (b) plot(time ~ height)   (c) par(height ~ time)   (d) par(time ~ height)   (e) None of them
   
   Note This is R related question, LEC6, p7. Answer=A

13. There are 12 balls in the box. 1 ball= Winning ball (W), 11 balls = Looser balls (L). 2 people draw a ball without replacement. Let \( X_i = W \) means i-th person gets the winning ball and \( X_i = L \) means i-th person gets a looser ball (i=1, 2). Then which is true?
   (a) \( P(X_1 = W) = P(X_2 = W) \)   (b) \( P(X_1 = W) < P(X_2 = W) \)
   (c) \( P(X_1 = W) > P(X_2 = W) \)   (d) \( P(X_1 = L) \neq P(X_2 = L) \)   (e) None of them

   Note This is LEC7, p4. Answer=A. \( P(X_1 = W) = 1/12 \). \( P(X_2 = W) = P(X_2 = W|X_1 = W)P(X_1 = W) + P(X_2 = W|X_1 = L)P(X_1 = L) = 0 + (1/11) \cdot (11/12) = 1/12. \)

14. Suppose \( X \sim N(5,5) \) Then which is true?
   (a) \( P(X = 5) = 0.5 \)   (b) \( P(-5 \leq X \leq 5) < P(-5 < X < 5) \)   (c) \( P(-5 \geq X) < P(-5 > X) \)
   (d) \( P(X = 5) = \phi \)   (e) None of them

   Note This is LEC9, p13. Answer=E. (a) is wrong because X is continuous, and \( P(X = 5) = 0. \) (b) is wrong because \( P(X = -5) = 0 \) and \( P(-5 \leq X \leq 5) = P(-5 < X < 5). \) (c) is wrong because \( P(X = -5) = 0 \) and \( P(-5 \geq X) = P(-5 > X) \) (d) is wrong because \( P(X = 5) = 0. \) Remember, \( \phi \) is a set, 0 is a number, and \( P(X = 5) \) is a number.
15. The weights of students in STAT301 are to be grouped into the following classes (unit=pound): 100-120, 120-140, 140-160, 160-180, 180-200, greater than 200 (200 included), **where the left endpoint is included but not the right endpoint**. Which one of the following is able to be determined from this frequency distribution?

(a) Number of students with weight larger than 130
(b) Number of students with weight smaller than 125
(c) Number of students with weight smaller than 200
(d) Number of students with weight larger than 161
(e) None of them

Note: This is from p43 [3.9] ANS (c). The total number of students minus number of students with weight greater than 200 is the number of students with weight smaller than 200.

16. To investigate the economic impact of doing business with the state, a sample was taken of 10 small firms in the service sector that are vendors to the state. The data, on the percent of total sales due to sales to the state, are

27, 15, 30, 12, 1, 7, 8, 16, 26, 19

The first quartile is:

(a) 7 (b) 8 (c) 10 (d) 19 (e) None of them

Note: This is from p60 [4.22] ANS (b). sort data from smallest to largest. first quartile is the number in the third place.

17. An urn contains two black balls and three white balls. Suppose two balls will be drawn with replacement. Find the probability that a white ball appears in the second draw.

(a) 3/5 (b) 2/5 (c) 3/4 (d) 1/2 (e) 1/5

Note: This is from p180 [5.8] ANS (a). With replacement means the second draw is independent from the first draw.

18. A sample of eight compact discs at the music store stated the following performance times (in minutes) for Beethoven’s Ninth Symphony.

64, 65, 66, 66, 69, 71, 75, 81

The interquartile range is:

(a) 9 (b) 13 (c) 10 (d) 12 (e) None of them

Note: This is from p74 [5.16] ANS:(e). sort data from smallest to largest. Interquartile=number in the 6th place-number in the 2nd place=71-65=6.

19. Sun visors come in red (r), orange (o), yellow (y), green (g), blue (b), purple (p), and white (w). For the next visor sold, consider the two events \(A=\{r, b, g, w\}\) and \(B=\{b, o, y\}\). Then, which of the following events has the exact composition as \(\{o, y\}\)

(a) \(AB\) (b) \(A\overline{B}\) (c) \(\overline{A}\) (d) \(\overline{A}B\) (e) None of them

Note: This is from p160 [3.1] ANS:(d). \(\overline{A}=\{o, y, p\}\) and \(\overline{A}B=\{o, y\}\)
20. For two events $A$ and $B$, the following probabilities are specified.

\[ P(A) = 0.25 \quad P(B) = 0.4 \quad P(AB) = 0.10 \]

What’s the probability of $P(\overline{A} \overline{B})$?
(a) .37  (b) .45  (c) .55  (d) .39  (e) None of them

Note: This is from p161 [3.10] ANS (b). A and B are independent. 
$P(\overline{A} \overline{B}) = (1-0.25)(1-0.4) = 0.45$

21. If $P(A) = 0.7$ and $P(B) = 0.4$, $P(AB) = 0.28$ which statement of the following is true?
(a) $A$ and $B$ are mutually exclusive
(b) $A$ and $B$ are independent
(c) $A$ and $B$ are not only mutually exclusive, but also independent
(d) $A$ and $B$ are unrelated
(e) None of them

Note: This is from p161 [3.12] ANS (b). since $P(A) \times P(B) = P(AB)$

22. Out of 10 people are applying for a job, 3 cannot do the work. Suppose 2 people will be hired. If two persons are chosen in a random manner, what is the probability that neither will be able to work?
(a) $1/9$  (b) $2/15$  (c) $4/15$  (d) $2/9$  (e) None of them

Note: This is from p188 [6.5] ANS (e). rule of combination. number of choose 2 from 3 who cannot work=3, number of choose 2 from 10=45. $P$(neither will be able to work)=$3/45$

23. Given the two probability distributions:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$y$</th>
<th>$f(y)$</th>
</tr>
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</table>

Which one is correct?
(a) The expected value of $x$ is larger than the expected value of $y$.
(b) The standard deviation (sd) of $x$ is larger than the sd of $y$.
(c) Both distributions have the same mean.
(d) The variance of $y$ is smaller than the variance of $x$.
(e) None of them

Note: This is from p221 [3.16] ANS (a). $E(X) = 2.2$, $E(Y) = 2$
24. A backpacking party carries five emergency signal flares, each of which will light with a probability of 0.8. Assuming that the flares operate independently. What is the probability that at least one flares light?

- (a) \( \binom{5}{1}0.2^40.8^1 \)  
- (b) \( \binom{5}{1}0.2^40.8^1 \)  
- (c) \( \binom{5}{0}0.2^40.8^1 + \binom{5}{2}0.2^20.8^3 \)  
- (d) \( 1 - 0.2^5 \)  
- (e) None of them

Note: This is from p226 [4.8] ANS (d). \( P(\text{at least one flares light}) = 1 - P(\text{none of the flares light}) \)

25. There are five tickets numbered 1, 2, 3, 4, 5. Suppose a two-digit number will be formed by first drawing one ticket at random and then drawing a second ticket at random from the remaining four. (For instance, if the first ticket drawn shows 4 and the second shows 1, the number recorded is 41.) Find the probability that the number is between 23 and 45 (both end points excluded).

- (a) 11/20  
- (b) 9/20  
- (c) 9/10  
- (d) 7/20  
- (e) None of them

Note: This is from p191 [7.7] ANS (b). we can get 4*5=20 different numbers from the two drawings. Of these numbers, 24,25,31,32,34,35,41,42,43 are between 23 and 45.

26. If a coin is tossed 10 times, the outcome can be recorded as a 10-character sequence of H’s and T’s according to the results of the successive tosses. In how many ways can there be 6 H’s and 4 T’s?

- (a) \( \binom{10}{6} \)  
- (b) \( \binom{10}{6} \)  
- (c) \( \binom{14}{4} \)  
- (d) \( \binom{10}{6}\binom{4}{4} \)  
- (e) None of them

Note: This is from p188 [6.4] ANS (a). rule of combination. \( \binom{10}{6} = \binom{10}{4} \)

27. The probability distribution of a random variable \( X \) is given by the function

\[
f(x_i) = \frac{6}{11x_i} \quad x_i = 1, 2, 3
\]

Please calculate the mean of this distribution.

- (a) 18/11  
- (b) 11/18  
- (c) 6/11  
- (d) 11/6  
- (e) None of them

Note: This is from p219 [3.10] ANS (A). \( E(x) = 1 \times (6/11) + 2 \times (6/22) + 3 \times (6/33) = 18/11. \)

28. You have a box containing 9 balls, 3 are red and 2 are blue, and 4 are pink. You want to draw two balls from the box without replacement. Find \( P(\text{Second ball is red} | \text{First ball is blue}) \)?

- (a) 1/4  
- (b) 3/8  
- (c) 3/11  
- (d) 1/12  
- (e) None of them

Note: This is from p.180 4.5.7. Answer=B The answer is trivial. Suppose we take one blue ball out, then there are 8 balls left. Among those 8 balls, three are red. Then the probability is \( \frac{3}{8} \).

29. You have a box containing 9 balls, 3 are red and 6 are blue. You want to draw two balls from the box without replacement. Find \( P(\text{Second ball is red} \cap \text{First ball is blue}) \).

- (a) 1/4  
- (b) 3/8  
- (c) 3/11  
- (d) 1/12  
- (e) None of them
Note: This is from p.180 4.5.7. Answer=A. For this question, we need to use the formula for conditional probability. $P[A|B] = \frac{P[A \cap B]}{P[B]}$. The probability of Second ball is red given that First ball is blue is actually equal to $P[\text{Second ball is red} \mid \text{First ball is blue}] \times P[\text{First ball is blue}] = \frac{3}{8} \times \frac{6}{9} = \frac{1}{4}$

30. You have a box containing 11 balls, 3 are red and 4 are blue, and 4 are pink. You want to draw two balls from the box without replacement. Find $P[\text{Second ball is red}]$.

(a) $1/4$ (b) $3/8$ (c) $3/11$ (d) $1/12$ (e) None of them

Note: This is from p.180 4.5.7. Answer=C This question is more complicated. We have to have the prior knowledge that $P[\text{Second ball is red}] = P[\text{First ball is blue} \cap \text{Second ball is red}] + P[\text{First ball is red} \cap \text{Second ball is red}] + P[\text{First ball is pink} \cap \text{Second ball is red}]$ so we have to calculate $P[\text{First ball is blue} \cap \text{Second ball is red}] = P[\text{Second ball is red} \mid \text{First ball is blue}] \times P[\text{First ball is blue}] = \frac{3}{10} \times \frac{6}{11} = \frac{12}{110}$; then use the same trick, we can get $P[\text{First ball is red} \cap \text{Second ball is red}] = \frac{6}{110}$, $P[\text{First ball is pink} \cap \text{Second ball is red}] = \frac{12}{110}$ so $P[\text{Second ball is red}] = \frac{12}{110} + \frac{6}{110} + \frac{12}{110} = \frac{30}{110}$

31. Suppose that $P[A] = 0.6$, $P[B] = 0.5$ $P[A \cup B] = 0.8$. Find the conditional probability that $B$ does not occur given that $A$ occurs.

(a) $1/3$ (b) $1/2$ (c) $1/4$ (d) $1/25$ (e) None of them

Note: This is from p.179 4.5.1 Answer=B $P[A \cap B] = 0.6 + 0.5 - 0.8 = 0.3$ and $P[\overline{B} \cap A] = P[A] - P[AB] = 0.6 - 0.3 = 0.3$; therefore, $P[B \mid A] = \frac{P[\overline{B} \cap A]}{P[A]} = 0.3/0.6 = 1/2$

32. Approximately 60% of the Wisconsin population has black hair. If 3 persons are selected at random, find $P[\text{at least one black hair}]$.

(a) $1/216$ (b) $1/64$ (c) $215/216$ (d) $63/64$ (e) None of them

Note This is from p.182 4.5.20 Answer=E Use the binomial model. The trick here is $P[\text{at least one}] = 1 - P[\text{none of them}] = 1 - (0.4)^3$

33. Find $P[x = 0]$, given that mean of the distribution is 1.5.

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<th>f(x)</th>
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<tbody>
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<td>0.3</td>
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<tr>
<td>2</td>
<td>0.3</td>
</tr>
<tr>
<td>4</td>
<td>?</td>
</tr>
</tbody>
</table>

(a) 0.1 (b) 0.15 (c) 0.25 (d) 0.3 (e) None of them

Note This is from p.218 5.3.2 Answer=C We first have to know, the sum of probability is 1. so in the question, $f(0) + 0.3 + 0.3 + f(4) = 1$ we know the mean is 1.5, which tells us $0 \times f(0) + 1 \times f(1) + 2 \times f(2) + 4 \times f(4) = 1.5$; $0 + 0.3 + 0.6 + 4 \times f(4) = 1.5$; $f(4) = 0.6/0.4 = 0.15$ as a result of sum of probability is 1, $f(0) = 1 - 0.3 - 0.3 - 0.15 = 0.25$
34. Suppose a 8 face die has its even-numbered faces painted red, and the odd-numbered faces are white. Consider the experiment of rolling the die once and the events. \( A = \{3 \text{ or } 5 \text{ or } 8 \text{ shows up}\} \) \( B = \{A \text{ red face shows up}\} \) Which of the following is true?

(a) \( P[A] = 0.4 \)  
(b) \( P[A \text{ or } B] = 0.7 \)  
(c) \( P[A \mid B] = 0.25 \)  
(d) \( P[A \text{ and } B] = 0 \)  
(e) None of them

Note This is from p.194, 4.7.24 Answer=C \( P[A] = \frac{3}{8} ; \ P[A \text{ or } B] = P[3 \text{ or } 5 \text{ or } 8 \text{ shows up or red color}] = P[3 \text{ or } 5 \text{ or } 8 \text{ shows up or even}] = \frac{6}{8} ; \ P[A \mid B] = \frac{(1/8)}{(1/2)} = 0.25 \)

35. Two of the integers \( \{0,1,3,5,6\} \) are chosen at random without replacement. Let \( X \) denotes the sum of the two integers. Find \( P[X=6] \).

(a) 0.1  
(b) 0.2  
(c) 0.3  
(d) 0.4  
(e) None of them

Note This is from p.209 5.2.8 Answer=B we can generate total 10 pairs and the only two case to generate are pair \((1,5)\) and \((0,6)\) so the \( P[X = 6] = \frac{2}{10} \)