Solving Huge QAPs with Condor

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LOTS OF PEOPLE IN THIS ROOM
Quadratic Assignment Problem

- Assign facilities to locations
- QAP is NP-Hard
  - No known algorithm is “significantly better” than complete enumeration
  - Examining \(10^9\) configurations / second, for \(n = 27\) would take 345,283,785,211 years, or > 17 Universe Lifetimes.
Mathematically, the QAP can be stated as

$$\min_{\pi \in \Pi} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} b_{\pi(i)\pi(j)} + \sum_{i=1}^{n} c_{i\pi(i)}$$

or

$$\min_{X \in \Pi} \text{trace}(AXB + C)X^T,$$

where $\pi$ is a permutation of $\{1, 2, \ldots, n\}$ and $\Pi$ is the set of permutation matrices of size $n \times n$.

The QAP has a rather long history, and historically it is one of the “hardest” NP-Complete problems. Practical applications are in facility layout, the optimal design of typewriter keyboards, telecommunication network design, and others. For references on the QAP, see the QAPLIB page.

http://www.imm.dtu.dk/~sk/qaplib/
How Patient are You?

- If 345,283,785,211 years seems a bit long to wait, you might try Branch and Bound.
  - Feasible solution $\Rightarrow$ upper bound
  - Relaxed problem $\Rightarrow$ lower bound

A detailed algorithmic description of branch and bound

1. Is solution to relaxed problem feasible?
   Yes? YAHOO!
Branch and Bound

- Conceptually, there is a search tree than must be explored
- Different nodes are different *Independent* searches
  ⇒ Parallel Computing to the Rescue!

* The size of the tree depends on how good the upper and lower bounds are!
Be Afraid. Be Very Afraid.

The lower bound for the QAP is obtained (in part) by solving the (convex) quadratic optimization problem:

\[
\begin{align*}
\min & \quad \text{vec}(X)^T [(B \otimes A) - (I \otimes S) - (T \otimes I)] \text{vec} X + \text{trace}(C X^T) \\
\text{such that} & \quad X e = X^T e = e, \quad X \geq 0.
\end{align*}
\]

\((S\text{ and }T\text{ are obtained from the spectral decompositions of }A\text{ and }B)\).

If you don’t understand this, join the club! (And I also tried to simplify the exposition! :–). See [AB00] for the gory details.

The point is that this quadratic programming problem can be (approximately) solved efficiently, and it produces a lower bound that is “pretty close” to the solution value of the real problem. It is (in order of importance) the combination of the strength of bound, computational efficiency of the bound, and the power of high-throughput computing that make solving big QAPs possible.
MW-QAP – Task definition

- What should a task be?
  - One node – *Way* too small (milliseconds)
  - One subtree – (Perhaps) too big (*Universe Lifetimes*)
  - Compromise – “do subtree for $t$ (300) seconds”
- Result of task – unfinished nodes and improved solution value
- What does the master do? `act_on_completed_task()`
  - Puts unfinished tasks in the list
  - Removes unnecessary tasks if improved solution
Building an Efficient Branch and Bound Solver

- A “Grainsize Analysis” for the QAP showed that...
  - 36% of the tasks have a grainsize smaller than 0.05s!
  - 56% of the tasks have a grainsize smaller than 0.5!
  - 87% of the tasks < 0.5s rooted at nodes deeper than 6.

- Thus we implemented a “finish up” strategy...
  - Workers work for max_cpu (300) seconds
  - If not finished, allow for extra time to finish up “deep” tasks.

- Short tasks reduced from 56% to 26%. But still too many!
  - For a given level to do the “fast” nodes first
  - Order children in DFS stack based on bound information!

- Hardly any short tasks! Parallel efficiency was up to 95%!
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<th>OS-Arch</th>
<th>Where</th>
<th>How</th>
<th>(Peak) GFLOPS</th>
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<td>Glide-in</td>
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<td>502</td>
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<td>-</td>
<td>-</td>
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Jeff’s Guide to Flocking and Glide-In

- Flocking
  0. Be nice to Miron – he will get you permission to flock with different pools.
  1. Modify `condor_config` file to set `LOCAL_DIR` and `FLOCK_HOSTS`.
  2. Start your own schedd: `condor_schedd -f -name linderot`
     ✴ Your jobs will magically be scheduled on machines in the `FLOCK_HOSTS` pools!

- Glide-In
  0. Get account(s) on Globus-enabled remote resources.
  1. Set up Globus identification on local machine. (Copy user certificate and key to known, private location).
  2. `grid-proxy-init`.
  3. `glidein.pl <machine> --count <number>`
     ✴ `<number>` processors from `<machine>` will magically appear in your Condor pool. If not, contact jfrey@cs.wisc.edu and go to 3.
World Records!

- Previous “record” [HHJ$^+99$] ($n = 25$).
  - Previous time, 66 days (sequential)
  + Our time – 6.7 hours! (Avg. 93 processors, Efficiency $\approx 97\%$.

- New “record” QAP solved [ABG$^+00$] ($n = 27$).

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<td>Wall Clock Time</td>
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<td>CPU Time</td>
<td>152:14:56:40</td>
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<td>Nodes</td>
<td>567,793,866</td>
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<td>Avg. Workers</td>
<td>205</td>
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<td>Max Number of Workers</td>
<td>286</td>
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<td>Parallel Efficiency</td>
<td>$&gt; 81%$</td>
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That’s All Folks!

- Conclusions
  - MW can do useful work
  - Condor with Flocking and Glide-in can give you lots of computational power
- Future Work
  - Make MW a little like MS (no need to restart every 12 hours) :-)
  - Size $n = 28$ and beyond!!
- Interactive Problem Solving Environment (iMW). Demo?
References

