ST 301 (AKI)  
Mid1 SOLUTION

A1:C  
Key concepts: A and B can be denoted as AB; not A can be denoted as \( \bar{A} \). So the answer is \( AB \bar{C} \).

A2:A  
\[ P(ABC) + P(AB\bar{C}) + P(B\bar{A}C) = P(\bar{A}C) + P(AB\bar{C}) + P(B\bar{A}), \]
where \( P(\bar{A}C) = 0.1, P(AB) = P(B) - P(AB) = P(B) - P(A|B)P(B) = 0.1. \) Since \( B \) and \( C \) are mutually exclusive, \( P(ABC) = P(A) - P(AB) - P(AC) = 0.25. \) So the answer is 0.45.

A3:B  
Refer to A2, \( P(AC) = 0.15 = P(A)P(C) \), so \( A \) and \( C \) is independent.

A4:B  
Expected winnings = \( 300 \times \frac{1}{300} + 30 \times \frac{1}{30} + 10 \times \frac{1}{10} + 5 \times \frac{1}{5} = 4 \)

A5:B  
This is a binomial distribution with \( n = 3 \) and \( p = 0.99 \), then \( P(X = 2) = \binom{3}{2}0.99^20.01 \).

A6:D  
\[ P(-1 \leq Z \leq 0) = P(0 \leq Z \leq 1) = P(Z \leq 1) - P(Z < 0) = 0.8413 - 0.5 = 0.3413. \]

A7:B  
Let \( A \) be \{third child is the first son\} and \( B \) be \{the first child is girl\}. Then \( P(A|B) = P(AB)/P(B) = \frac{0.53}{0.5} = 0.25. \)

A8:D  
- \( \mu = 1 \) not 2.
- The probability histogram is symmetric.
- \( \sigma = 0.1 \) not 0.6.

A9: B  
The desired option is supposed to cover one or several groups entirely.

A10: B  
The first quartile is one-quarter of the way down the list.

A11: A  
Sample median is the middle value of the ordered data.

A12: A
\[ \overline{x} = \frac{1}{n} \sum x_i = \frac{4+6+8}{3} = 6 \]

\[ s^2 = \frac{1}{n-1} \sum (x_i - \overline{x})^2 = \frac{4+0+4}{2} = 4. \]

A13: A
Interquartile range = Third quartile - First quartile = 2.5 - 1 = 1.5.

A14: B
\( \overline{B} = \{r, o, y, g\} \), hence \( A \overline{B} = \{r, y\} \).

A15: D
\[ P(\overline{A}B) = P(A) - P(AB) = .35 - .23 = .12; \]
\[ P(\overline{B}) = 1 - P(B) = 1 - .58 = .42 \]
\[ P(AB) = P(B) - P(\overline{AB}) = .42 - .12 = .3 \]

A16: B
\[ P(AB) = P(A) + P(B) - P(A \text{ or } B) = 1.1 - P(A \text{ or } B) > 0, \]
hence \( A \) and \( B \) are not exclusive.

A17: D
\[ P(\overline{A}) = 1 - P(A) = 1 - .6 = .4; \]
\[ P(\overline{AB}) = P(B) - P(AB) = .5 - .2 = .3; \]
\[ P(A|\overline{B}) = \frac{P(\overline{AB})}{P(\overline{A})} = \frac{3}{4} = .75; \]

A18: B
Let \( B \) denote \( \{ \text{A red ball appears in the first draw}\} \).
Then \[ P(A) = P(B)P(A|B) + P(\overline{B})P(A|\overline{B}) = \frac{6}{10} \times \frac{5}{9} + \frac{4}{10} \times \frac{6}{9}. \]

A19: D
\[ P(AB) = P(B)P(A|B) = .35; \quad P(A \text{ or } B) = P(A) + P(B) - P(AB) = .6 + .7 - .35 = .95 \]

A20: B
\[ P(AB) = P(B)P(A|B) = .24; \]
\[ P(\overline{AB}) = P(B) - P(AB) = .8 - .24 = .56; \]

A21: B
Since \( AB \subset A \subset A \text{ or } B \), we have \( P(AB) \leq P(A) \leq P(A \text{ or } B) \).
A22: A
10 choose 3.

A23: E
The number of all possible outcomes is \( \binom{20}{3} \), the number of outcomes in A is \( \binom{15}{2} \binom{5}{1} \), hence the correct answer should be \( \frac{\binom{15}{2} \binom{5}{1}}{\binom{20}{3}} \).

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A24:C
\[ P(M_0) = P(\text{[face 2] or [face 3 and B appears]}) = P(\text{[face 2 up]}) + P(\text{[face 3 up and B appears]}) \text{ (mutually exclusive)} \]
\[ = \frac{1}{5} + P(B \text{ appears| face 3 up})P(\text{ face 3 up}) \text{ (Bayes rule)} \]
\[ = \frac{1}{5} + \left(\frac{3}{10}\right)\left(\frac{1}{5}\right) = \frac{13}{50}. \]

A25: C
Let \( X_i = i\)-th throw result. Observe \( P(X_i = B) = P(M_0) = \frac{13}{50} \). Since each throw is independent, \( P(M_1) = P(X_1 = B, X_2 = B, X_3 = B, X_4 = B, X_5 = B) \)
\[ = P(X_1 = B)P(X_2 = B)P(X_3 = B)P(X_4 = B)P(X_5 = B) \]
\[ = \left(\frac{13}{50}\right)^5. \]

A26: C
Observe \( P(M_2) = P(M_1) = 1 - P(M_1) = 1 - \left(\frac{13}{50}\right)^5. \)

A27: E
Observe, \( P(X_1 = C) = P(X_i = \text{[face 3 up and C appears]}) \)
\[ = P(C \text{ appears| face 3 up})P(\text{ face 3 up}) \text{ (Bayes rule)} \]
\[ = \left(\frac{7}{10}\right)\left(\frac{1}{5}\right) = \frac{7}{50}. \]
Hence
\[ P(X_1 = A) = \frac{1}{5} \]
\[ P(X_1 = B) = \frac{13}{50} \]
\[ P(X_1 = C) = \frac{7}{50} \]
\[ P(X_1 = D) = \frac{1}{5} \]
\[ P(X_1 = E) = \frac{1}{5} \]

Because each throw is independent, for \( i = 1 \ldots 5 \), \( P(X_i = A) = \frac{1}{5} \)
\[ P(X_i = B) = \frac{13}{50} \]
\[ P(X_i = C) = \frac{7}{50} \]
\[ P(X_i = D) = \frac{1}{5} \]
\[ P(X_i = E) = \frac{1}{5} \]
Hence,
\[ P(X_1 = A, X_2 = B, X_3 = C, X_4 = D, X_5 = E) = P(X_1 = A)P(X_2 = B)P(X_3 = C)P(X_4 = D)P(X_5 = E) \text{ (independence)} \]
\[ = \left( \frac{1}{5} \right)^3 \frac{3}{5} \frac{7}{50} \]

Now there are 5! ordering of ABCDE, i.e. the number of outcome with 5 different letter is ABCDE

ABCD

ABDEC

etc..

They are equally likely. ie, \( P(X_1 = A, X_2 = B, X_3 = C, X_4 = D, X_5 = E) \)
\[ = P(X_1 = A, X_2 = B, X_3 = C, X_4 = E, X_5 = D) \]
\[ = P(X_1 = A, X_2 = B, X_3 = D, X_4 = E, X_5 = C) \]
\[ = \text{ etc...} \]

Hence, \( P(M_3) = P(X_1 = A, X_2 = B, X_3 = C, X_4 = D, X_5 = E) \cdot 5! \)
\[ = 5! \left( \frac{1}{5} \right)^3 \frac{3}{5} \frac{7}{50} \]

A28: D

A29: B

small \( p=0.5 \) means symmetric around the center (\( n/2 \)). We have \( n=100^{100} \).

Hence, symmetric around \( (100^{100})/2 \).

A30: A

Because \( p=0.51 \), \( \binom{100^{100}}{0} + 1 = \binom{100^{100}}{100^{100}} + 1 \),
\[ P(X = 0) = \binom{100^{100}}{0} + 1 \left( \frac{0.49}{100^{100}} + 1 \right) \]
\[ P(X = 100^{100} + 1) = \binom{0}{0} + 1 \left( \frac{0.51}{100^{100}} + 1 \right) \]
Hence, \( P(X = 0) < P(X = 100^{100} + 1) \).