Maximizing System Lifetime in Wireless Sensor Networks

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ABSTRACT

Maximizing system lifetime in battery-powered wireless sensor networks with power aware topology control protocols and routing protocols has received intensive research. In the past, this problem has been mostly studied from the indirect perspective of energy conservation. Although this leads to solutions that help extend network lifetime, energy conservation is not the same problem as network lifetime maximization. Some researchers have formally studied network lifetime maximization problems, based on the assumption that energy is only consumed by packet transmission. However, it is well known that in many cases energy is significantly consumed during overhearing and idle periods. In this paper, we try to present formal analysis of a variety of network lifetime maximization problems in different energy consumption models. In particular, we identify different energy consumption models, define a variety of fundamental network lifetime maximization problems in individual energy consumption models, and formally analyze their complexity. Polynomial time algorithms are presented for tractable problems, and NP-hardness proofs are presented for intractable problems.

1. INTRODUCTION

Multi-hop, ad hoc, wireless sensor networks (WSNs) are considered a promising technology to change our physical environment and hence our life in this environment. WSNs are typically deployed using battery-powered stationary sensor nodes equipped with sensing, computing and wireless communicating modules. In a broad range of potential applications, inexpensive sensors can be embedded into buildings or scattered into spaces to collect, process, store and send out relevant information for various civilian or military purposes. When a data sink (e.g. a base station) is out of reach of a data source sensor node, they can rely on intermediate sensor nodes to relay data packets.

A salient feature of battery-powered WSN is its extremely constrained source of energy supplied by batteries coming with sensor nodes, because sensor nodes are typically small and thus use tiny batteries. In many scenarios, it seems infeasible to replace or recharge batteries of sensor nodes. For example, NASA plans to deploy sensor networks in areas of interest on Mars [1]. Meanwhile, in WSNs, wireless communication is considered much more energy consuming than sensing and computing [2]. All these factors make it essential to develop efficient routing and topology control protocols to maintain requested network properties (e.g. connectivity) for as long a network lifetime as possible.

In the literature, there have been two different approaches to maximizing network lifetime. One indirect approach aims to minimize energy consumption, while the other approach directly aims to maximize network lifetime. Although the indirect approach can help extend network lifetime, it does not address precisely the problem of maximizing network lifetime. Therefore, some researchers have aimed to directly maximize network lifetime.

- Chang and Tassiulas [3, 4] considered the problem of maximizing the time to the first node failure for a unicast session, where each data source generates data for delivery at a fixed rate.
- In [5, 6], optimal solutions are presented for maximizing the time to the first node failure for a static broadcast tree.
- In the more general multicast paradigm, Das *et al.* [7] presented an optimal solution for maximizing the time to the first node failure for a static multicast tree. Floréen *et al.* [8] investigated the problem of maximizing the lifetime of a multicast session over a network of energy constrained nodes, where the multicast tree can be dynamically adjusted to utilize any node with available energy.

While these efforts are based on the energy consumption model where energy is consumed only when transmitting packets, it is well known that wireless transceivers consume a significant amount of energy during overhearing and idle periods as well [9, 10, 11].

The contribution of this paper is the formal analysis of a number of network lifetime maximization problems, under different energy consumption models. In particular, we identify representative energy consumption models, define a variety of fundamental network lifetime maximization problems under these models, and formally analyze their complexity. Polynomial time algorithms are presented for tractable problems, and NP-hardness proofs are presented for intractable problems. Despite significant research in this area, we do not know of any optimal solutions to these fundamental problems identified in this paper, and the complexity of these problems remain unknown. To the best of our knowledge, this paper is the first to present such a formal analysis.

The rest of the paper is organized as follows. In Section 2, we identify representative energy consumption models and define network lifetime in individual energy consumption models. In Section 3, various network lifetime maximization problems are defined under individual energy consumption models. The complexity of these problems is formally analyzed. Finally, we conclude the paper in Section 4.

2. MODELS AND DEFINITIONS

In most of the past research efforts aiming to extend network lifetime, energy consumption is completely attributed to packet transmission. Wireless transceivers are assumed to consume power only when transmitting packets, and energy is thus consumed on a per packet basis. This model is simple and neat. In this paper, we also include this energy consumption model in our analysis. For simplicity, we refer to this model as the *packet based model*.

Despite the prevalence of the packet based model, it has been well known that energy is also significantly consumed during overhearing and idle periods [9, 10, 11]. In particular, wireless transceivers are powered to receive every incoming packet and decode to decide if the packet should be accepted, forwarded, or discarded. Although many packets turn out to be simply discarded, their reception has already consumed a significant amount of energy. In addition, wireless transceivers also consume energy during idle periods, because they have to be powered to detect if there are packets being transmitted at all. Researchers [9, 11] have shown that in some cases, energy consumption during overhearing and idle periods can be comparable to energy consumption due to transmitting/receiving packets. In many applications, WSNs are presumed to be densely deployed, and this has two implications. On one hand, pair-wise distance between sensor nodes is small, and thus packet transmission between sensor nodes consumes less energy. On the other hand, each sensor node covers more sensor nodes in its transmission range, and thus more energy will be consumed due to overhearing.

In the extreme case where wireless transceivers stay idle and no communication happens at all, energy is completely consumed in the idle state, on a per time unit basis. We hereby refer to this energy consumption model as the *time based model*. In a broad range of applications where sensor nodes only need sporadic (and possibly asynchronous) communication, power consumption is dominated by idle time and transceivers consume almost the same amount of energy. For example, sensor nodes may be configured to send back environment information once per hour. The time based model fits well into such scenarios. In such scenarios, to effectively conserve energy and extend network lifetime, it is no longer adequate to simply optimize transmission power as has been done by most researchers. Instead, *we need to turn off as many transceivers as much as possible*. When a sensor node's transceiver is turned off, it is considered *sleeping*. In the sleeping state, energy consumption during overhearing and idle periods is avoided. Communication is handled by a *backbone* composed of nodes that do not sleep, connecting every pair of nodes in the network. A sleeping node may occasionally wake up to send out packets over the backbone. That part of the energy consumption can be addressed by the packet based model.

In cases where communication is relatively frequent, energy consumption can be divided into two parts. On one hand, (homogeneous) sensor nodes consume as much power as each other on a per time unit basis, due to overhearing and staying idle. On the other hand, they may consume significantly different amounts of energy on a per packet basis, due to packet transmission/reception. We refer to this case as the *mixed model*.

Various definitions of network lifetime have been proposed for different scenarios. In [12], Blough and Santi present a discussion on defining network lifetime, and outline the principle that *network lifetime should refer to the capability of the network to serve its design purpose*. In this paper, we define network lifetime for a number of network lifetime maximization problems according to this general principle. For problems in the packet based model, we define network lifetime as the number of packets (to be perfectly accurate, the number of bits) that can be delivered by the network. This definition applies to all routing paradigms including unicast, multicast and broadcast.

In the time based model, the design purpose is to maintain an always active communication backbone connecting every pair of nodes in the network. Accordingly, we define network lifetime to be the time until no such backbone can be formed. This definition is also motivated by the following features of WSNs.

- On one hand, sensor nodes are presumed to be densely deployed and sensor networks are thus highly redundant. Even if some sensor nodes fail due to battery depletion, the whole sensor network is most likely still in good order to serve its purpose.
- On the other hand, wireless communication is considered the primary cause of energy consumption in WSNs, especially in many applications where sensor nodes only need to conduct modest data collecting and processing. Even if this assumption is not true in some cases, we may reserve a certain amount of energy for sensing, processing and sending data, and reserve the rest of available energy for staying active in the backbone and relaying packets. Thus, even if some sensor node

has run out of its energy for relaying packets, it can still collect, process and send out data as usual. As long as there exists such a backbone, the functionality of the whole sensor network remains intact to serve its design purpose.

3. ANALYSIS

In this section, we formally analyze a variety of network lifetime maximization problems in the time based model as well as the intensively researched packet based model. Definitions and complexity analysis of problems specific to individual models are presented. Note that the time based model and the packet based model are both special cases of the mixed model, thus their hardness results trivially apply to the mixed model.

In our network model, stationary sensor nodes are assumed to be equipped with an omnidirectional antenna. A wireless sensor network is denoted by a weighted directed graph $G = (V, A)$, where V is the set of sensor nodes and A is the set of directed links. Each node is labeled with a unique ID $i \in [1..|V|]$ and has a maximum transmission power of P^i_{max}. Let P_{ij} denote the minimum transmission power required to maintain a reasonably good quality link from node i to node j. G contains link (i, j) (i.e., the link from node i to node j) if and only if $P_{ij} \leq P^i_{max}$. Initially, each sensor node $i \in V$ has an energy of p_i. Time is divided into discrete time slots, denoted by $t \geq 1$, $t \in Z^+$.

3.1 The time based model

In this section, we investigate the problem of maximizing network lifetime in the time based model and prove it to be NP-hard. When proving the NP-hardness of intractable problems identified in this section, we actually prove stronger results that the problems remain NP-hard even if they are restricted to the special case where all sensor nodes have the same maximum transmission power P_{max} and $P_{ij} = P_{ji}$ for each node pair (i, j). In this case, a stationary wireless sensor network can be modelled as a weighted undirected graph $G = (V, E)$, where E is the set of undirected edges and G contains edge (i, j) if and only if $P_{ij} \leq P_{max}$.

To maintain network connectivity, what we need is a backbone, which is represented as a connected dominating set (CDS) [13]. In an undirected graph $G(V, E)$, a dominating set is defined as a subset $S \subseteq V$ of nodes such that each node $i \in V$ is either in S or adjacent to some node $v \in S$. A connected dominating set S is a dominating set such that the subgraph $G' = (S \subseteq V, E' \subseteq E)$ induced by S is connected. Here, we shall prove an even stronger result that the problem of maximizing network lifetime while preserving connectivity in undirected graphs remains NP-hard even if we restrict it to the special case where during each time step, each node $i \in V$ consumes p_i energy. In this case, each node has a *battery life* of one (time slot) and can be used in exactly one CDS. The problem of maximizing network lifetime thus becomes the *connected domatic number (CDN)* problem, which is defined as follows.

> Connected Domatic Number (CDN)
>
> **INSTANCE** Graph $G = (V, E)$. Positive integer K.
>
> **QUESTION** Does G contain at least K disjoint CDSs?

Theorem 1. *Connected domatic number is NP-hard.*

Proof. We prove the NP-hardness of CDN by reducing from the *3-dimensional matching (3DM)* problem, which is known to be NP-hard [14] and formally defined as follows.

> 3-Dimensional Matching (3DM)
>
> **INSTANCE** Set $M = \{m_1, m_2, \ldots, m_m\} \subseteq W \times X \times Y$, where $W = \{w_1, w_2, \ldots, w_q\}$, $X = \{x_1, x_2, \ldots, x_q\}$, and $Y = \{y_1, y_2, \ldots, y_q\}$ are disjoint sets having the same number q of elements and $|M| = m$.
>
> **QUESTION** Does M contain a matching, i.e., a subset $M' = \{m'_1, m'_2, \ldots, m'_q\} \subseteq M$ such that $|M'| = q$ and no two elements of M' agree in any coordinate?

Given an instance of 3DM, we construct a graph $G = (V, E)$ as shown in Fig. 2, where nodes are distributed into four layers and edges exist only between nodes in the same layer or adjacent layers. The graph in Fig. 2 is constructed from the following instance of 3DM.

> $W = \{w_1, w_2\}$, $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ and $M = \{m_1, m_2, m_3, m_4\}$, where $m_1 = (w_1, x_2, y_1)$, $m_2 = (w_1, x_1, y_1)$, $m_3 = (w_2, x_2, y_2)$, and $m_4 = (w_2, x_1, y_2)$.

In the top layer, there are 3 disjoint groups of *set nodes*, $\mathscr{W} = \{W_1, W_2, \ldots, W_{m-q}\}$, $\mathscr{X} = \{X_1, X_2, \ldots, X_{m-q}\}$, and $\mathscr{Y} = \{Y_1, Y_2, \ldots, Y_{m-q}\}$. In the second layer, there are 3 corresponding disjoint groups of *element nodes*, $\mathbb{W} = \{w_1, w_2, \ldots, w_q\}$, $\mathbb{X} = \{x_1, x_2, \ldots, x_q\}$, and $\mathbb{Y} = \{y_1, y_2, \ldots, y_q\}$. $\mathbb{W}$, $\mathbb{X}$, and $\mathbb{Y}$ represent W, X, and Y in the 3DM instance, respectively. $\mathscr{W} \cup \mathbb{W}$ forms a clique of size m, and so do $\mathscr{X} \cup \mathbb{X}$ and $\mathscr{Y} \cup \mathbb{Y}$. Besides the element nodes, the second layer also contains a group $\mathbb{B} = \{b_1, b_2, \ldots, b_{m-q}\}$ of *bridge nodes*. Each bridge node is adjacent to every set node in the top layer. In the third layer, there is a group $\mathbb{M} = \{m_1, m_2, \ldots, m_m\}$ of *triplet nodes* representing the elements in M. Each bridge node in the second layer is adjacent to every triplet node as well. Each triplet node is also adjacent to the 3 element nodes that occur in the element in M that it represents. In the bottom layer, there is a group $M' = \{m'_1, m'_2, \ldots, m'_q\}$ of *matching nodes* representing a potential 3-dimensional matching M'. Each matching node is adjacent to every triplet node in the third layer. The transformation is clearly polynomial, and we prove

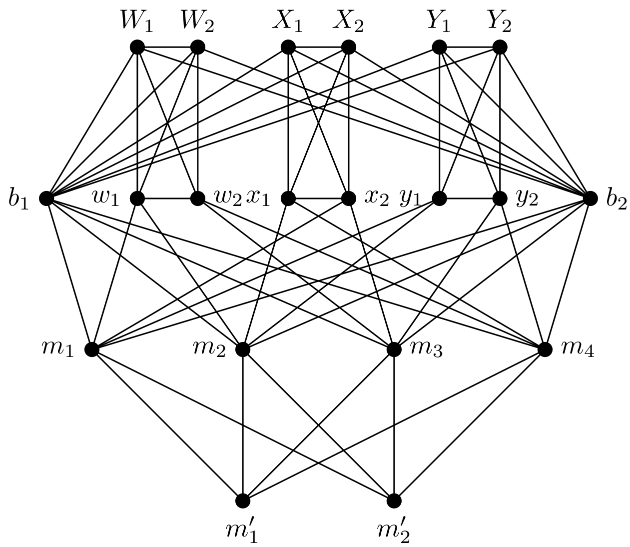

Figure 1: Reduction from 3DM to CDN.

that M contains a 3-dimensional matching of size q if and only if G contains m disjoint CDSs.

We start with the "only if" direction. If M contains a matching of size q, each triplet node in the matching, its associated element nodes, and a matching node form a CDS of G. Each of the other $m-q$ CDSs is comprised of one bridge node, one set node from each of W, X, Y, and one of the remaining triplet nodes.

We proceed to prove the "if" direction. If G contains m disjoint CDSs, each CDS must contain exactly one triplet node because matching nodes are only adjacent to triplet nodes.

Recall that each one of $\mathscr{W} \cup \mathbb{W}$, $\mathscr{X} \cup \mathbb{X}$, and $\mathscr{Y} \cup \mathbb{Y}$ forms a clique comprised of q element nodes and $m-q$ set nodes. Since each CDS only contains one triplet node, it can dominate at most one element node in each clique via its triplet node. Therefore, in non-trivial cases where $q \geq 2$, each CDS also has to contain at least one node from each clique as well. On the other hand, each CDS can have at most one node from each clique since each clique only has m nodes to be shared by m CDSs. Clearly, each CDS also contains exactly one node from each clique.

If a CDS contains a set node, the set node can only be connected to its triplet node via some bridge node, since we have proven above that a CDS can not have another node from the same clique to connect the set node to its triplet node. Given $m-q$ bridge nodes, it is clear that at most $m-q$ CDSs can contain a set node. On the other hand, each CDS can contain at most one set node from each clique, which means at least $m-q$ CDSs have to contain some set node. Therefore, it must be the case that there are exactly $m-q$ CDSs each containing one set node from each clique, while each of the other q CDSs contains one element node from each clique. Note that in each of these q CDSs, each element node has to be directly connected to the triplet node since there can not be another node from the same clique. Thus, these q CDSs form a 3-dimensional matching of size q we need. □

3.2 The packet based model

In this section, we analyze network lifetime maximization problems in the intensively researched packet based model. In particular, we analyze the complexity of a number of network lifetime maximization problems in different routing paradigms, i.e., unicast, multicast and broadcast. NP-hardness proofs are presented for intractable problems, and polynomial time algorithms are given for tractable problems.

We start with the problem of maximizing the lifetime of a broadcast session over energy constrained WSNs, which is formally defined as follows.

> BROADCAST LIFETIME
>
> **INSTANCE** Directed graph $G = (V, A)$. Specified source s. Positive integer K.
>
> **QUESTION** Does G have enough power to broadcast K packets from s to all other nodes?

The problem of minimum energy broadcast has been well researched in the literature and proved to be NP-hard [15, 16]. However, the complexity of broadcast lifetime remains open. Floréen *et al.* [8] investigated the problem of maximizing the lifetime of a *multicast* session over a network of energy-constrained nodes, where the network contains some critical nodes that have to be included in every steiner tree. Therefore, [8] did not address our problem.

THEOREM 2. *Broadcast lifetime is NP-hard.*

PROOF. The NP-hardness of broadcast lifetime can be proved by slightly adapting the proof of Theorem 1. In particular, we shall also prove by reducing from the 3-dimensional matching (3DM) problem.

Given an instance of 3DM, we construct a graph $G = (V, E)$ as shown in Figure 2, where nodes are distributed into four layers and edges exist only between nodes in the same layer or adjacent layers. Nodes in each layer is the same as defined in the proof of Theorem 1. The only difference is that, in the bottom layer, there is only the source node s, which has an energy of m and is adjacent to all the triplet nodes. Each node other than s has one unit energy. It is clear that the transformation is polynomial, and we prove that M contains a 3-dimensonal matching of size q if and only if m packets can be broadcast.

We start with the "only if" direction. If M contains a matching of size q, each triplet node in the matching, its associated element nodes plus the source node s form a broadcast tree. Each of the other $m-q$ broadcast trees is composed of one bridge vertex, one set node for each of W, X, Y, one triplet node, plus the source node s. All these broadcast trees are node-disjoint (except s) and s

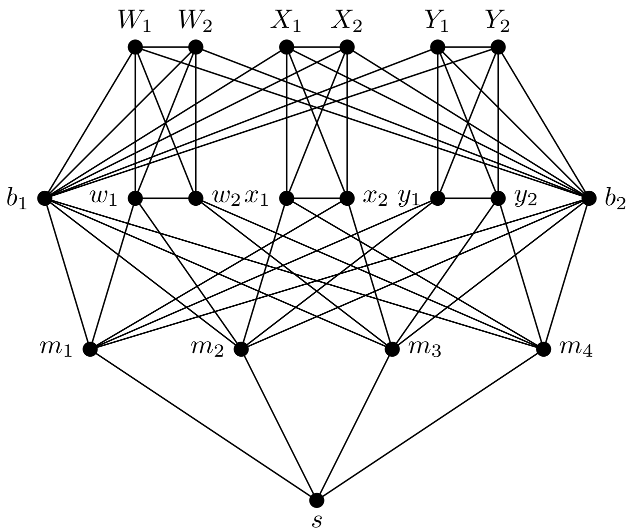

Figure 2: The graph is constructed from the following 3DM instance. $W = \{w_1, w_2\}$**,** $X = \{x_1, x_2\}$**, and** $Y = \{y_1, y_2\}$**.** $M = \{m_1, m_2, m_3, m_4\}$**, where** $m_1 = (w_1, x_2, y_1)$**,** $m_2 = (w_1, x_1, y_1)$**,** $m_3 = (w_2, x_2, y_2)$**, and** $m_4 = (w_2, x_1, y_2)$**.**

has enough power. Thus, one packet can be broadcast over each of these m broadcast trees, respectively.

We then prove the "if" direction. If m packets can be broadcast, it is clear that there have to be m node-disjoint broadcast trees each containing exactly one triplet node, since s relies on the triplet nodes to forward its packets. Consequently, in non-trivial cases where $q \geq 2$, each broadcast tree has to contain at least one node from each clique to broadcast a packet to the nodes in the cliques. Since each clique has m nodes, each broadcast tree has exactly one node from each clique. If a broadcast tree contains a set node, the set node can only be connected to the triplet node of that broadcast tree via a bridge node, since there can not be another node from the same clique in the broadcast tree. Given $m - q$ bridge nodes and $3(m - q)$ set nodes, it must be the case that there are $m - q$ broadcast tree each containing one bridge node and one set node from each clique. Therefore, each of the other q broadcast trees contains one element node from each clique and the element nodes have to be adjacent to the triplet node in the broadcast tree. These q triplet nodes thus form a 3-dimensional matching of size q. □

Similarly, in the problem of *multicast lifetime*, we want to maximize the number of packets that can be multicast from a specified source s to a specified group T of terminals. Since broadcast is just a special case of multicast, the NP-hardness of multicast lifetime directly follows.

We then proceed to investigate the problem of maximizing lifetime of unicast sessions. Similarly, we also define network lifetime as the maximum number of packets that can be delivered by the network. Because even if some nodes fail due to battery depletion, the network may still be able to deliver packets for a unicast session.

There are four different cases in unicast: *one-to-one* unicast, *one-to-many* unicast, *many-to-one* unicast and *many-to-many* unicast, of which many-to-many unicast is the most general case. Meanwhile, there are two different flow models in unicast, i.e., the *multiple commodity model* and the *single commodity model*. In the multiple commodity model, packets to be delivered between each source-sink pair are considered a separate commodity. In the more relaxed single commodity model that has been previously studied by Chang and Tassiulas [3], all packets are considered the same commodity and each sink is satisfied if and only if it receives the *number* of packets it requests, no matter which source sends the packets. The most general case of many-to-many unicast lifetime is formally defined in each model, respectively. The definitions of the other three cases can be easily induced as special cases of many-to-many unicast lifetime.

> MANY-TO-MANY UNICAST LIFETIME (MULTIPLE COMMODITY MODEL)
>
> **INSTANCE** Directed graph $G = (V, A)$. Specified set of sources $S = \{s_1, s_2, \ldots, s_m\} \subseteq V$ and specified set of sinks $D = \{t_1, t_2, \ldots, t_n\} \subseteq V$. Each source s_i has N_{ij} packets to be delivered to sink t_j. Positive integer K.
>
> **QUESTION** Does G have enough power to deliver K packets?

> MANY-TO-MANY UNICAST LIFETIME (SINGLE COMMODITY MODEL)
>
> **INSTANCE** Directed graph $G = (V, A)$. Specified set of sources $S = \{s_1, s_2, \ldots, s_m\} \subseteq V$ and specified set of sinks $D = \{t_1, t_2, \ldots, t_n\} \subseteq V$. Each source s_i has N_i^s packets to be delivered and each sink t_j requests for N_j^t packets. Positive integer K.
>
> **QUESTION** Does G have enough power to deliver K packets?

It is clear that the definition of many-to-one unicast lifetime, one-to-many unicast lifetime, and one-to-one unicast lifetime remain the same in the multiple commodity model and the single commodity model.

LEMMA 1. *One-to-one unicast lifetime is NP-hard.*

PROOF. It suffices to prove the NP-hardness of the special case where every packet is to be sent from a source node s to a sink node t. Again, we reduce from 3DM and we illustrate the reduction in Figure 3 with the same 3DM instance as used in Figure 2. Nodes are still distributed into four layers. In the top layer, there is the sink node t. In the second layer, there are 3 disjoint groups of *element nodes*, $\mathbb{W} = \{w_1, w_2, \ldots, w_q\}$,

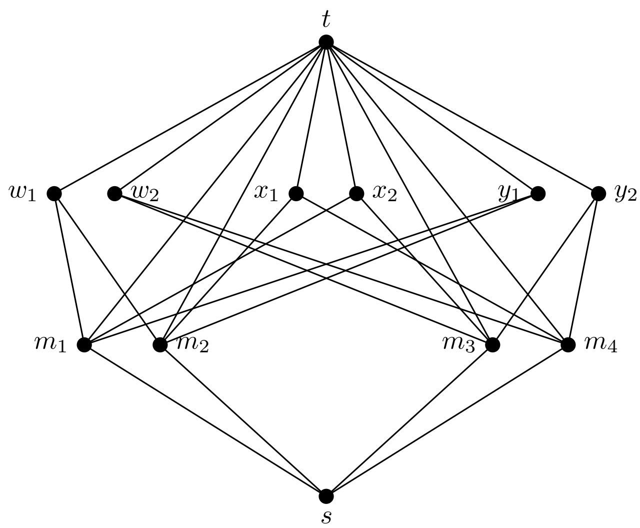

Figure 3: Reduction from 3DM to one-to-one unicast.

$\mathbb{X} = \{x_1, x_2, \ldots, x_q\}$, and $\mathbb{Y} = \{y_1, y_2, \ldots, y_q\}$. $\mathbb{W}$, $\mathbb{X}$, and $\mathbb{Y}$ represent W, X, and Y, respectively. Each element node has an energy of 1 and is adjacent to t. In the third layer, there is a group of m *triplet nodes* $\mathbb{M} = \{m_1, m_2, \ldots, m_m\}$ representing the elements in M. Each triplet node has an energy of 3 and is adjacent to t as well as the three element nodes occurring in the element in M that it represents. In the bottom layer, there is the source node s with an energy of $m + 2q$. Each triplet node is also adjacent to s. Edges between triplet nodes and t have a weight of 3, while the other edges have a weight of 1. The transformation is clearly polynomial, and we prove that M contains a 3-dimensional matching of size q if and only if $m + 2q$ packets can be gathered from s to t.

If M contains a 3-dimensional matching of size q, $3q$ packets can be delivered through the q triplet nodes in the matching and the $3q$ element nodes. The other $m - q$ packets can be delivered through the other $m - q$ triplet nodes.

If $m + 2q$ packets can be delivered from s to t, it is clear that the only way to achieve that is the same as described above, since every packet has to be forwarded by some triplet node. Thus, the q triplet nodes adjacent to the $3q$ element nodes form a 3-dimensional matching of size q. □

Since one-to-one unicast lifetime is a special case of the other three unicast lifetime problems, the following theorem directly follows.

THEOREM 3. *In both multiple commodty model and single commodity model, many-to-many unicast lifetime, many-to-one unicast lifetime, one-to-many unicast lifetime and one-to-one unicast lifetime are all NP-hard.*

Although the unicast lifetime problems are proven to be NP-hard, it turns out that in cases where each node i has a fixed transmission power of $P_{max}(i)$ (e.g. tiny sensor nodes may not be able to adjust their transmission power), we may be able to solve them in polynomial time.

THEOREM 4. *If each node has a fixed transmission power, one-to-one unicast lifetime can be solved in polynomial time.*

PROOF. Given an instance of one-to-one unicast lifetime, for each node $i \in V$, define its *capacity* to be $c_i = p_i/P_{max}(i)$, where p_i is its initial energy. An algorithm for the *node-capacitated network flow* problem [17] can be applied to compute the maximum number of packets that can be delivered from s to t. □

THEOREM 5. *If each node has a fixed transmission power, many-to-one unicast lifetime can be solved in polynomial time.*

PROOF. Given that one-to-one unicast lifetime is polynomially solvable, it suffices to reduce many-to-one unicast lifetime to one-to-one unicast lifetime. First of all, we point out that for many-to-one unicast lifetime, we can safely assume without loss of generality that $t \notin S$.

Given an instance of many-to-one unicast lifetime, we transform it into an instance of one-to-one unicast lifetime as follows. For each source node s_i, generate a mirror node s'_i with an energy of n_i, where n_i is the number of packets to be delivered from s_i to the sink t. Then, add a directed link of weight 1 from s'_i to s_i. Finally, add a *super source* s and a directed link of weight 0 from s to each mirror node. All packets are now to be delivered from s to t. It is clear that the transformation is polynomial.

Assume that K packets can be delivered to t in the given instance of many-to-one unicast lifetime, where each source node s_i has $n'_i \le n_i$ packets delivered to t. In the constructed instance of one-to-one unicast lifetime, s can safely dispatch the n'_i packets to s_i via s'_i, and all the K packets can be delivered to t along the same paths as they travel along in the given instance of many-to-one unicast lifetime.

On the other hand, if K packets can be delivered from s to t in the constructed one-to-one unicast lifetime instance, each packet has to travel through some source node s_i. The available energy at mirror nodes guarantees that for each $1 \le i \le m$, at most n_i packets first reaches s_i among the source nodes. Thus, in the given instance of many-to-one unicast lifetime, K packets can travel from the sources to t along the same paths as they travel along in the constructed instance of one-to-one unicast lifetime. □

THEOREM 6. *If each node has a fixed transmission power, one-to-many unicast lifetime can be solved in polynomial time.*

PROOF. We similarly prove by reducing to one-to-one unicast lifetime and point out that for one-to-many unicast lifetime, we can also safely assume without loss of generality that $s \notin D$.

Given an instance of one-to-many unicast, we transform it into an instance of one-to-one unicast lifetime as follows. For each sink node t_i, generate a mirror node t'_i with an energy of n_i, where n_i is the number of packets to be delivered from the source node s to t_i. And add a directed link of weight 0 from t_i to t'_i. Then, add a *super sink* t and a directed link of weight 1 from each mirror node to t. All packets are now destined to t. It is clear that the transformation is polynomial.

Assume that K packets can be delivered in the given instance of one-to-many unicast lifetime, where each sink node t_i receives $n'_i \leq n_i$ packets. Then in the constructed instance of one-to-one unicast lifetime, each sink node t_i can simply forward the n'_i packets to t via t'_i, and all the K packets are thus delivered to t.

On the other hand, if K packets can be delivered from s to t in the constructed instance of one-to-one unicast lifetime, each packet has to travel through some sink node t_i. The available energy at mirror nodes guarantees that for each $1 \leq i \leq n$, at most n_i packets travel to t via t'_i. Thus, in the given instance of one-to-many unicast lifetime, K packets can be delivered along the same paths as they travel along in the constructed instance of one-to-one unicast lifetime. □

THEOREM 7. *In the single commodity model, if each node has a fixed transmission power, many-to-many unicast lifetime is polynomially solvable.*

PROOF. In the single commodity model, packets can be delivered from any source to any sink. Thus, we only need to consider non-trivial cases where $S \cap D = \phi$. Recall that many-to-one unicast lifetime remains the same in the multiple commodity model and the single commodity model. Given Theorem 5, it suffices to reduce many-to-many unicast lifetime to many-to-one unicast lifetime.

Given an instance of many-to-many unicast lifetime, generate a mirror node t'_i with an energy of n_i for each sink t_i, where n_i is the number of packets requested by t_i. Add a directed link (t_i, t'_i) of weight 0. Then, add a *super sink* t and a directed link of weight 1 from each mirror node to t. All packets are now destined to t. The transformation is clearly polynomial.

Assume that K packets can be delivered in the given instance of many-to-many unicast lifetime, where each sink node t_i receives $n'_i \leq n_i$ packets. Then in the constructed instance of many-to-one unicast lifetime, each sink node t_i can simply forward the n'_i packets to t via t'_i, and all the K packets are thus delivered to t.

On the other hand, if K packets can be delivered to t in the constructed instance of many-to-one unicast lifetime, each packet has to travel through some sink node t_i. The available energy at mirror nodes guarantees that for each $1 \leq i \leq n$, at most n_i packets travel to t via t'_i. Thus, in the given instance of many-to-many unicast lifetime, K packets can be delivered along the same paths as they travel along in the constructed instance of many-to-one unicast lifetime. □

THEOREM 8. *In the multiple commodity model, even if each node has a fixed transmission power, many-to-many unicast lifetime remains NP-hard.*

PROOF. We prove by reducing from the NP-hard *disjoint connecting paths* problem [14], which is defined as follows.

> DISJOINT CONNECTING PATHS
>
> **INSTANCE** Graph $G = (V, E)$, where V is the set of nodes and E is the set of edges. Disjoint set of sources $S = \{s_1, s_2, \ldots, s_m\} \subseteq V$ and set of sinks $D = \{t_1, t_2, \ldots, t_m\} \subseteq V$.
>
> **QUESTION** Does G contain m node disjoint paths, each connecting one pair of source and sink (s_i, t_i) for all $1 \leq i \leq m$?

Given an instance of disjoint connecting paths, assign each edge a weight of 1. Assign each sink node an energy of 0 and each non-sink node an energy of 1. Let each s_i have one packet to be delivered to the corresponding sink t_i. The transformation is clearly polynomial, and we show that m packets can be delivered from the sources to the sinks if and only if there are m node disjoint paths each connecting one pair of source and sink (s_i, t_i) for all $1 \leq i \leq m$.

If G contains m node disjoint paths each connecting one pair of source and sink (s_i, t_i) for all $1 \leq i \leq m$, each s_i can deliver its packet to t_i along the path connecting them. And all of the m packets can thus be delivered.

Assume that all of the m packets can be delivered. Since each edge has a weight of 1 and non-sink nodes have an energy of 1, each non-sink node is on the delivery path of at most one packet. Sink nodes do not have energy and there is only one packet destined to each sink, thus each sink node is on the delivery path of at most one packet as well. Therefore, the delivery paths of the m packets are node disjoint, each connecting one pair of source and sink (s_i, t_i) for all $1 \leq i \leq m$. □

4. CONCLUSIONS

We have presented formal analysis of a variety of network lifetime maximization problems in different energy consumption models. An analysis of energy consumption in wireless sensor networks leads to two energy consumption models for formal analysis, i.e., the time based model and the intensively researched packet based model. Various network lifetime maximization problems are identified in individual models. The complexity of these problems are formally analyzed.

Most of the past research efforts aiming to extend network lifetime are based on the packet based model, while it is well known that in many applications energy consumption in the time based model is comparable to that in the packet based model. On the other hand, there are two different approaches to network lifetime maximization, and most of the past research efforts followed the indirect approach of energy conservation. Although helpful to extend network lifetime, energy conservation is not precisely the same problem as network lifetime maximization.

In this paper, we directly investigate the problem of network lifetime maximization in individual energy consumption models as well as routing paradigms. In the time based model, we study the problem of maximizing network lifetime while preserving connectivity and prove that it is NP-hard. In the packet based model, we formally define the following problems: broadcast lifetime, multicast lifetime, many-to-many unicast lifetime, many-to-one unicast lifetime, one-to-many unicast lifetime and one-to-one unicast lifetime. Broadcast lifetime and multicast lifetime are NP-hard, even if each node has a fixed transmission power. We show that the unicast lifetime problems are NP-hard in both the multiple commodity model and the single commodity model. However, we show that in cases where each node has a fixed transmission power, many-to-one unicast lifetime, one-to-many unicast lifetime, and one-to-one unicast lifetime are polynomially solvable. Many-to-many unicast lifetime is also polynomially solvable in the single commodity model, but remains NP-hard in the multiple commodity model.

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