Homework 2
Due on March 18

A: SIZE BOUNDS FOR JOINS [25%]

1. [15%] Compute the maximum possible output size for the following queries. Assume that all relations have the same size N.

   (a) \( q(x, y) : -R(x), S(x, y), T(y). \)
   (b) \( q(x, y, z, w) : -R(x, y), S(y, z), T(z, w), U(w, x), V(x, z). \)
   (c) \( q(x, y, z, w) : -R(x, y, z), S(x, z, w), T(x, y, w), U(y, z, w). \)

2. [10%] Suppose that relation \( R_i \) has size \( N_i \). Compute the maximum output size for the following query. (Hint: there will be different cases depending on the specific \( N_i \)).
   \( q(x_1, x_2, x_3, x_4) : -R_1(x_1, x_2), R_2(x_2, x_3), R_3(x_3, x_4), R_4(x_4, x_1). \)

B: DATALOG [75%]

1. [15%] We are given a directed graph represented by the binary relation \( R(x, y) \).
   Write a Datalog program that computes the following unary relations:
   (a) \( OddCycle(x) \): there exists a cycle of odd length that goes through node \( x \),
   (b) \( EvenCycle(x) \): there exists a cycle of even length that goes through node \( x \).
   (c) \( Cycle(x) \): there exists a cycle that goes through node \( x \).

2. [10%] Show that every Datalog query is monotone.

3. [10%] Is the following Datalog program equivalent to a UCQ query? If so, write the query. If not, prove why it is not the case.
   \[ B(X, Y) : - L(X, Y). \]
   \[ B(X, Y) : - T(X), B(Z, Y). \]

4. [10%] Consider the following Datalog program:
T(x, y) :- F(x, y).
T(x, y) :- up(x, z1), T(z1, z2), down(z2, y).

What is the immediate consequence operator for semi-naive evaluation? Run the semi-naive evaluation on the following instance:

\[ F(c, g), F(c, c), up(f, e), up(e, d), up(d, c), down(c, d), down(c, e), down(e, h). \]

5. [20%] Perform the magic set transformation for the following Datalog program:

\[
\begin{align*}
T(x, y) & :\ F(x, y). \\
T(x, y) & :\ up(x, z1), T(z1, z2), F(z2, z3), T(z3, z4), down(z4, y). \\
q(y) & :\ T(a, y).
\end{align*}
\]

6. [10%] Find all possible stratifications for the following Datalog program with negation:

\[
\begin{align*}
T(x) & :\ S(x), \text{ not } R(x) . \\
S(x) & :\ T(x), \text{ not } R(x) . \\
U(x) & :\ R(x), \text{ not } T(x), \text{ not } S(x) . \\
V(x, y) & :\ V(x, y), \text{ not } U(x), \text{ not } U(y) , .
\end{align*}
\]

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Submit a single PDF document using the Dropbox in learn@uw (Homework 2). It is strongly suggested to use \LaTeX{} to write your solution.