An Analysis of Extremes: Semiparametric Efficiency in Regression

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Extremes are of interest in connection with earthquakes, radiation, floods, wind, breaking strength, rare events, and longevity, among others.

We consider statistical inference for non-regular semiparametric models where extremes are represented as the right boundary parameter $\theta$ of the support $[0, \theta]$ of the density of the random variable whose extreme value $\theta$ is of interest.

The extreme value theorem can be applied for upper boundary estimation. One of the most obvious estimators of the upper boundary for one-sample case is the sample maximum. The sample maximum to estimate the boundary when the extreme value index satisfies $\xi < -1/2$ (our model has $\xi = -1$) is approximately equivalent to the moment related estimator for the endpoint (Fraga Alves, Neves, and Rosario (2017) and de Haan and Ferreira (2006, Remark 4.5.5).) However such simple estimator produces a bias and can be improved by our approach. Another approach is to use the Generalized Pareto Distribution (Smith 1987). However it has slower convergence rate than our approach and does not produce semiparametrically efficient estimators.

By using an extension of Le Cam’s theory of limits of experiments to semiparametric models, we derive a lower bound on estimation risk and construct estimators whose risk reaches this lower bound. Our models include regression experiments where the boundary parameter depends on predictors.

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