Verifying Concurrent Programs via Bounded Context-Switching and Induction

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Verifying Programs

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At the end of this talk, I will know a way of...

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Verifying Programs

Background: Software Verification

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Assertions specify *Properties*

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Challenge!

Infinite / large type domains

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[2,0,*p6*]

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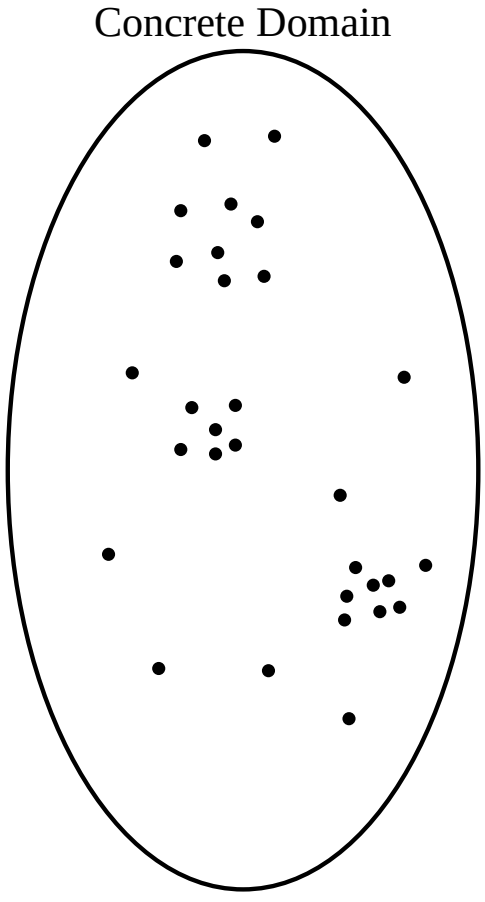
[2,0,*p5*]

[3,0,*p6*]

...

Background: Abstract Interpretation



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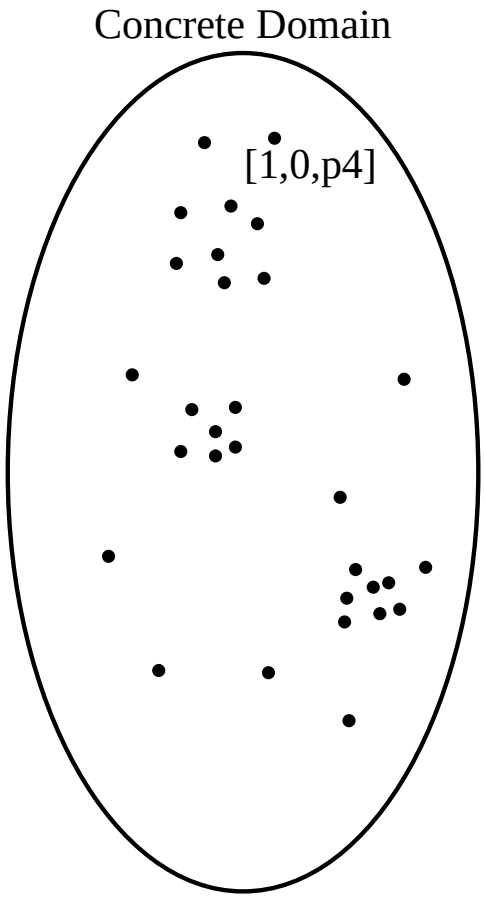

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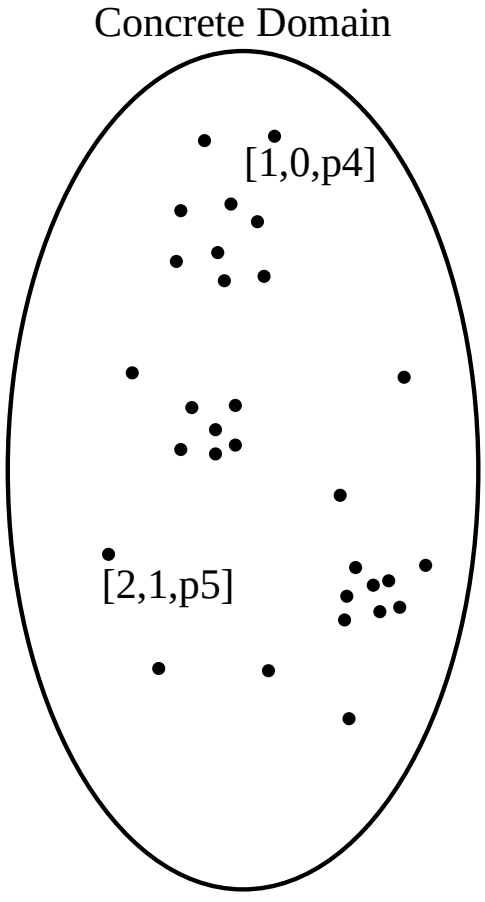

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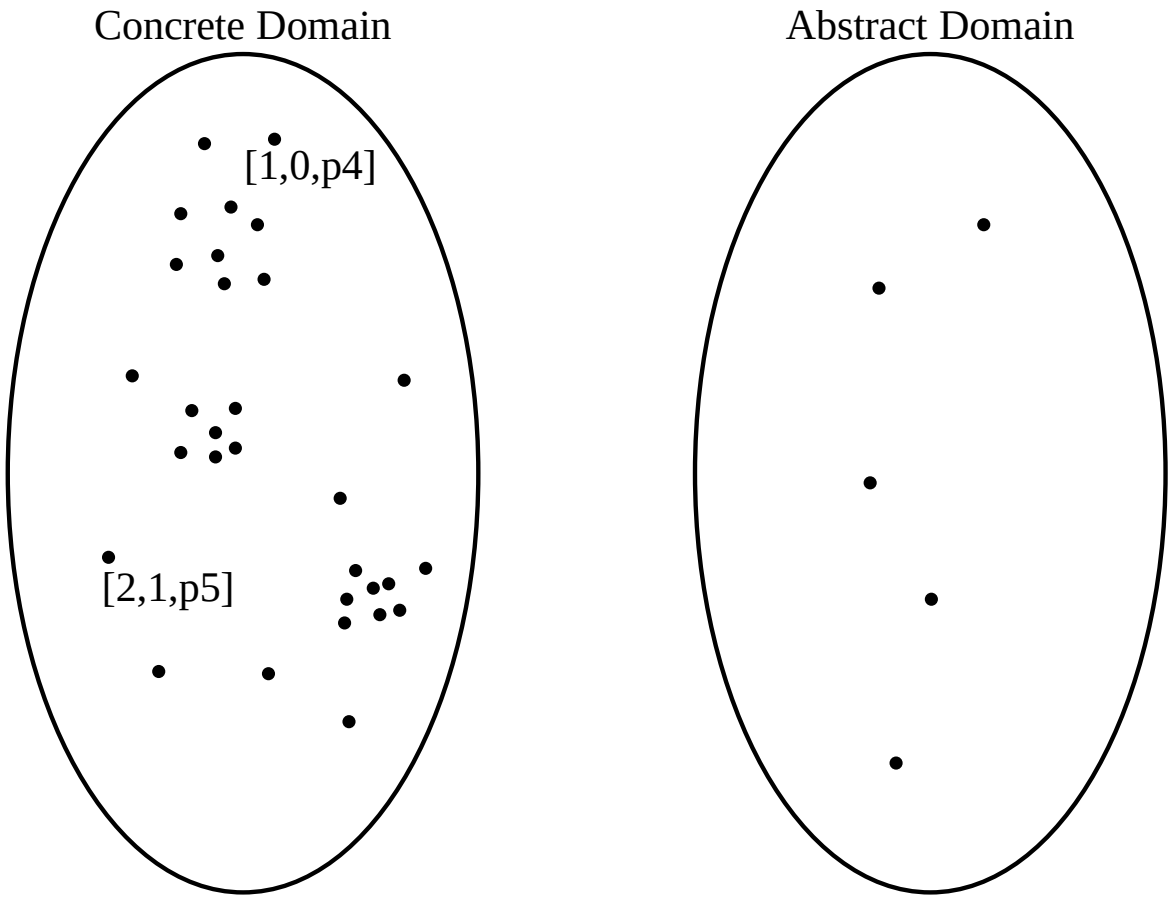

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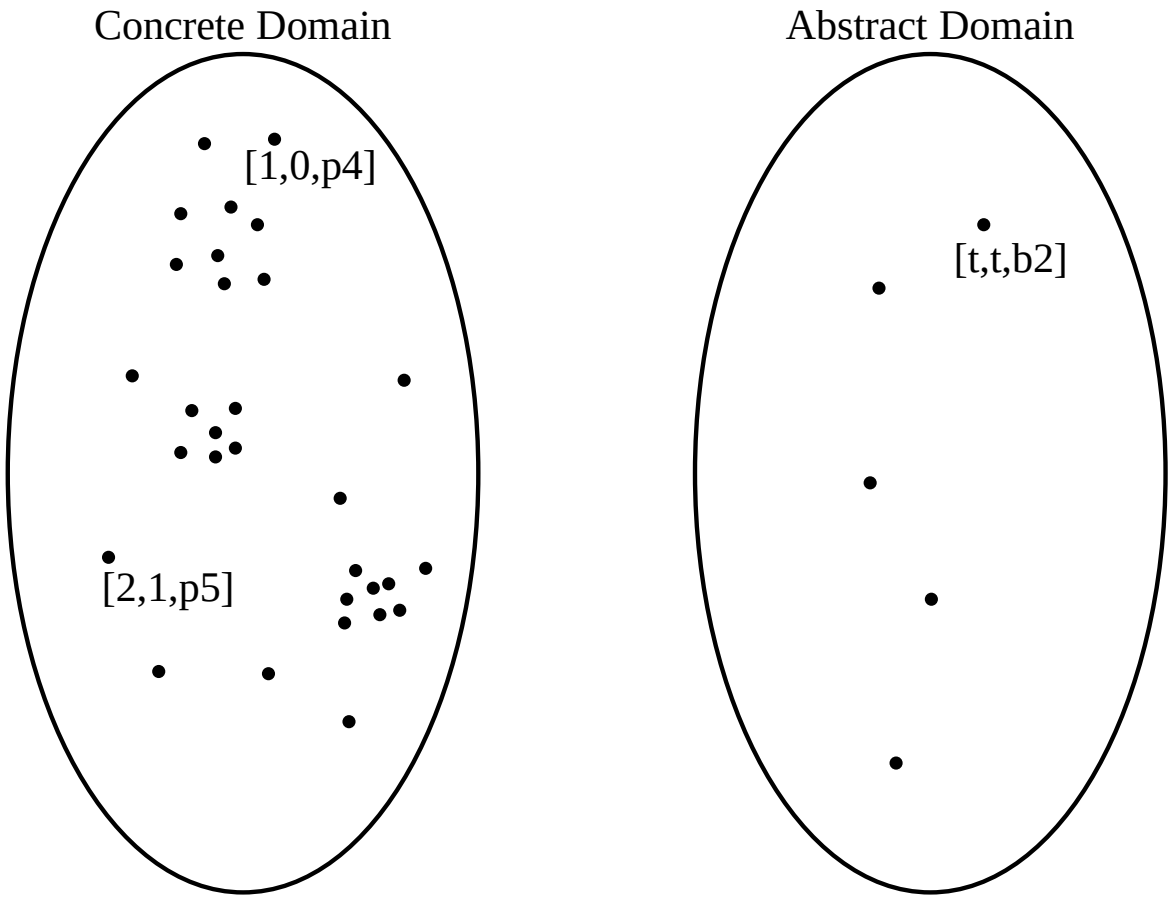

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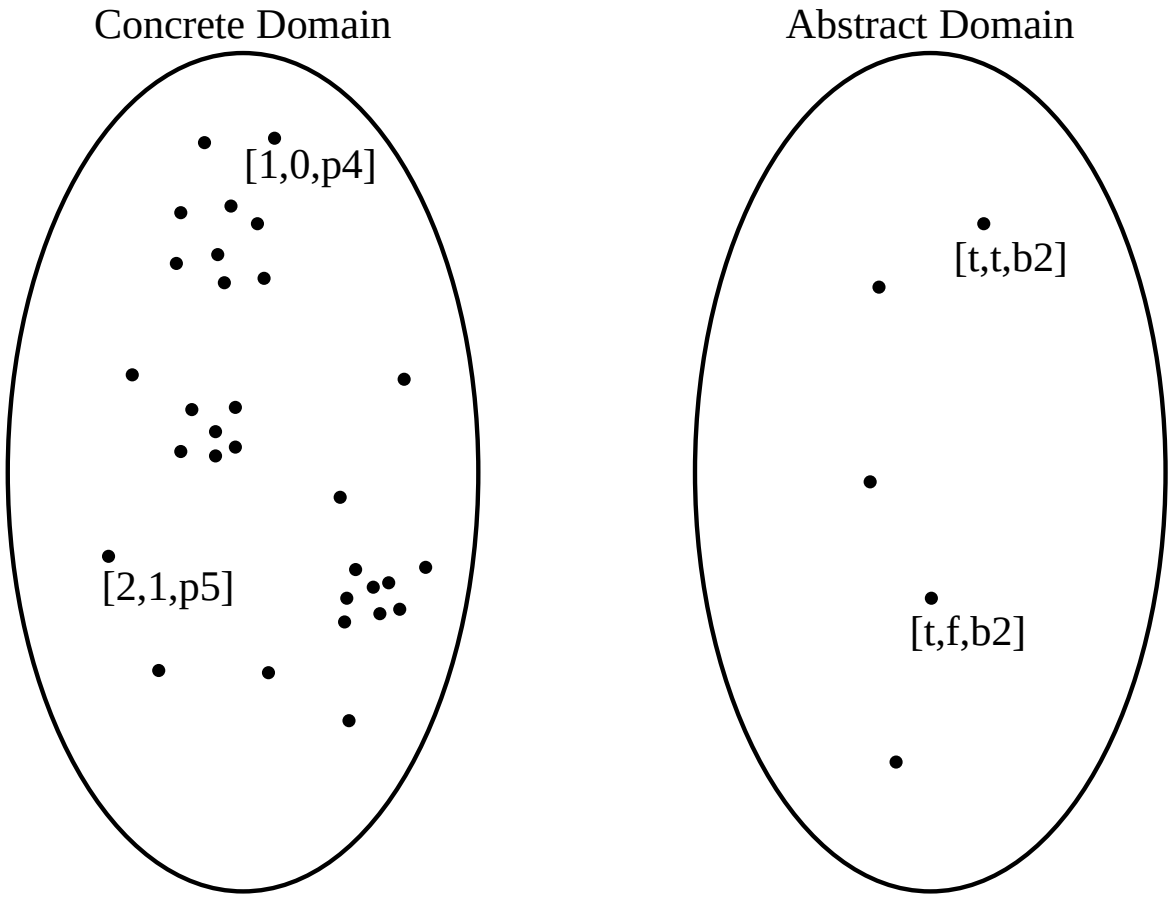

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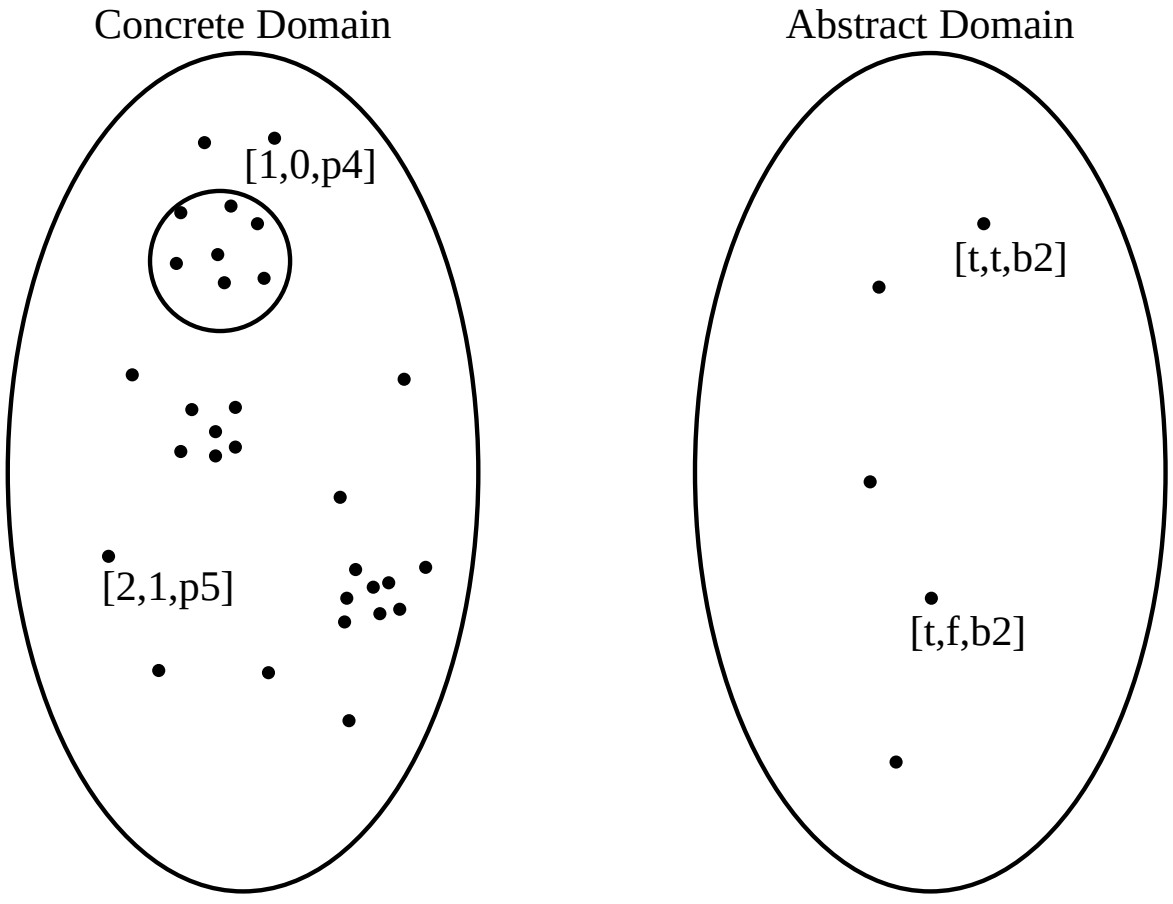

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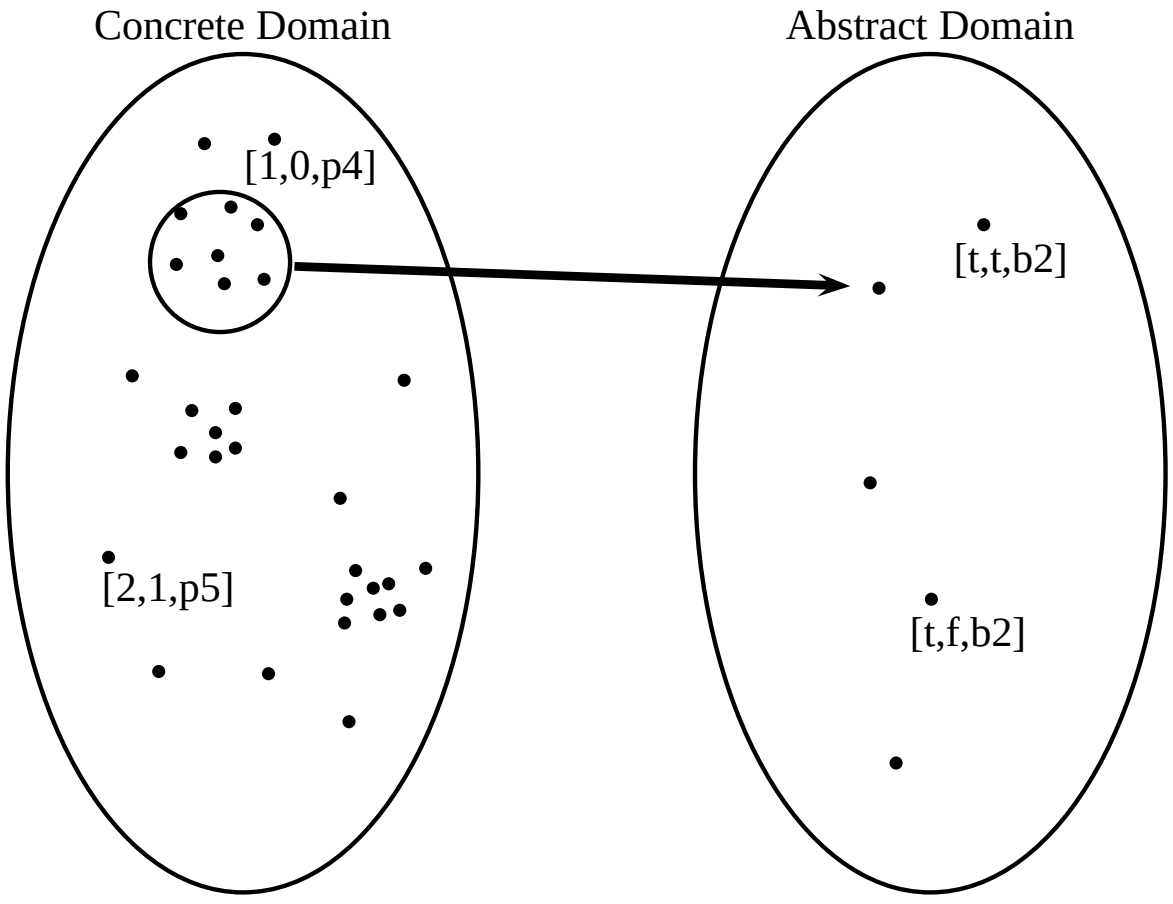

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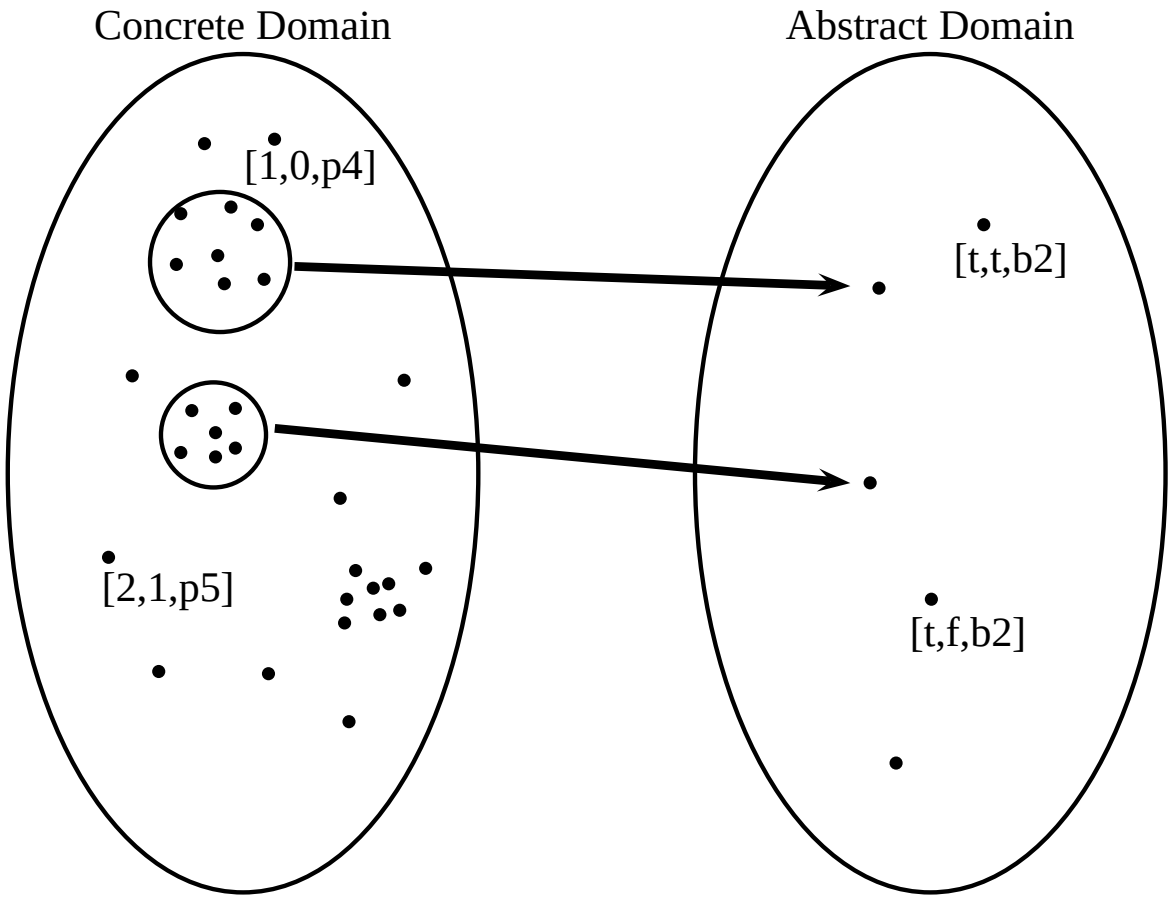

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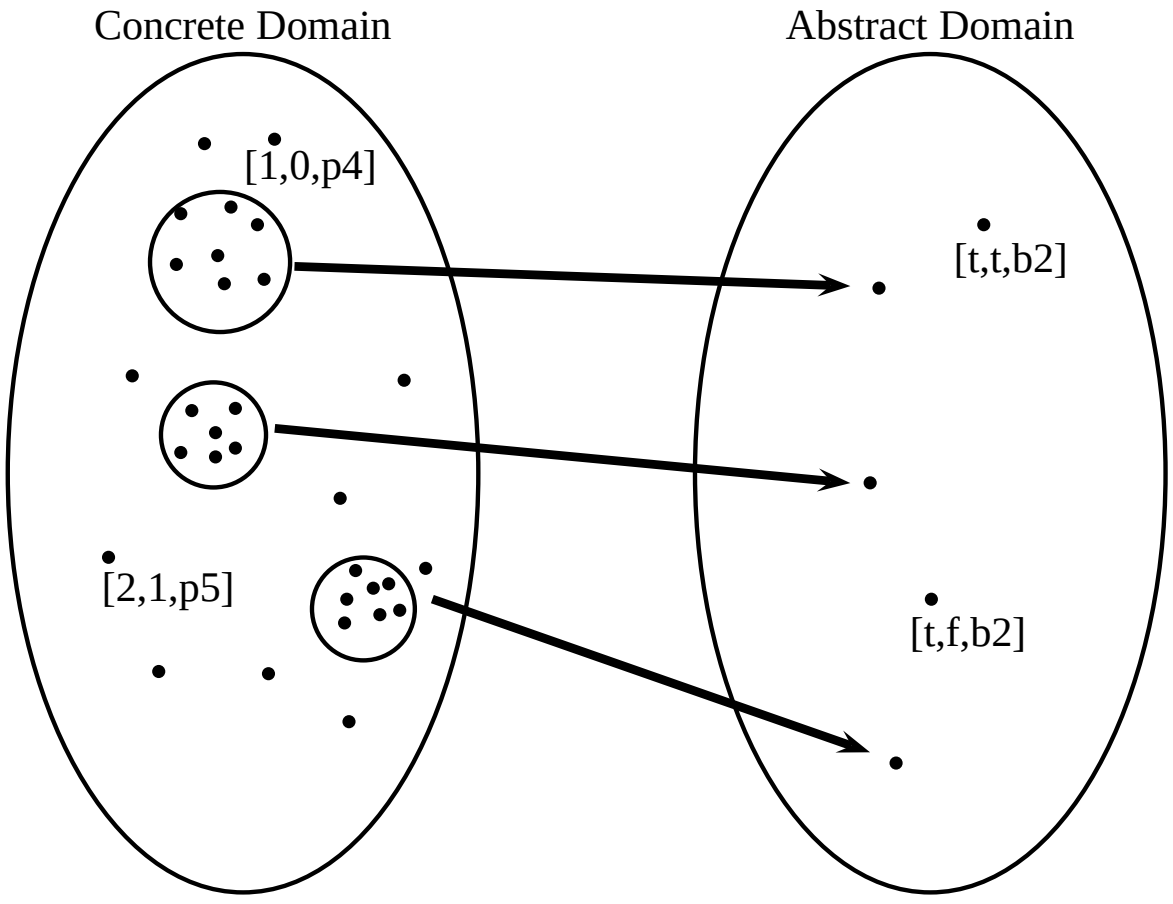

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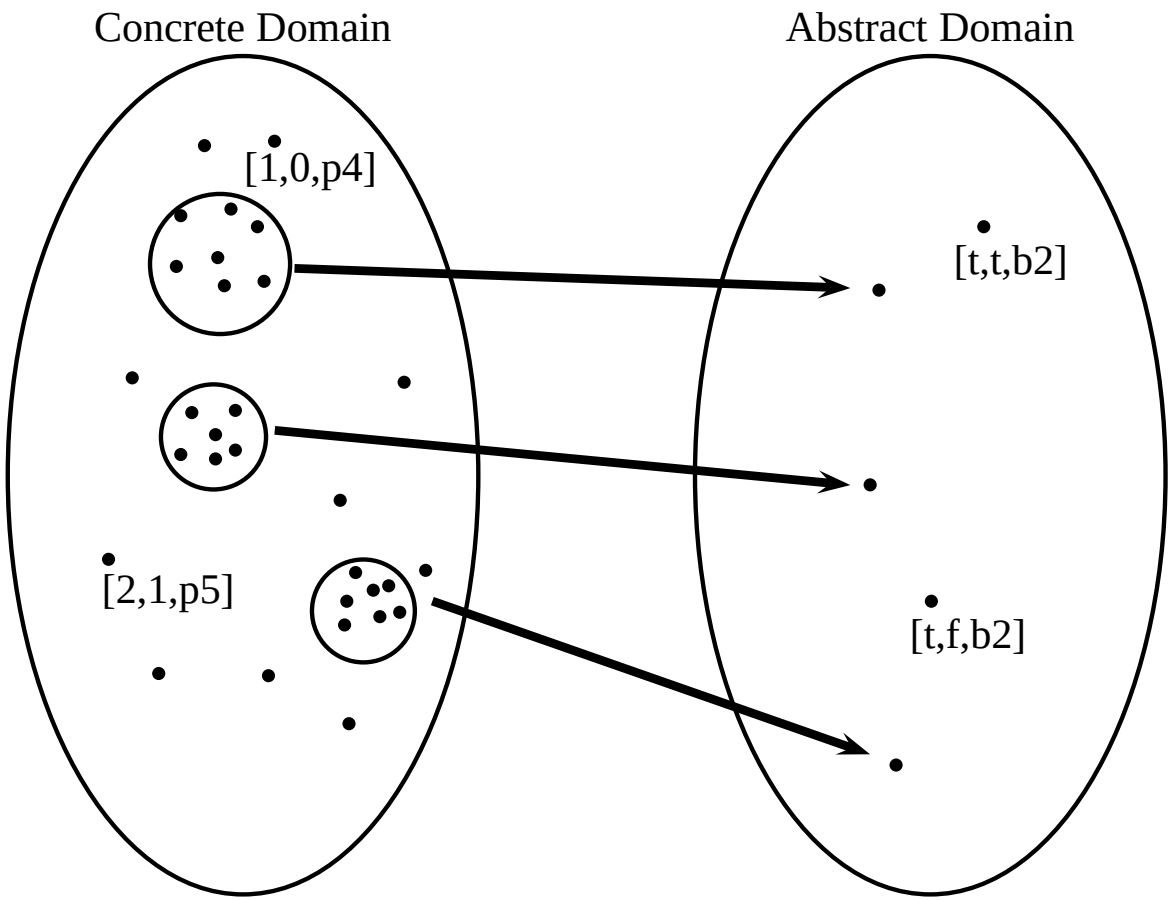

Predicate Abstraction

Background: Abstract Interpretation - Boolean Programs

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Only Boolean Variables

a := *true*
| *false*
| b
| ! b
| b $\vee$ c
| b $\wedge$ c
| *

assume(expr)
if(*) {}
do(*) {}
label:
skip

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States of the transition system

$\mathbb{S} = \{(v, e) | v \in \mathbb{V}ar \wedge e \in \{t, f\}\} \times pc$

e.g.: $[t, t, b1]$

Background: Abstract Interpretation - Boolean Programs

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| b
| ! b
| b ∨ c
| b ∧ c
| *

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e.g.: $[t, t, b1]$

$\Delta[a := true](pc, \overline{v}) = (pc', \overline{v}[a \mapsto t])$

$\Delta[a := *](pc, \overline{v}) = (pc', \overline{v}[a \mapsto t]) \cup (pc', \overline{v}[a \mapsto f])$

$\Delta[if(*)\{pc_{in}\}pc_{out}](pc, \overline{v}) = (pc_{in}, \overline{v}) \cup (pc_{out}, \overline{v})$

$\Delta[assume(\text{expr})](pc, \overline{v}) = \text{if } [\text{expr}](\overline{v}) = true \text{ then } (pc', \overline{v}) \text{ else } \phi$

Background: Boolean Programs - example

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[p1] void main(){
[p2]     int a, b;
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Predicate

$X = a > b$

Background: Boolean Programs - example

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$X = a > b$

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[b3]     X := *;
[b4]     do(*){
[b5]         assume(X);
[b6]         if(*){
[b7]           assume(X);
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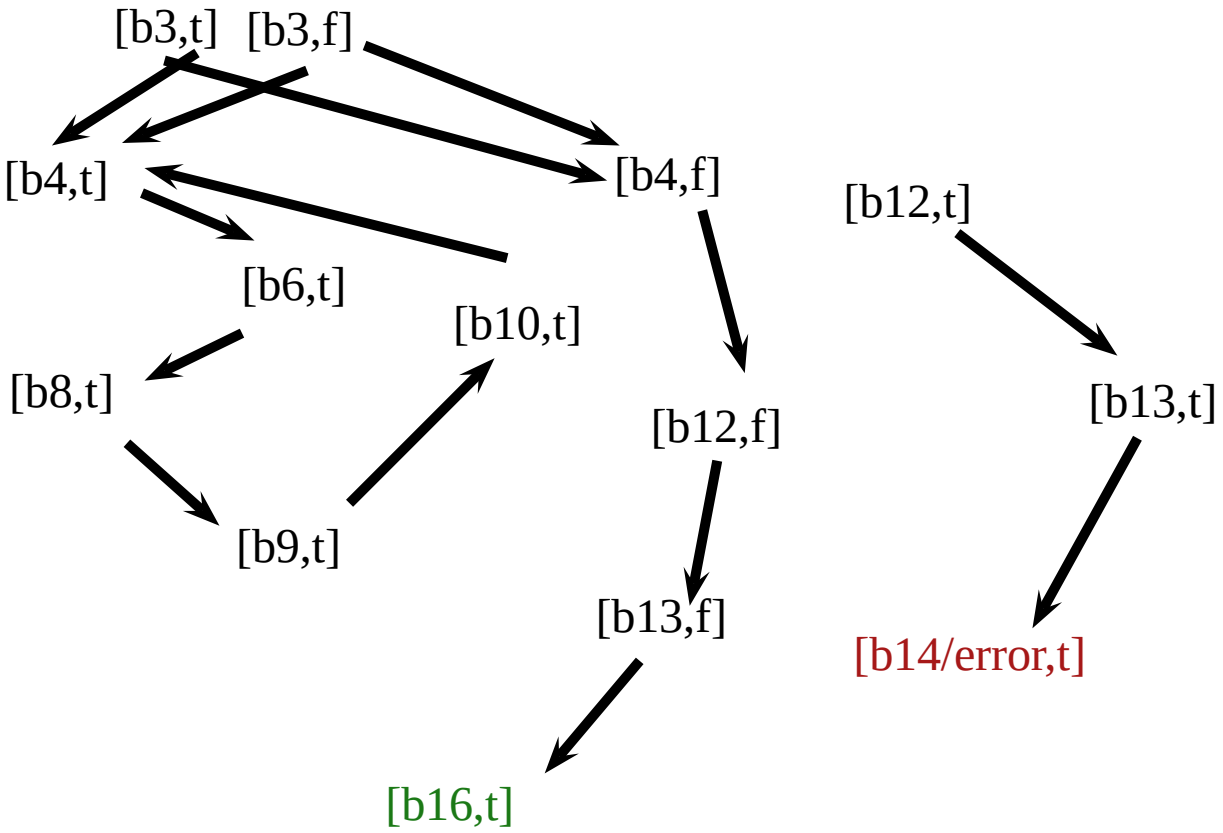
Background: Boolean Programs - model checking

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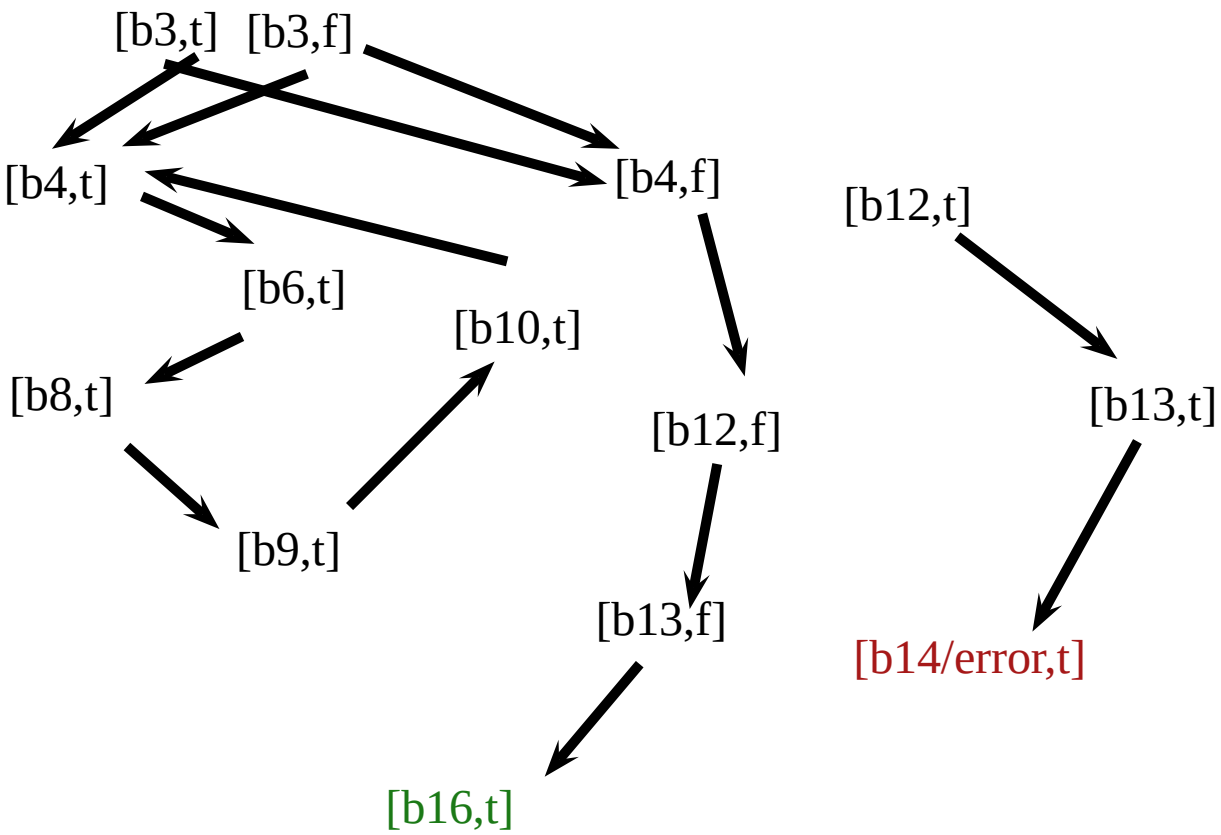

Takeaways

- Use predicate abstraction and develop verification methods for boolean programs

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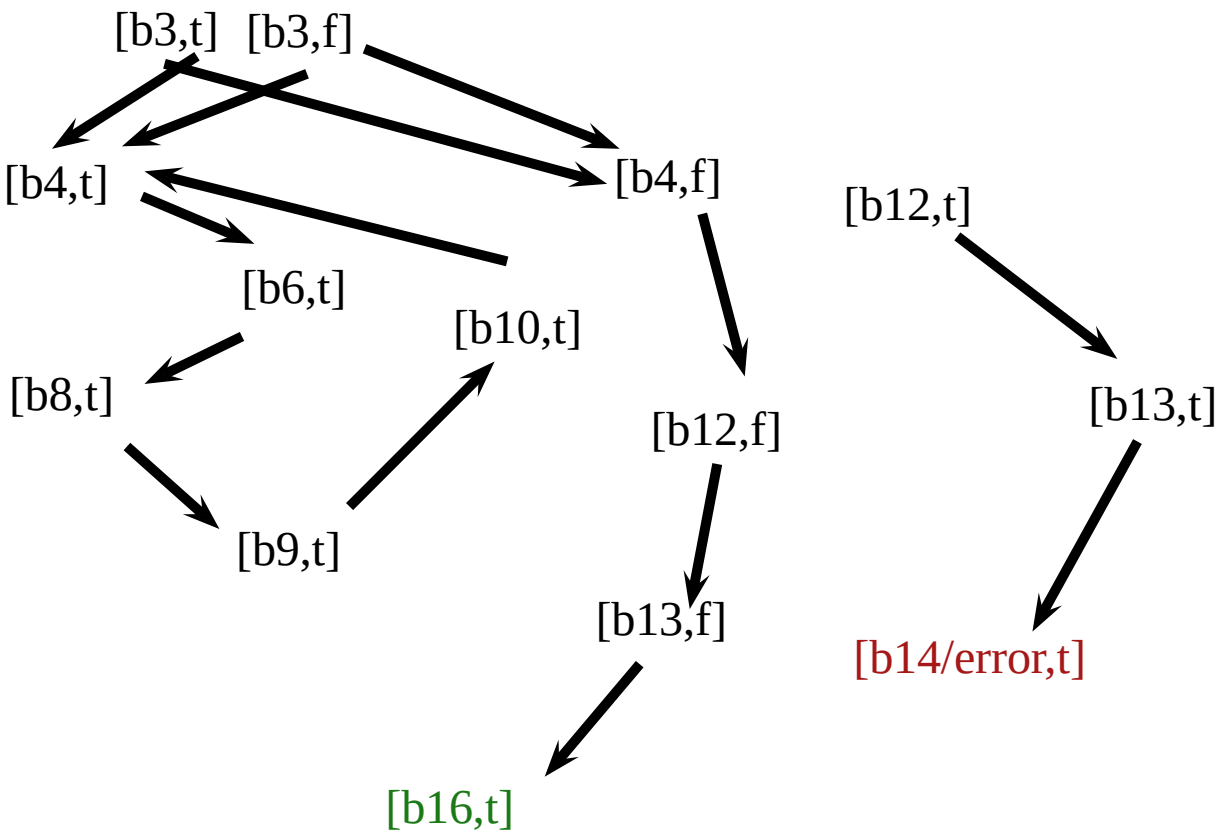

Takeaways

- Use predicate abstraction and develop verification methods for boolean programs
- Restricted to safety properties

At the end of this talk, I will know a way of...

Verifying Programs

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Verifying Concurrent Programs

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Verifying Recursive Concurrent Programs

Extending the model to cover more realistic programs

Recursion

Extending the model to cover more realistic programs

Recursion

- Add function calls to the modelling language, *bool foo(bool . . .)* { . . . }

Extending the model to cover more realistic programs

Recursion

- Add function calls to the modelling language, *bool foo(bool . . .)* { . . . }
- State space no longer finite

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Concurrency

- Multiple threads with designated entry functions

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Concurrency

- Multiple threads with designated entry functions
- Shared memory model - global variables treated as shared variables

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Concurrency

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- Add an atomic construct to the language *atomic* { . . . }

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Concurrency

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Tough luck!

Proving safety properties for *concurrent recursive boolean programs* undecidable[7]

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Verifying Concurrent Programs

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Verifying Concurrent Programs via Induction

Inductive proofs on transition systems

Sound and Incomplete proof technique

Inductive proofs on transition systems

Sound and Incomplete proof technique

Transition System: $T = (S, \Delta \subseteq S \times S, I)$
Property: $P : S \rightarrow \{t, f\}$
$\forall s \in S_{Reach} \quad P(s) \iff$

1. $\forall i \in I \quad P(i) i$
2. $\forall s, s' \in S, P(s) \wedge (s, s') \in \Delta \Rightarrow P(s')$

Inductive proofs on transition systems

Sound and Incomplete proof technique

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1. $\forall i \in I \quad P(i) i$ (base case)
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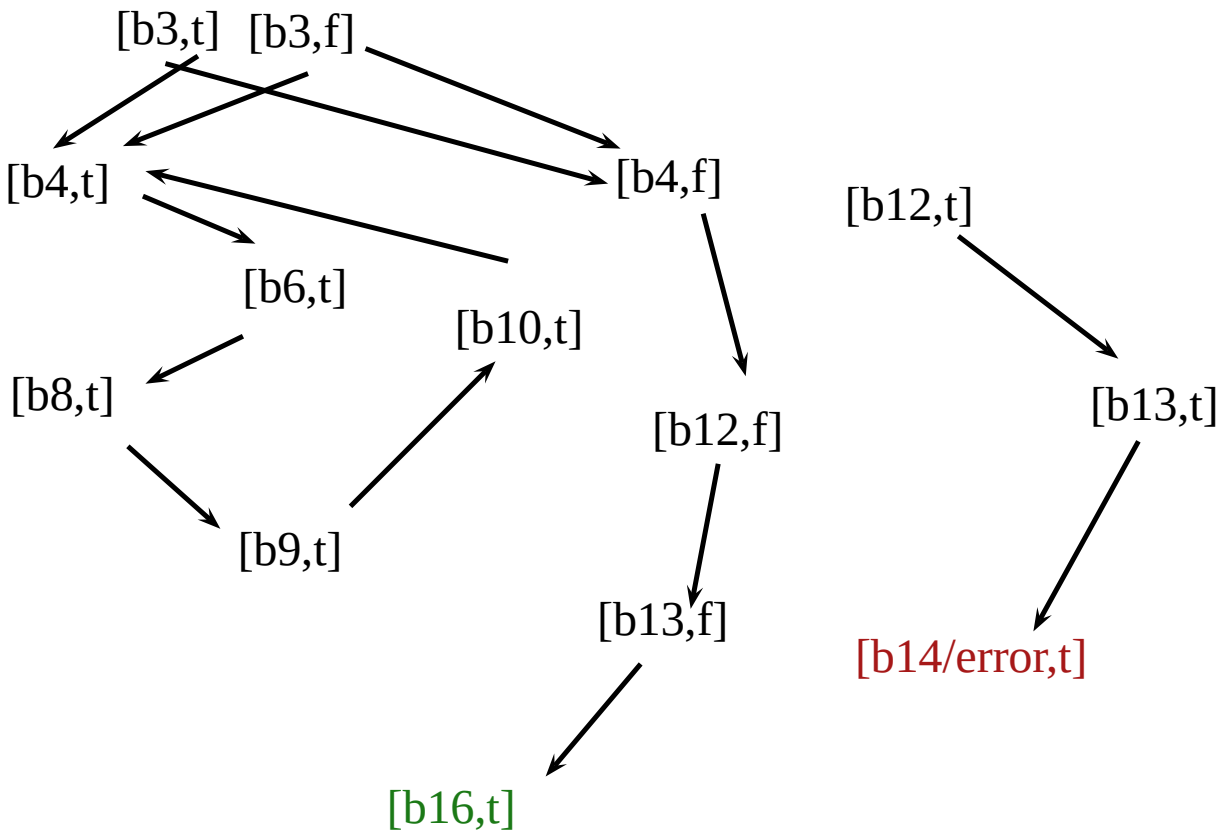
2. $\forall s, s' \in S, P(s) \wedge (s, s') \in \Delta \Rightarrow P(s')$ (induction step)

Inductive proofs on transitions systems - example



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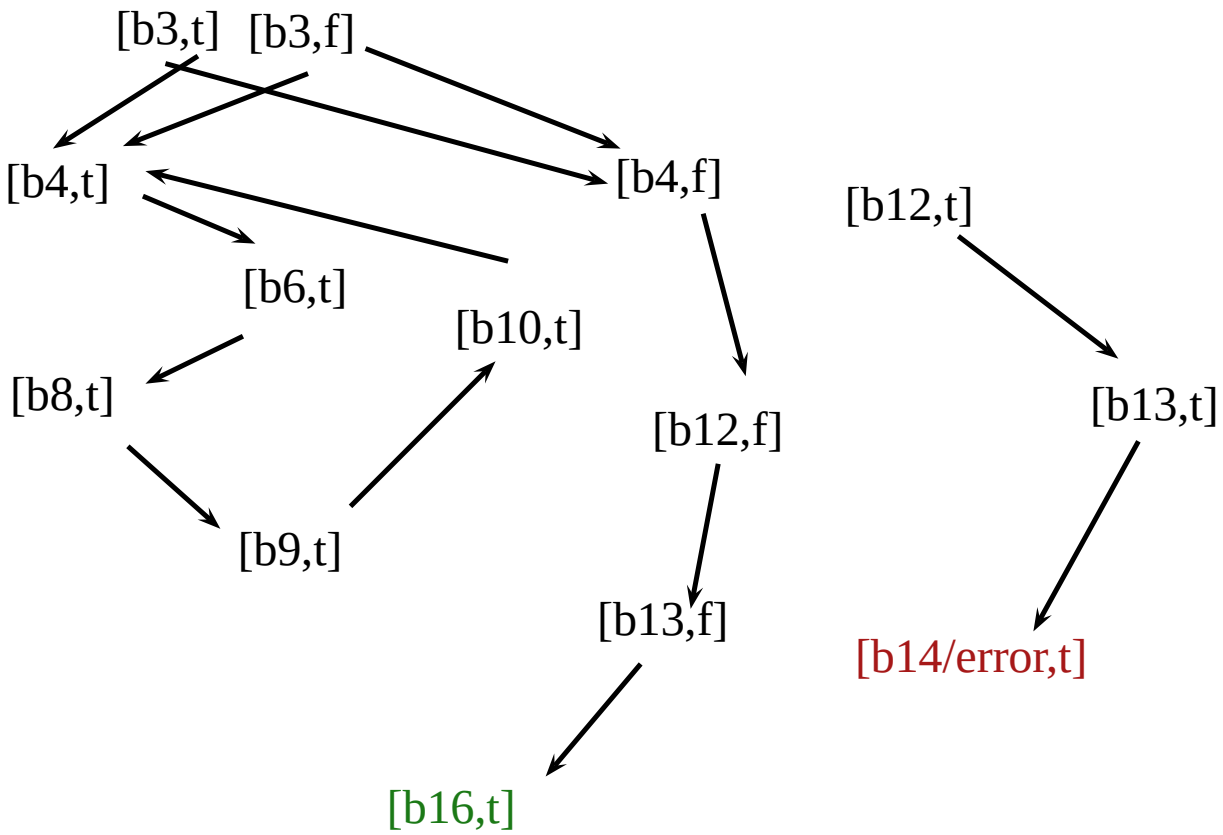
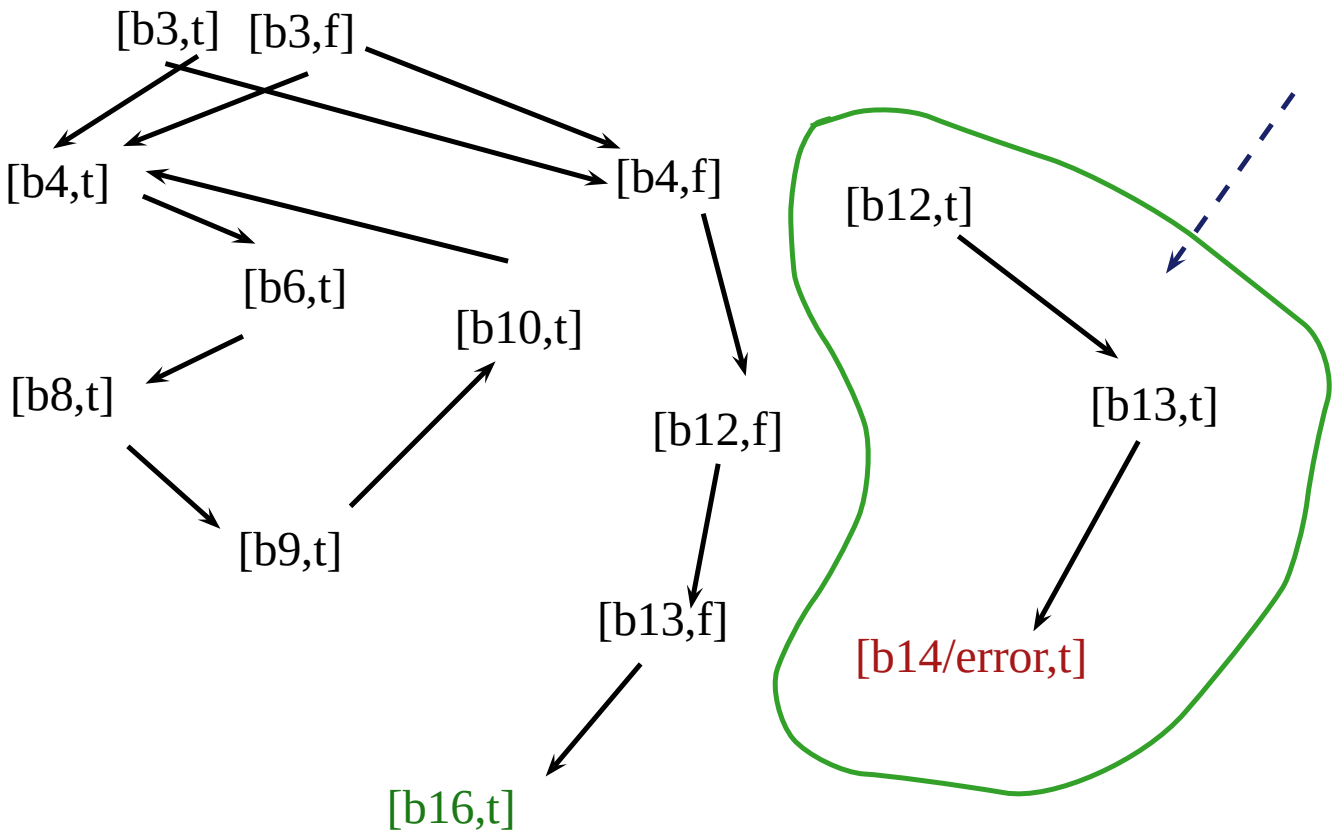


$P = S \backslash \{[b12, t], [b13, t], [error, t]\}$

Inductive proofs on transitions systems - example



$P = S \backslash \{[b12, t], [b13, t], [error, t]\}$

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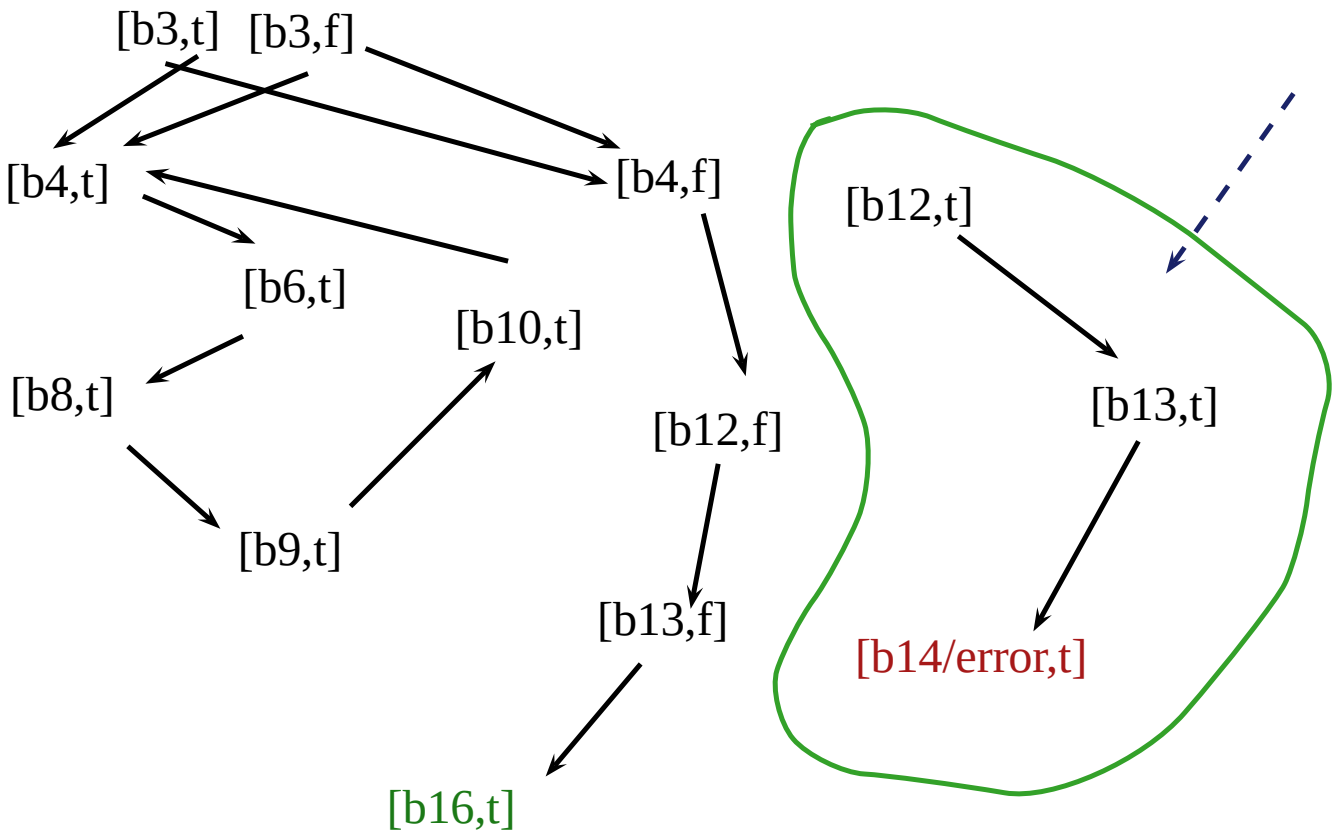

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$P_{error} = S \backslash \{[error, t], [error, f]\}$

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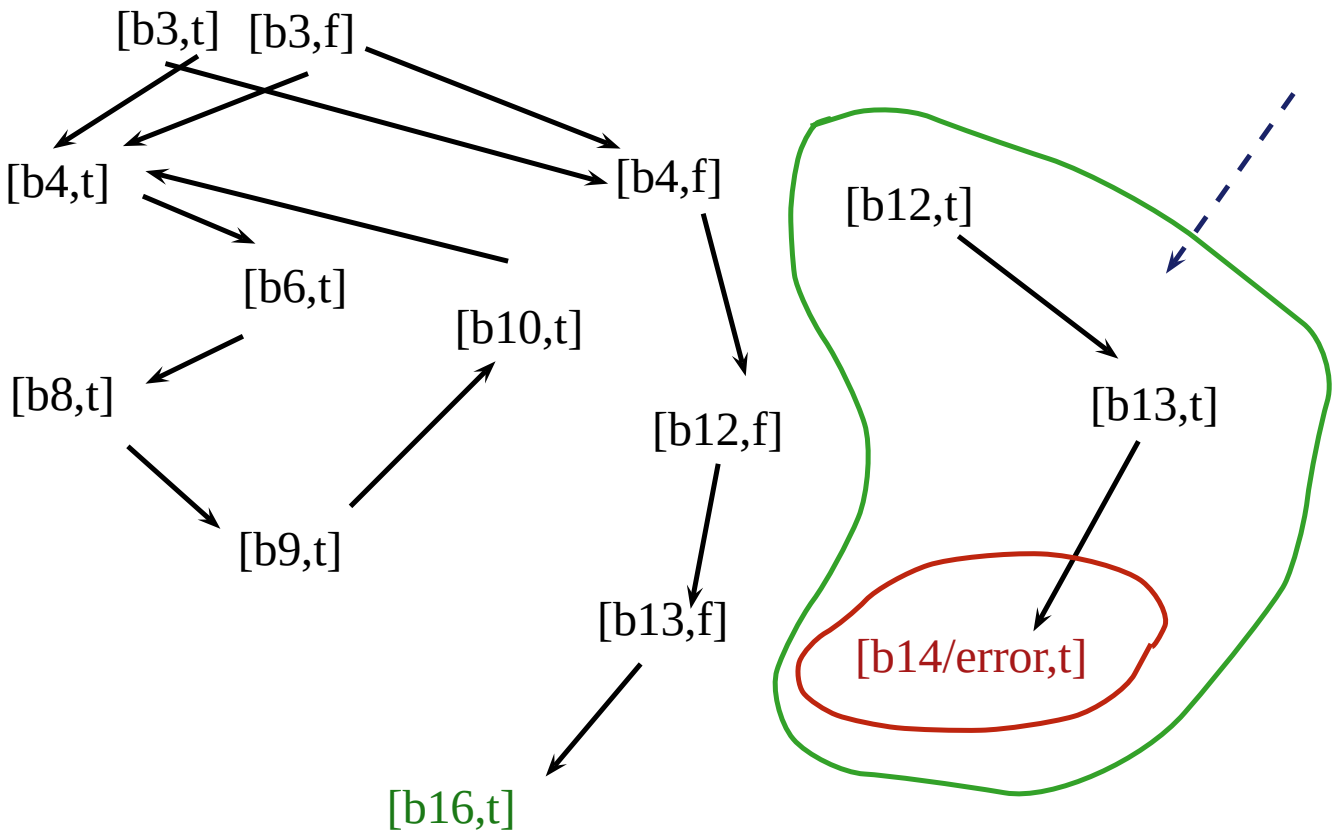

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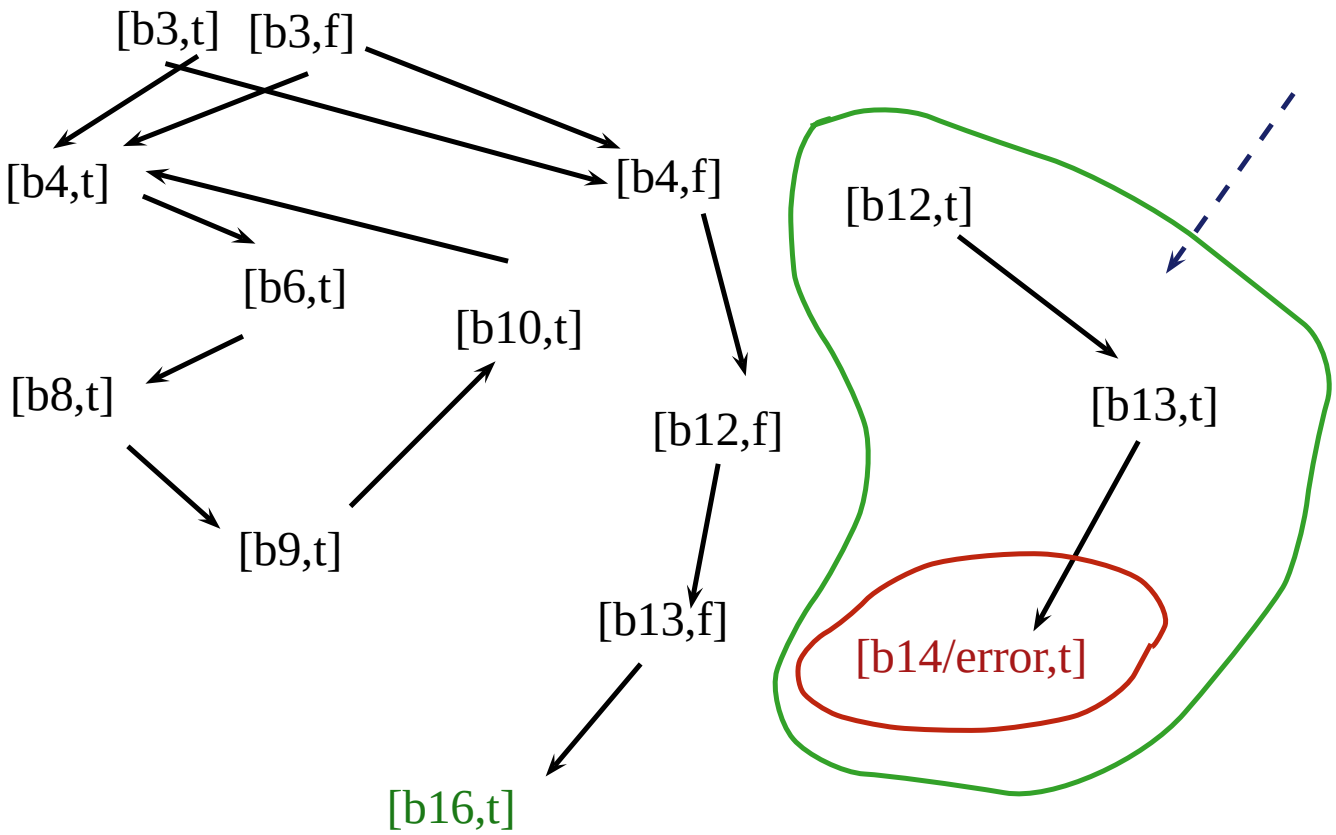

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$P \Rightarrow P_{error}$ (Strengthening the property)

K-induction proofs on transition systems

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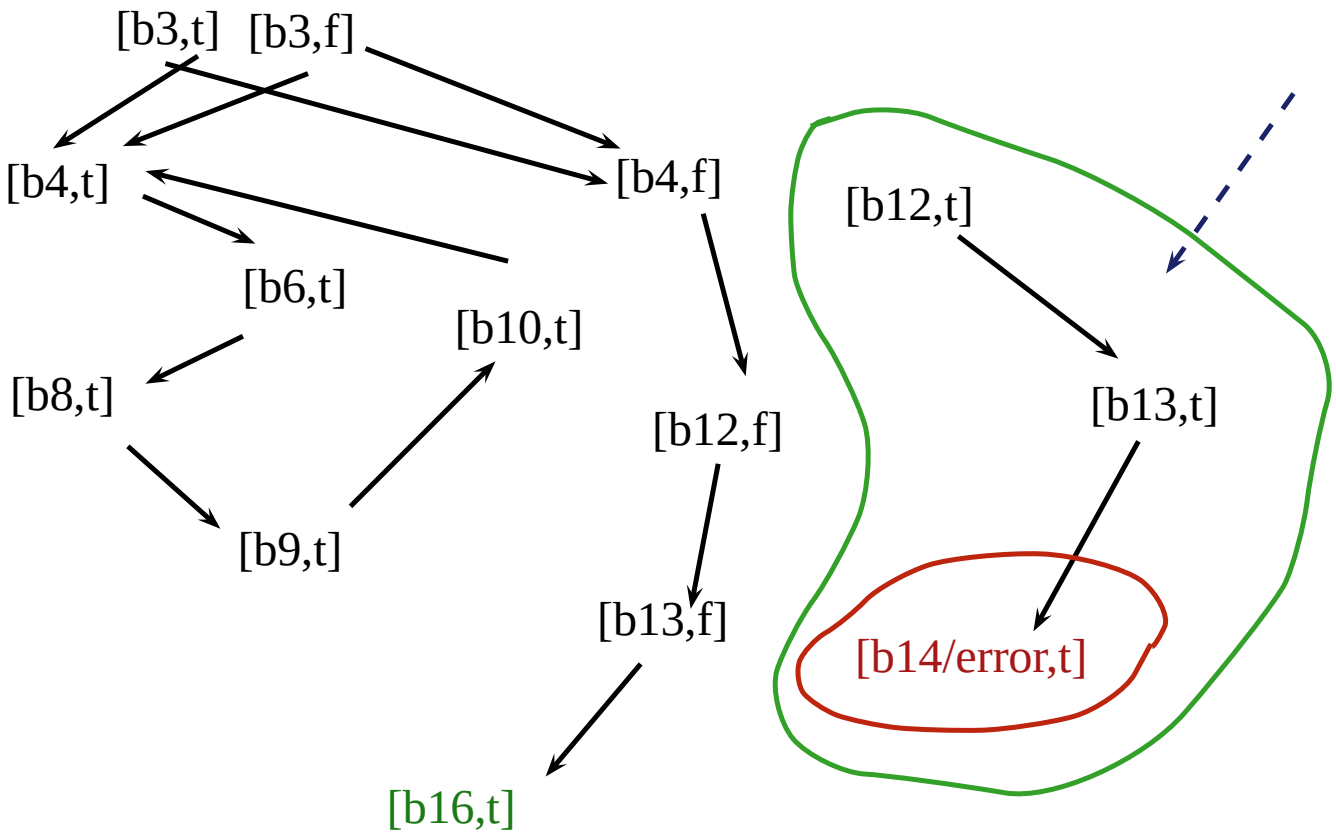
Transition System[2]: $T = (S, \Delta \subseteq S \times S, I)$
Property: $P : S \rightarrow \{t, f\}$
$\forall s \in S_{Reach} \quad P(s) \iff$

1. $\forall i_1, s_2 \ldots s_K \quad i_1 \in I \wedge i, s_2 \ldots s_K \in \rho \Rightarrow P(i_1) \wedge P(s_2) \ldots P(s_K)$
2. $\forall s_1, s_2, s_3 \ldots s_K, s_{K+1} \in \rho \quad P(s_1) \wedge P(s_2) \wedge P(s_3) \ldots P(s_K) \Rightarrow P(s_{K+1})$

K-induction proofs on transition systems - example



$P = S \backslash \{[b12, t], [b13, t], [error, t]\}$

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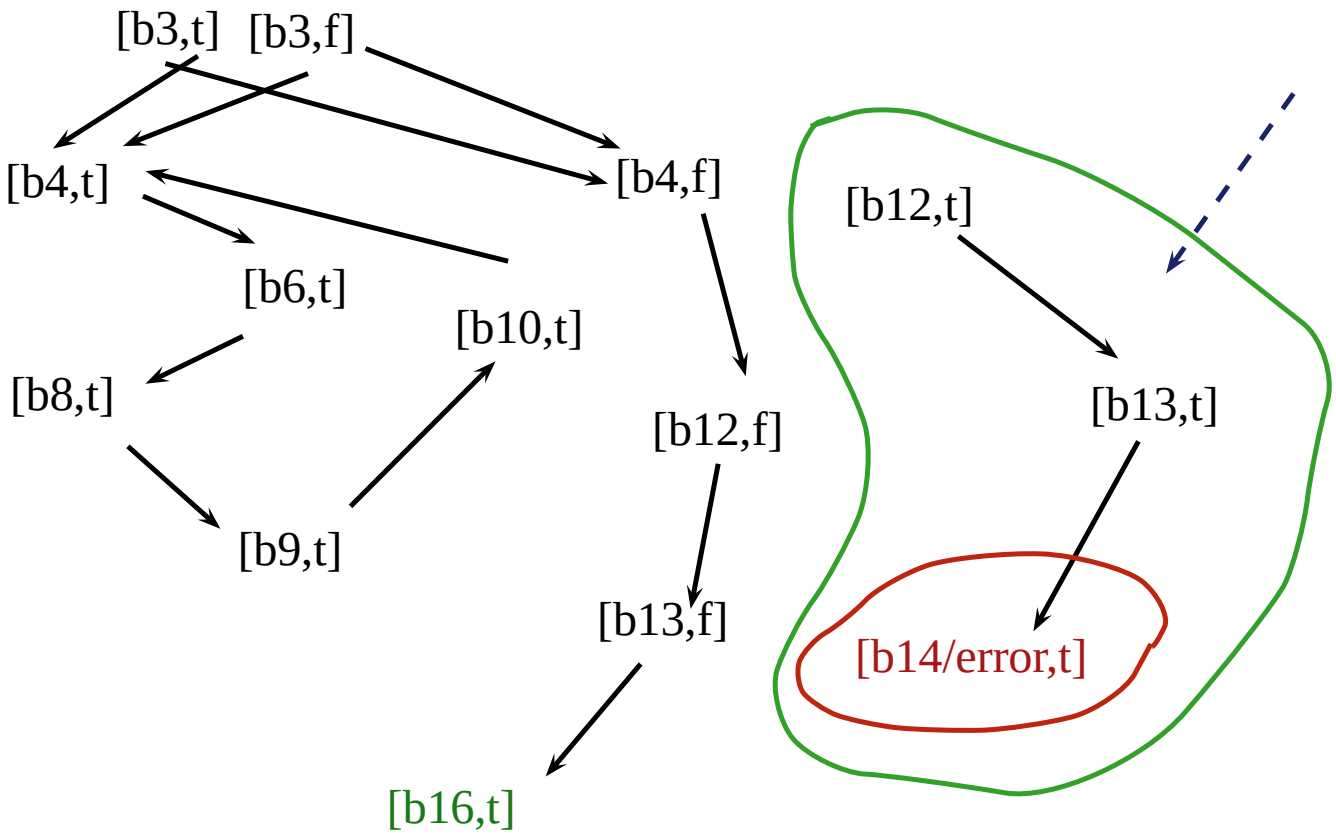
K-induction proofs on transition systems - example



$P = S \backslash \{[b12, t], [b13, t], [error, t]\}$ K = 1

$P_{error} = S \backslash \{[error, t], [error, f]\}$

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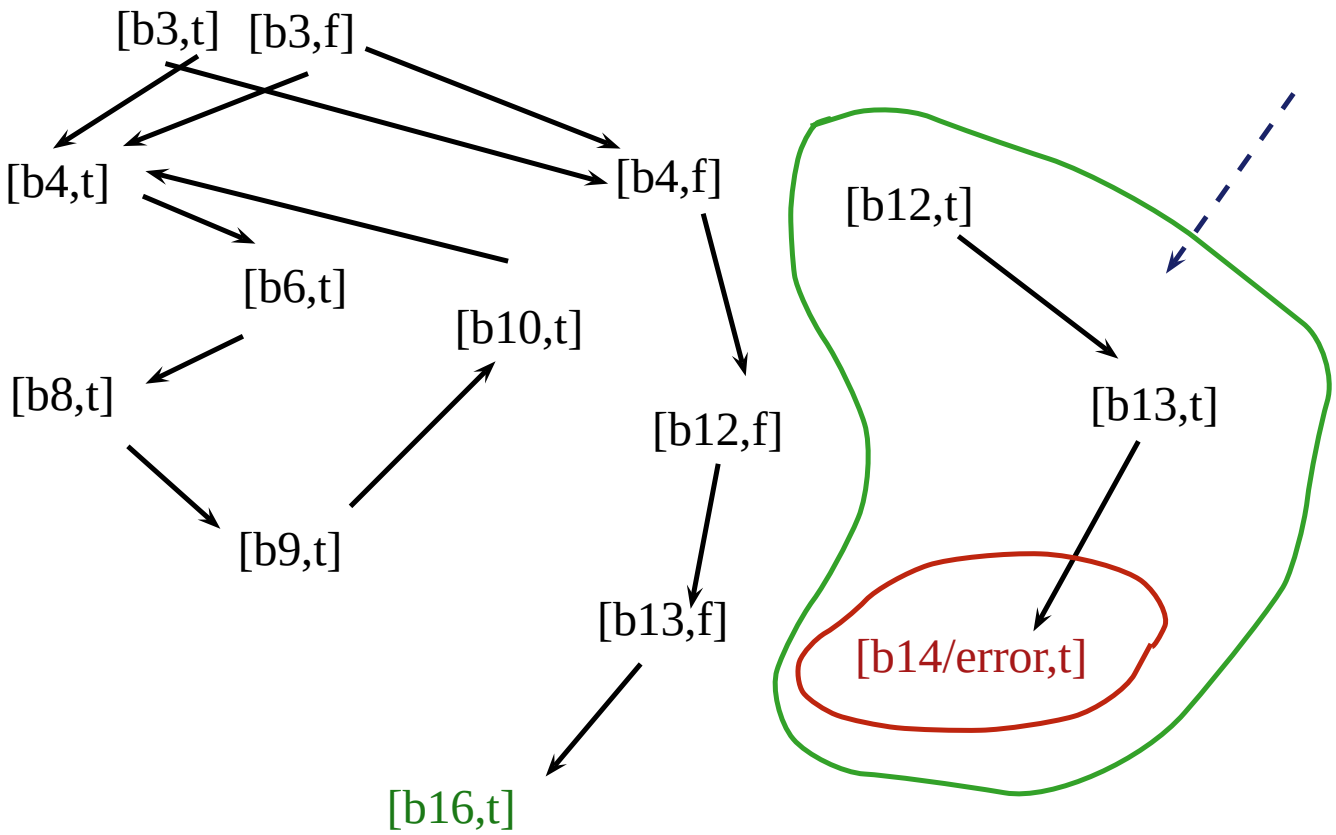
K-induction proofs on transition systems - example



$P = S \backslash \{[b12, t], [b13, t], [error, t]\}$ K = 1

$P_{error} = S \backslash \{[error, t], [error, f]\}$ K = 3

$P \Rightarrow P_{error}$ (Strengthening the property)

Extended example

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[l1] bool x,y;
[l2] lock()
[l3]   atomic
[l4]     do(*)
[l5]       assume(x);
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[p1] p()
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[q1] q()
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States of the transition system

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Activation stack of each of the threads - (local variable values + program counter)

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$$\mathbb{G}lobals \rightarrow \{t, f\} \times \Pi_i^{i \in \mathbb{T}hreads}(pc_i \times \mathbb{L}ocals \rightarrow \{t, f\})^*$$

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$$[x, y, pc_p, pc_q]$$

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Induction case

K=2:

$[f, t, p6, q3] \rightarrow [f, t, p6, q4] \rightarrow [f, t, p6, q3] \rightarrow [f, t, error, q3]$

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$[f, t, p6, q3] \rightarrow [f, t, p6, q4] \rightarrow [f, t, p6, q3] \rightarrow [f, t, error, q3]$
K=3:
$[f, t, p6, q4] \rightarrow [f, t, p6, q3] \ldots$

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K-induction - extended example

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[l1] bool x,y;
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Haven't *really* faced concurrency yet!

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At the end of this talk, I will know a way of...

Verifying Concurrent Programs via Induction

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Verifying Concurrent Programs via Bounded Context-Switching and Induction

Context Bounded Analysis[5, 6]

- under-approximation of the behaviour of the program
- Bug-finding technique

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Context Switch

A conceptual step in which the scheduler chooses the next thread to continue its execution among all the active threads.

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A conceptual step in which the scheduler chooses the next thread to continue its execution among all the active threads.

Idea

Check the required property for all states reachable from the start state within C context switches.

$\cdot \overset{p}{\rightsquigarrow} \cdot \overset{CS}{\rightarrow}$
$\cdot \overset{q}{\rightsquigarrow} \cdot \overset{CS}{\rightarrow}$
$\cdot \overset{p}{\rightsquigarrow} \ldots$

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Not with C=2

$$[f, f, p1, q1] \rightarrow [f, f, p2, q1] \rightarrow [f, f, l1, q1] \overset{CS}{\rightsquigarrow}$$
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Context Bounded Analysis - Continued

- Efficient algorithms exist to check safety properties within bounded contexts
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Black Box

$$\tau(P, C, error) = P_C^{Seq}$$

$$P_C^{Seq}(i) :^? i \overset{P}{\underset{C}{\rightsquigarrow}} error$$

Call this check $CBA[P, C, i]$

CBA + K-induction = Verification?

Technique

K-induction while inducting over the number of context switches instead of program steps

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Proof of correctness??

CBA + K-induction = Verification?

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Proof of correctness??

Base Case

$$\forall i_1, s_2 \ldots s_K \quad i_1 \in I \wedge i, s_2 \ldots s_K \in \rho \Rightarrow P(i_1) \wedge P(s_2) \ldots P(s_K)$$

Context Bounded Analysis with C = K, $CBA[P, K, i]$

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Context Bounded Analysis with C = K, $CBA[P, K, i]$

Induction Step

$$\forall s_1, s_2, s_3 \ldots s_K, s_{K+1} \in \rho \quad P(s_1) \wedge P(s_2) \wedge P(s_3) \ldots P(s_K) \Rightarrow P(s_{K+1})$$

- Generate an arbitrary program state, s.
- Modify $CBA'[P, K, s]$: Check if *error* is reached within K context switches. If not, verify that *error* isn't reached within the next context.

CBA + K-induction = Verification? - example

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CBA + induction step : Challenge I

Reduction to program steps

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K-induction

Transition System: $T = (S, \Delta \subseteq S \times S, I)$
Property: $P : S \rightarrow \{t, f\}$, $P(s)$ true iff the required property holds for state s.
$\forall s \in S_{Reach} \quad P(s) \iff$

1. $\forall i_1, s_2 \ldots s_K \quad i_1 \in I \wedge i, s_2 \ldots s_K \in \rho \Rightarrow P(i_1) \wedge P(s_2) \ldots P(s_K)$
2. $\forall s_1, s_2, s_3 \ldots s_K, s_{K+1} \in \rho \quad P(s_1) \wedge P(s_2) \wedge P(s_3) \ldots P(s_K) \Rightarrow P(s_{K+1})$

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K-induction with amplification - example

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[l1] bool x,y;
[l2] lock()
[l3]   atomic
[l4]     do(*)
[l5]       assume(x);
[l6]     assume(!x);
[l7]     x = true;
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K=2

$$[f, f, p4, q3] \overset{1}{\rightsquigarrow} [f, f, p4, q3] \overset{CS}{\rightarrow} [f, f, p4, q3] \overset{2}{\rightsquigarrow}$$
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```
[p1] p()
[p2]   lock()
[p3]   y = false
[p4]   do(*)
[p5]     assume(!y);
[p6]   assume(y);
[p7]   error: skip;
```

```
[q1] q()
[q2]   lock();
[q3]   do(*)
[q4]     skip;
[q5]   y = true;
```

K-induction with amplification - example

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CBA + induction : Challenge II

$$\forall s \in S \quad ! \; CBA[P, K, s] \vee CBA[P, K+1, s]$$

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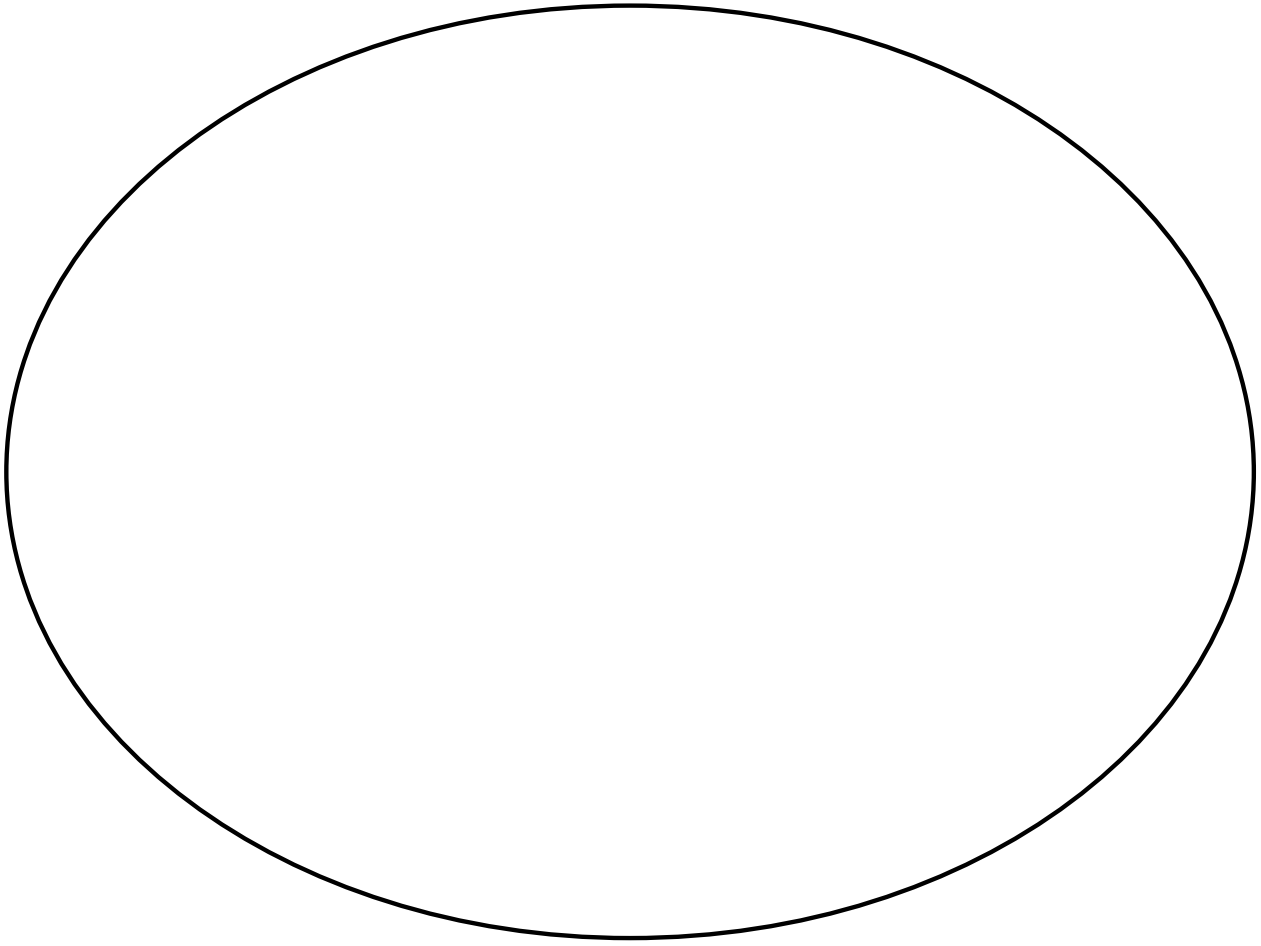
Express in terms of backward reachability and set difference

$$CBA^{rev}[P, K+1, error] - CBA^{rev}[P, K, error] = \emptyset$$

K-induction explained



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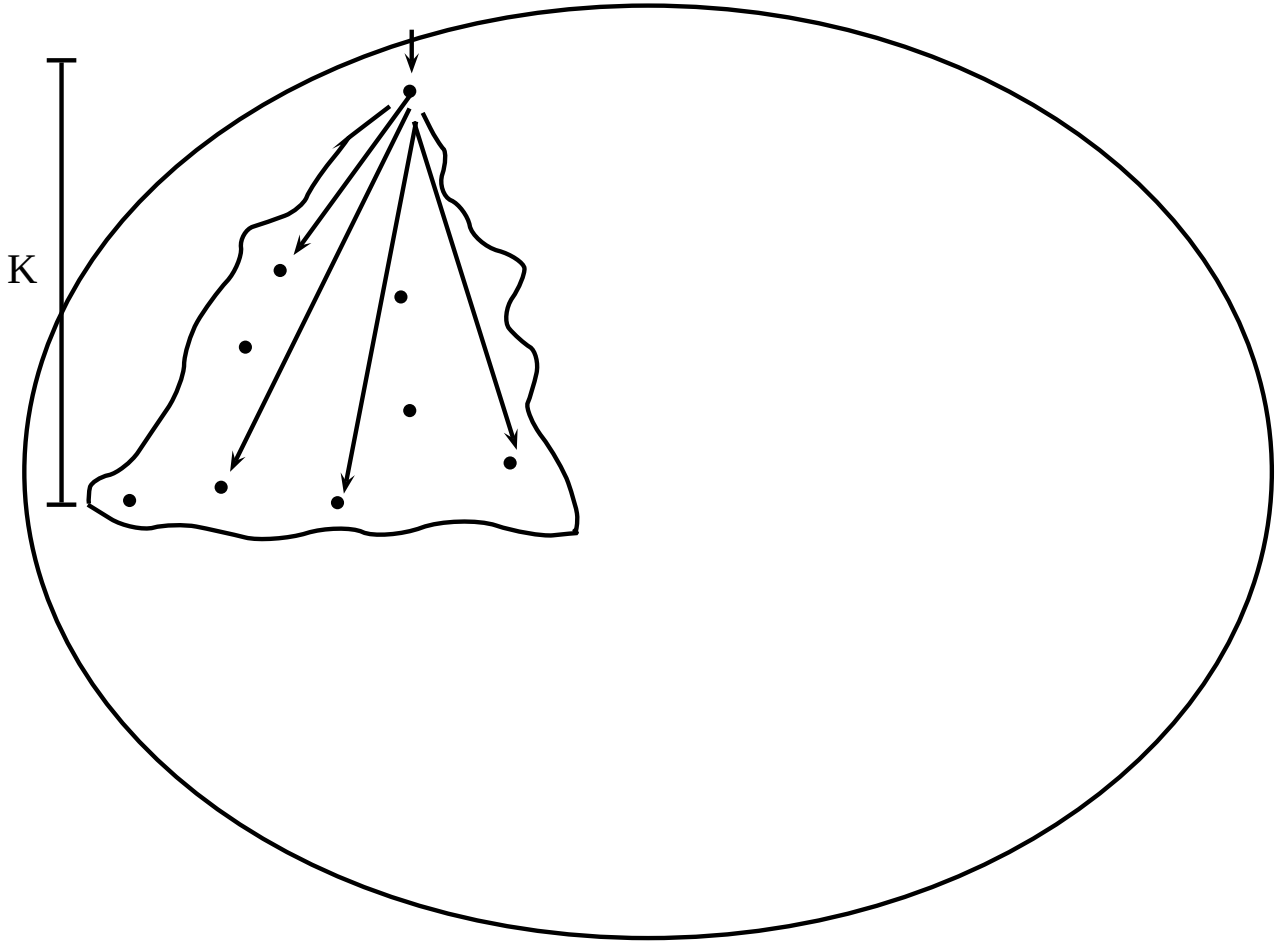

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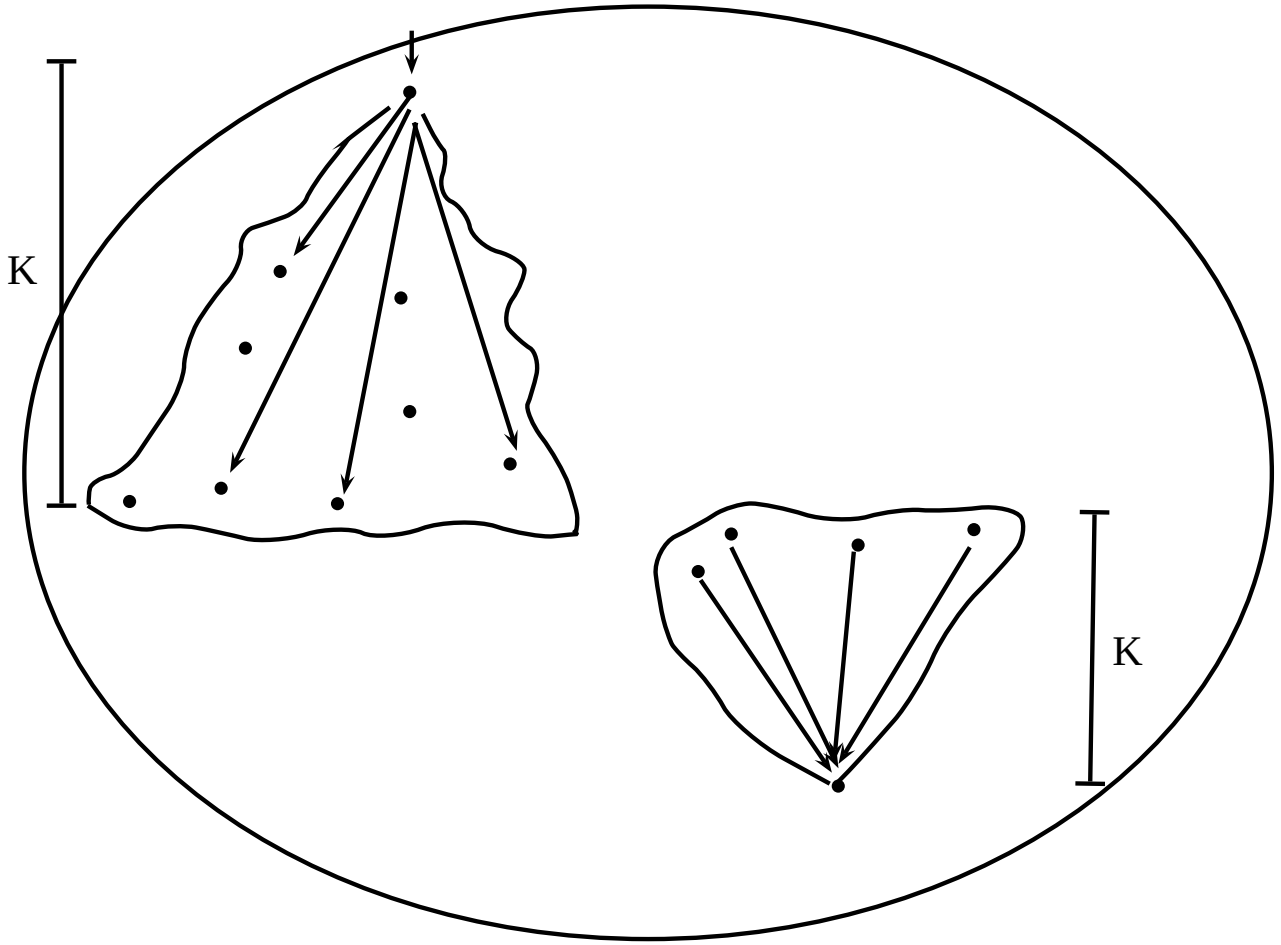

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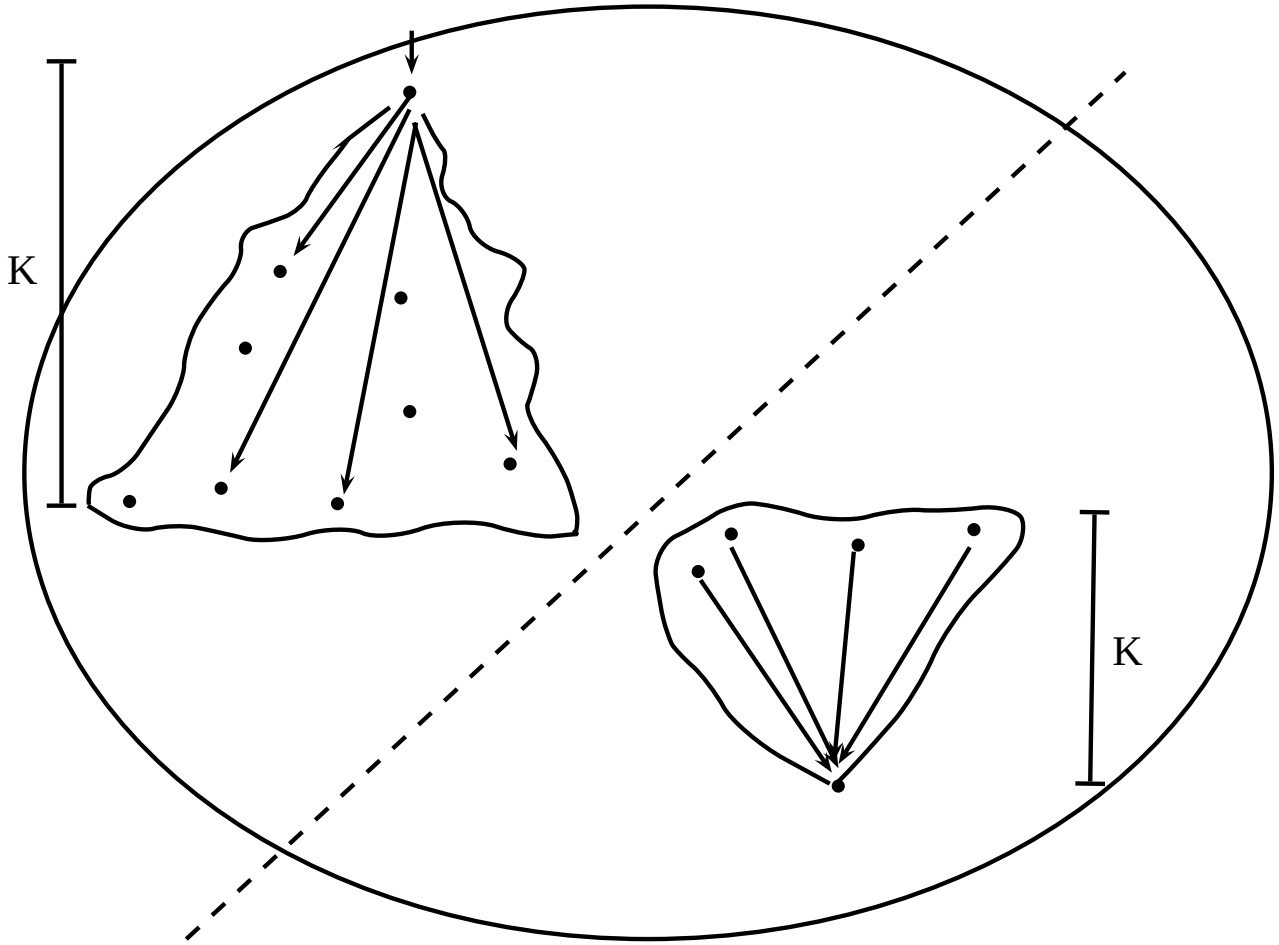

The algorithm

Semi-decision procedure for concurrent recursive boolean programs:

K-induction-amplification

1: **for** $K = 1; true; K = K + 1$ **do**
2: **if** CBA^K(P) returns reachable **then**
3: // Error is reachable: bug found
4: EXIT
5: // Compute the set of states that reach error in K contexts, M^K
6: $M^K = prune(CBA^{Rev}[P, K, error])$
7: // Compute the set of states that reach error in $K + 1$ contextx, M$^K + 1$
8: $M^{K+1} = prune(CBA^{Rev}[P, K + 1, error])$
9: **if** $M^{K+1} - M^K = \emptyset$ **then**
10: // Both obligations passed: program proved correct
11: EXIT

Gotcha!

Reality Check - Incompleteness

For a give program P, a K that works may not exist

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Reality Check - Hardness

$$\forall s \in S \quad CBA[P, K, s] \vee! \; CBA[P, K+1, s]$$

- Quantifier alternation

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- Quantifier alternation

$$CBA^{rev}[P, K+1, error] - CBA^{rev}[P, K, error] = \emptyset$$

- Set subtraction
- Finite State Machine Complementation

Experiments

Experiments on an implementation using MOPED[3]. Models from a Bluetooth driver model and some mutual exclusion protocols[4].

Program	#threads	#globals	#locals	K	Time(s)
blue2	2	5	6	3	32.44
dekkers	2	4	1	3	3.74
lock_unlock_K	2	2	1	4	0.6
lock_unlock_3	3	4	3	3	44.02
petersons	2	4	1	3	0.3
petersons_loop	2	4	1	3	6.05

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Need symbolic methods for the set (automata) subtraction in the last step to make the technique efficient.

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- There exist bug founding approaches that only explore bounded executions of the program.
- Induction built over these Bounded Context methods lets us directly address the complexity arising from interleaving of threads.
- A stronger version of K-induction is needed to effectively induct over these path blocks. Proving the induction step in this modified induction is harder.

Further Reading I


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Thank you!

Thank you for listening...

Questions?