ST 301 (AKI)  

Mid2 SOLUTION

1: This is from p275 [5.12]. \( Z = \frac{b-\mu}{\sigma} = \frac{b+1}{2} = z_{0.025} = 1.96. \) Hence \( b=2.92. \) The answer is b.

2: This is from p275 [5.12]. The period with \( \mu = 20 \) as its center has largest area, i.e. highest probability. Hence the answer is b.

3: This is from p281 [6.1]. Normal approximation to binomial distribution requires each person is independent, however, persons in same family are dependent. The answer is d.

4: This is from p281 [6.5]. To apply normal approximation to binomial, two conditions should be satisfied: \( np > 15 \) and \( n(1-p) > 15. \) The answer is c.

5: This is from p282 [6.14]. \( \text{mean}=np = 2400 \times .4 = 960; \text{sd}=\sqrt{2400 \times .4 \times .6} = 24; \) Hence the probability=\( P\left(\frac{912-960}{24}\right) < Z < \frac{1008-960}{24}\right) = P(-2 < Z < 2) = 0.9544. \) The answer is c.

6: This is from p287 [8.6]. Since standard normal distribution is symmetrical, the answer is a.

7: This is from p288 [8.17]. Two conditions are satisfied, hence normal approximation to binomial is appropriate. \( \text{mean}=15000 \times 0.01 = 150; \text{sd}=\sqrt{15000 \times 0.01 \times 0.99} = 12.186. \) Hence the probability=\( P(Z \geq \frac{165-150}{12.186}) = P(Z > 1.23) = 0.1093. \) The answer is a.

8: This is from p317 [4.12]. The probability=\( P(|X| > 5) = P(|Z| > \frac{5}{60/15}) = P(|Z| > 1.25) = 0.2112. \) The answer is a.

9: This is from p329 [2.1]. a and b are interval estimate; c is point estimate. Hence the answer is c.
10: This is from p360 [5.15]. From the problem we know that \( P(\overline{X} - 50 \leq -c \text{ or } \overline{X} - 50 \geq c) = \alpha = 0.02 \). Hence \( P(\overline{X} - 50 \geq c) = 0.01 \). So, Hence \( P(Z > \frac{c}{5/10}) = 0.01 \Rightarrow c = 1.165 \). The answer is a.

A11: The test statistic is \( Z_{obs} = \frac{\bar{x} - 76}{s/\sqrt{n}} = 2 \). Then \( P-value = P(|Z| > 2) = 2 \times P(Z > 2) = 2 \times 0.0228 = 0.0456 \). The answer is b.

A12: Since \( z_{\alpha/2} = 0.98 \), then \( z_{\alpha/2} = 1.96 \), which implies \( \alpha = 0.05 \). The answer is a.

A13: 18.01 is located in the confidence interval. The answer is b.

A14: For (a), \( P(X > 1.6) = P(Z > 0.05) \) should be very close to 0.5
For (b), \( P(X > 5.5) = P(Z > 2) \neq P(Z > 1) \)
For (c), \( \frac{\overline{X} - 1.5}{2} \) is standard normal
For (d), \( P(X > 2) = P(Z > 0.25) = P(Z < -0.25) = P(X < 1) \)
The answer is d.

A15: Both \( np \) and \( n(1 - p) \) is larger than 15. \( \frac{X - np}{\sqrt{np(1-p)}} \) is approximately standard normal. \( P(67.5 < X < 75) < P(X < 75) = P(Z < 0) = 0.5 \). The answer is a.

A16: For (a)(d), \( sd(\bar{X}) = \sigma/\sqrt{n} = 0.7 \)
For (b), \( E(\bar{X}) = \mu = 99 \)
For (c), \( sd(\bar{X}) = \sigma/\sqrt{n} \) decrease with sample size
The answer is d.

A17: For (a), \( P(\bar{X} - \mu < k) < P(\bar{X} - \mu < 0) = 0.5 \), then \( k \) should be negative.
For (b), \( P(\bar{X} < 3) \) can not be calculated, since we don’t know \( \mu \)
For (c), \( P(1 < \bar{X} - \mu < 2) < P(0 < \bar{X} - \mu) = 0.5 \), then can not be 0.6
For (d), \( \frac{\bar{x} - \mu}{2} \) is standard normal.
The answer is a.

A18: For (a)(c), The length is \( 2 \times z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 0.392 \)
For (b)(d), The error margin is \( z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 0.196 \)
The answer is b.
A19: The answer is a.

A20: The answer is b. because qnorm(0.025) gives lower 0.025 quantile. By symmetry, put ”minus” gives upper 0.025 quantile.

A21: (d). Because $\mu_2 = E[Y_1] = E[-100X_{100}] = -100E[X_{100}] = -100\mu_1$.

A22: (e).

$\mu_{20} \equiv (200 + 1)(-k)$ where k=100. Observe $\bar{Y} = -k \cdot \bar{X}$.

Hence,

$$Z_{Nancy} = \frac{\bar{Y} - \mu_{20}}{sd_Y} = \frac{-k\bar{X} - ((200 + 1)(-k))}{ksd_X} = \frac{-\bar{X} + 200 + 1}{sd_Y}$$

$$= \frac{-\bar{X} + 200}{sd_Y} + \frac{1}{sd_Y} = -Z_{Mike} + \frac{1}{sd_Y} = -(-1.8) + \frac{1}{sd_Y} = 1.8 + \frac{1}{sd_Y}$$

(1)

Remember, $\frac{1}{sd_Y} > 0$ Hence $1.8 < Z_{Nancy}$. Also remember, we don’t care $\alpha = 0.15$ because to get p-value, you do not need $\alpha$.

This is C2 test, hence, (just draw a picture...)

Nancy’s $p-value = P(Z < Z_{Nancy}) > P(Z < 1.8) = 1 - P(Z > 1.8) = 1 - P(Z < -1.8) = 1 - 0.036 = 0.964$. Hence Nancy’s $p-value > 0.964$.

A23: (a). From A22, (now we have C1 test. So p-value is opposite to the A22.)

Nancy’s $p-value = P(Z > Z_{Nancy}) < P(Z > 1.8) = 0.036$. (Just draw a picture.)

Also remember, we don’t care $\alpha = 0.15$ because to get p-value, you do not need $\alpha$. 
A24: (b) \(0.25/2=0.125\). Hence rejection region is wider than Mike’s one sided hypothesis test if you look at left side (negative side) of rejection region. By symmetry, same for right side (positive side) Hence, ”Reject \(H_0\)” In addition, we know from A22, \(1.8 < Z_{Nancy}\).

A25: (d).