ST 301 (AKI)

Choose one answer for each question.

1. Express the event “A and B occur and C does not” in terms of the three events A, B and C.
   (a) ABC (b) $A \cup B \cup C$ (c) $AB\bar{C}$ (d) $A \cup B \cup \bar{C}$ (e) None of them

2. Suppose three events $A$, $B$ and $C$ are such that $B$ and $C$ are mutually exclusive and $P(A) = 0.6$, $P(B) = 0.3$, $P(C) = 0.25$, $P(A|B) = \frac{2}{3}$, and $P(\bar{A}C) = 0.1$. What is the probability that only one of the three events occurs?
   (a) 0.45 (b) 0.25 (c) 0.1 (d) 0.2 (e) None of them

3. Suppose three events $A$, $B$ and $C$ are such that $B$ and $C$ are mutually exclusive and $P(A) = 0.6$, $P(B) = 0.3$, $P(C) = 0.25$, $P(A|B) = \frac{2}{3}$, and $P(\bar{A}C) = 0.1$. Which pair of events are independent?
   (a) $B$, $C$ (b) $A$, $C$ (c) $A\bar{B}$, $C$ (d) $A$, $B \cup C$ (e) None of them

4. A book club announces a sweepstakes to attract new subscribers. The prizes and the corresponding chances are listed here:
<table>
<thead>
<tr>
<th>Prize</th>
<th>Chance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$300</td>
<td>1 in 300</td>
</tr>
<tr>
<td>$30</td>
<td>1 in 30</td>
</tr>
<tr>
<td>$10</td>
<td>1 in 10</td>
</tr>
<tr>
<td>$5</td>
<td>1 in 5</td>
</tr>
</tbody>
</table>

What is the expected winnings?
   (a) $100 (b) $4 (c) $0.13 (d) Can not be calculated from above information. (e) None of them

5. A backpacking party carries three emergency signal flares, each of which will light with a probability of 0.99. Assuming that the flares operate independently. What is the probability that exact two flares light?
   (a) $\binom{3}{2}0.01^20.99^1$ (b) $\binom{3}{2}0.01^10.99^2$
   (c) $\binom{3}{2}0.01^10.99^2 + \binom{3}{1}0.01^20.99^1$ (d) $1 - 0.99^3$ (e) None of them

6. Suppose we know for standard normal distribution $Z$, $P(Z \leq 1) = 0.8413$, what is the probability of $P(-1 \leq Z \leq 0)$?
   (a) 0.8413 (b) 0.5 (c) 0.1587 (d) 0.3413 (e) None of them

7. The probability of having a male child is 0.5, and a young couple only have a girl. If they plan to have at least three children, what is the probability that their third child is the first son?
   (a) 0.125 (b) 0.25 (c) 0.5 (d) 0.875 (e) None of them

8. For the following probability distribution:
   \[
   \begin{array}{c|c}
   x & f(x) \\
   \hline
   0 & 0.3 \\
   1 & 0.4 \\
   2 & 0.3 \\
   \end{array}
   \]
Which one is correct?
(a) $\mu$ is 2.0 (b) The probability histogram of the distribution is not symmetric. (c) $\sigma$ is 0.6. (d) The probability of $x = 3$ is 0. (e) None of them.

9. The ages of people in Madison are to be grouped into the following classes: 0-6, 7-13, 14-20, 21-30, 31-60, older than 60. Which one of the following is able to be determined from this frequency distribution?
(a) Number of people older than 25
(b) Number of people younger than 21
(c) Number of people younger than 60
(d) Number of people older than 14
(e) None of them

10. A large mail-order firm employs numerous persons to take phone orders. Computers on which orders are entered also automatically collect data on phone activity. One variable useful for planning staffing levels is the number of calls per shift handled by each employee. From the data collected on 25 workers, calls per shift were:

66 76 77 85 123

The first quartile is:
(a) 66 (b) 76 (c) 77 (d) 85 (e) None of them

11. To investigate the economic impact of doing business with the state, a sample was taken of 15 small firms in the service sector that are vendors to the state. The data, on the percent of total sales due to sales to the state, are:

1, 1, 2, 4, 5, 7, 7, 8, 8, 10, 10, 10, 11, 14, 23

The sample median is:
(a) 8 (b) 7 (c) 7.5 (d) 8.5 (e) None of them

12. A sample of seven compact discs at the music store stated the following performance times (in minute) for Beethoven’s Ninth Symphony.

4, 6, 8

The sample mean $\bar{x}$ and sample variance $s^2$ are:
(a) $\bar{x}=6$ $s^2=4$ (b) $\bar{x}=6.5$ $s^2=6.67$
(c) $\bar{x}=6$ $s^2=6.67$ (d) $\bar{x}=6.5$ $s^2=4$
(e) None of them
13. In August, the Jump River Electric Co. suffered 11 power outages that lasted the following number of hours:

1, 1, 1, 1.5, 1.5, 1.5, 2, 2, 2.5, 3, 10

The interquartile range is:
(a) 1.5  (b) .5  (c) .75  (d) 1  (e) None of them

14. Sun visors come in red (r), orange (o), yellow (y), green (g), blue (b), purple (p), and white (w). For the next visor sold, consider the two events \( A = \{ r, y, b, w \} \) and \( B = \{ b, p, w \} \). Then, which of the following events has the exact composition as \( \{ r, y \} \)?
(a) \( AB \)  (b) \( A \overline{B} \)  (c) \( \overline{A} \)  (d) \( \overline{AB} \)  (e) None of them

15. For two events \( A \) and \( B \), the following probabilities are specified.

\[
P(A) = .35 \quad P(B) = .58 \quad P(AB) = .23
\]

What’s the probability of \( P(\overline{A} \overline{B}) \)?
(a) .77  (b) .12  (c) .35  (d) .30  (e) None of them

16. If \( P(A) = .7 \) and \( P(B) = .4 \), which statement of the following is true?
(a) \( A \) and \( B \) are mutually exclusive
(b) \( A \) and \( B \) are not mutually exclusive
(c) More information are needed to determine whether \( A \) and \( B \) are mutually exclusive or not
(d) \( P(AB) = .28 \)  (e) None of them

17. Suppose that \( P(A) = .6 \), \( P(B) = .5 \), \( P(AB) = .2 \).
Then \( P(B|A) = ? \)
(a) .50  (b) .33  (c) .40  (d) .75  (e) None of them

18. An urn contains four green balls and six red balls. Suppose two balls will be drawn at random one after another and without replacement (i.e., the first ball is not returned to the urn before the second one is drawn). Let \( A = \{ \text{a red ball appears in the second draw} \} \). Then, \( P(A) = ? \)
(a) \( \frac{6}{10} \times \frac{5}{9} \)  (b) \( \frac{6}{10} \times \frac{5}{9} + \frac{4}{10} \times \frac{6}{9} \)  (c) \( \frac{4}{10} \times \frac{5}{9} \)  (d) \( \frac{6}{10} \times \frac{4}{9} \times \frac{5}{10} \times \frac{6}{9} \)  (e) None of them

19. Suppose that \( P(A) = .6 \), \( P(B) = .7 \), \( P(A|B) = .5 \).
Then \( P(A \text{ or } B) = ? \)
(a) 1.0  (b) 1.3  (c) .35  (d) .95  (e) None of them

20. Suppose that \( P(A) = .4 \), \( P(B) = .8 \), \( P(A|B) = .3 \).
Then \( P(\overline{AB}) = ? \)
(a) .48  (b) .56  (c) .16  (d) .40  (e) None of them
21. Suppose each of the number .21, .37, and .57 represents the probability of one of the events $A$, $AB$, and $A$ or $B$. Which of the following correctly connect the probabilities to the appropriate events?

(a) $P(A) = .21$  $P(AB) = .37$  $P(A \text{ or } B) = .57$
(b) $P(A) = .37$  $P(AB) = .21$  $P(A \text{ or } B) = .57$
(c) $P(A) = .57$  $P(AB) = .37$  $P(A \text{ or } B) = .21$
(d) $P(A) = .37$  $P(AB) = .57$  $P(A \text{ or } B) = .57$
(e) None of them

22. If a coin is tossed 10 times, the outcome can be recorded as a 10-character sequence of H’s and T’s according to the results of the successive tosses. In how many ways can there be 3 H’s and 7 T’s?

(a) $\binom{10}{3}$  (b) $\binom{7}{3}$  (c) $\binom{13}{3}$  (d) $\binom{17}{7}$  (e) None of them

23. A batch of 20 items contains 5 defectives. If three items are sampled at random.

Let $A$= [Exactly one defectives appear], then $P(A) =$?

(a) $\binom{15}{1} \binom{5}{4} \binom{20}{5}$  (b) $\binom{15}{1} \binom{5}{4} \binom{20}{3}$  (c) $\binom{15}{4} \binom{5}{1} \binom{20}{5}$  (d) $\binom{15}{4} \binom{5}{1} \binom{20}{3}$  (e) None of them

Fair 5-face dice has 2 letters on each face:

face1: [1 A]
face2: [2 B]
face3: [3 ?] where "? = B" with 30%, "? = C" with 70%.
face4: [4 D]
face5: [5 E]

For example, if face 2 is up, you observe [2 B] all the time. If face 3 is up, you observe [3 B] with 30% or [3 C] with 70%. Define events:

$M = 1$ throw. "A" appears.
$M_0 = 1$ throw. "B" appears.
$M_1 = 5$ throws. "B" appears 5 times.
$M_2 = 5$ throws. "B" DOES NOT appear 5 times.
$M_3 = 5$ throws. Each letter $A, B, C, D,$ and $E$ appears once, i.e. "ADEBC", "CDEBA", "EDCBA", etc.

For example, $P(M) = P([1 A]$ appears) = $P$ (face1) = $1/5$.

24. $P(M_0) =$?

(a) 0.24  (b) $\frac{1}{5} + \frac{1}{5} \cdot \frac{4}{5}$  (c) $\frac{1}{5} + \frac{1}{5} \cdot \frac{3}{10}$  (d) $\frac{1}{5} + \frac{1}{5} + \frac{3}{10}$  (e) None of them
25. \( P(M_1) = ? \)
   (a) \( 5 \cdot \frac{13}{50} \)  
   (b) 0.0007963  
   (c) \( \left( \frac{13}{50} \right)^5 \)  
   (d) \( \left( \frac{1}{5} \right)^5 \)  
   (e) None of them

26. \( P(M_2) = ? \)
   (a) 0.9992037  
   (b) \( 1 - \left( \frac{1}{5} \right)^5 \)  
   (c) \( 1 - \left( \frac{13}{50} \right)^5 \)  
   (d) 0  
   (e) None of them

27. \( P(M_3) = ? \)
   (a) \( \left( \frac{5}{9} \right)^5 \left[ \frac{1}{5} + \frac{1}{5} \cdot \frac{3}{10} \right]^5 \{ 1 - \left( \frac{1}{5} + \frac{1}{5} \cdot \frac{3}{10} \right) \}^0 \)  
   (b) \( \frac{5}{9} \cdot 0.7 + \frac{1}{9} \cdot 0.3 \)  
   (c) \( \frac{5}{9} \cdot 0.7 \)  
   (d) \( \frac{5}{9} \cdot 0.7 \)  
   (e) None of them

28. You have 1000 observed data. We want to see \( Q_1, Q_2(\text{median}), \) and \( Q_3. \)  Which software R command is most appropriate?
   (a) \text{qbinom()} \  
   (b) \text{hist()} \  
   (c) \text{qqplot()} \  
   (d) \text{boxplot()} \  
   (e) None of them

29. Consider distribution \( X \sim \text{Binomial}(n = 100^{100}, p = 0.5) \)
   Which is true?
   (a) it is symmetric around \( x=(100^{100})/2 - 1 \)  
   (b) it is symmetric around \( x=(100^{100})/2 \)  
   (c) it is symmetric around \( x=(100^{100})/2 + 1 \)  
   (d) it is not symmetric.  
   (e) None of them

30. Consider distribution \( X \sim \text{Binomial}(n = 100^{100} + 1, p = 0.51) \) Which is true?
   (a) \( P(X = 0) < P(X = 100^{100} + 1) \)  
   (b) \( P(X = 0) > P(X = 100^{100} + 1) \)  
   (c) \( P(X = 0) = P(X = 100^{100} + 1) \)  
   (d) \( [P(X = 0) + P(X = 100^{100} + 1)]/2 = \frac{2}{100^{100} + 1} \)  
   (e) None of them