BLOCK CIPHERS
A function \( f : \{0, 1\}^\ell \rightarrow \{0, 1\}^\ell \) is a permutation if there is an inverse function \( f^{-1} : \{0, 1\}^\ell \rightarrow \{0, 1\}^\ell \) satisfying

\[
\forall x \in \{0, 1\}^\ell : f^{-1}(f(x)) = x
\]

This means \( f \) must be one-to-one and onto, meaning for every \( y \in \{0, 1\}^\ell \) there is a unique \( x \in \{0, 1\}^\ell \) such that \( f(x) = y \).
Permutations and Inverses

A permutation

Not a permutation
Permutations and Inverses

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A permutation

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Its inverse
Let \( E : \{0, 1\}^k \times \{0, 1\}^\ell \rightarrow \{0, 1\}^\ell \) be a function taking a key \( K \) and input \( x \) to return output \( E(K, x) \). For each key \( K \) we let \( E_K : \{0, 1\}^\ell \rightarrow \{0, 1\}^\ell \) be the function defined by
\[
E_K(x) = E(K, x)
\]

We say that \( E \) is a block cipher if

- \( E_K : \{0, 1\}^\ell \rightarrow \{0, 1\}^\ell \) is a permutation for every \( K \), meaning has an inverse \( E_K^{-1} \),
- \( E, E^{-1} \) are efficiently computable,

where \( E^{-1}(K, x) = E_K^{-1}(x) \).
The table entry corresponding to the key in row $K$ and input in column $x$ is $E_K(x)$.

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In this case, the inverse cipher $E^{-1}$ is given by the same table: the table entry corresponding to the key in row $K$ and output in column $y$ is $E_K^{-1}(y)$. 
Block Ciphers: Example

Let $\ell = k$ and define $E : \{0, 1\}^k \times \{0, 1\}^\ell \to \{0, 1\}^\ell$ by

$$E_K(x) = E(K, x) = K \oplus x$$

Then $E_K$ has inverse $E_K^{-1}$ where

$$E_K^{-1}(y) = K \oplus y$$

Why? Because

$$E_K^{-1}(E_K(x)) = E_K^{-1}(K \oplus x) = K \oplus K \oplus x = x$$

The inverse of block cipher $E$ is the block cipher $E^{-1}$ defined by

$$E^{-1}(K, y) = E_K^{-1}(y) = K \oplus y$$
Block cipher usage

- $K \leftarrow \{0, 1\}^k$
- $K$ (magically) given to parties S, R, but not to A.
- S, R use $E_K$

Algorithm $E$ is public! Think of $E_K$ as encryption under key $K$.

Leads to security requirements like:

- Hard to get $K$ from $y_1, y_2, \ldots$
- Hard to get $x_i$ from $y_i$
1972 – NBS (now NIST) asked for a block cipher for standardization

1974 – IBM designs **Lucifer**

Lucifer eventually evolved into DES.

Widely adopted as a standard including by ANSI and American Bankers association

Used in ATM machines

Replaced (by AES) only a few years ago
Key Length $k = 56$

Block length $\ell = 64$

So,

$$\text{DES}: \{0, 1\}^{56} \times \{0, 1\}^{64} \rightarrow \{0, 1\}^{64}$$

$$\text{DES}^{-1}: \{0, 1\}^{56} \times \{0, 1\}^{64} \rightarrow \{0, 1\}^{64}$$
DES Construction

function \( \text{DES}_K(M) \)  \( // |K| = 56 \text{ and } |M| = 64 \)

\( (K_1, \ldots, K_{16}) \leftarrow \text{KeySchedule}(K) \)  \( // |K_i| = 48 \text{ for } 1 \leq i \leq 16 \)

\( M \leftarrow \text{IP}(M) \)

Parse \( M \) as \( L_0 \parallel R_0 \)  \( // |L_0| = |R_0| = 32 \)

for \( i = 1 \) to \( 16 \) do

\( L_i \leftarrow R_{i-1} ; \quad R_i \leftarrow f(K_i, R_{i-1}) \oplus L_{i-1} \)

\( C \leftarrow \text{IP}^{-1}(L_{16} \parallel R_{16}) \)

return \( C \)

Round \( i \):

Invertible given \( K_i \):
function DES\(_K\)\((M)\)  //  \(|K| = 56\) and \(|M| = 64\)  
\((K_1, \ldots, K_{16}) \leftarrow \text{KeySchedule}(K)\)  //  \(|K_i| = 48\) for 1 ≤ i ≤ 16  
\(M \leftarrow \text{IP}(M)\)  
Parse \(M\) as \(L_0 \parallel R_0\)  //  \(|L_0| = |R_0| = 32\)  
for \(i = 1\) to 16 do  
  \(L_i \leftarrow R_{i-1}\) ;  \(R_i \leftarrow f(K_i, R_{i-1}) \oplus L_{i-1}\)  
\(C \leftarrow \text{IP}^{-1}(L_{16} \parallel R_{16})\)  
return \(C\)

function DES\(^{-1}\)_\(_K\)\((C)\)  //  \(|K| = 56\) and \(|M| = 64\)  
\((K_1, \ldots, K_{16}) \leftarrow \text{KeySchedule}(K)\)  //  \(|K_i| = 48\) for 1 ≤ i ≤ 16  
\(C \leftarrow \text{IP}(C)\)  
Parse \(C\) as \(L_{16} \parallel R_{16}\)  
for \(i = 16\) downto 1 do  
  \(R_{i-1} \leftarrow L_i\) ;  \(L_{i-1} \leftarrow f(K_i, R_{i-1}) \oplus R_i\)  
\(M \leftarrow \text{IP}^{-1}(L_{0} \parallel R_{0})\)  
return \(M\)
function DES_K(M)  // |K| = 56 and |M| = 64
(K_1, \ldots, K_{16}) \leftarrow \text{KeySchedule}(K)  // |K_i| = 48 for 1 \leq i \leq 16
M \leftarrow \text{IP}(M)
Parse M as L_0 \parallel R_0  // |L_0| = |R_0| = 32
for i = 1 to 16 do
    L_i \leftarrow R_{i-1} ;  \quad R_i \leftarrow f(K_i, R_{i-1}) \oplus L_{i-1}
C \leftarrow \text{IP}^{-1}(L_{16} \parallel R_{16})
return C

\begin{array}{cccccccc}
\text{IP} & 58 & 50 & 42 & 34 & 26 & 18 & 10 & 2 \\
       & 60 & 52 & 44 & 36 & 28 & 20 & 12 & 4 \\
       & 62 & 54 & 46 & 38 & 30 & 22 & 14 & 6 \\
       & 64 & 56 & 48 & 40 & 32 & 24 & 16 & 8 \\
       & 57 & 49 & 41 & 33 & 25 & 17 & 9 & 1 \\
       & 59 & 51 & 43 & 35 & 27 & 19 & 11 & 3 \\
       & 61 & 53 & 45 & 37 & 29 & 21 & 13 & 5 \\
       & 63 & 55 & 47 & 39 & 31 & 23 & 15 & 7 \\
\text{IP}^{-1} & 40 & 8 & 48 & 16 & 56 & 24 & 64 & 32 \\
       & 39 & 7 & 47 & 15 & 55 & 23 & 63 & 31 \\
       & 38 & 6 & 46 & 14 & 54 & 22 & 62 & 30 \\
       & 37 & 5 & 45 & 13 & 53 & 21 & 61 & 29 \\
       & 36 & 4 & 44 & 12 & 52 & 20 & 60 & 28 \\
       & 35 & 3 & 43 & 11 & 51 & 19 & 59 & 27 \\
       & 34 & 2 & 42 & 10 & 50 & 18 & 58 & 26 \\
       & 33 & 1 & 41 & 9 & 49 & 17 & 57 & 25 \\
\end{array}
function $f(J, R)$  // $|J| = 48$ and $|R| = 32$

\[ R \leftarrow E(R) ; \quad R \leftarrow R \oplus J \]

Parse $R$ as $R_1 \parallel R_2 \parallel R_3 \parallel R_4 \parallel R_5 \parallel R_6 \parallel R_7 \parallel R_8$  // $|R_i| = 6$ for $1 \leq i$

for $i = 1, \ldots, 8$ do

\[ R_i \leftarrow S_i(R_i) \quad \text{// Each S-box returns 4 bits} \]

\[ R \leftarrow R_1 \parallel R_2 \parallel R_3 \parallel R_4 \parallel R_5 \parallel R_6 \parallel R_7 \parallel R_8 \quad \text{// } |R| = 32 \text{ bits} \]

\[ R \leftarrow P(R) \]

return $R$

\[
\begin{array}{cccccc}
E & 32 & 1 & 2 & 3 & 4 & 5 \\
4 & 5 & 6 & 7 & 8 & 9 \\
8 & 9 & 10 & 11 & 12 & 13 \\
12 & 13 & 14 & 15 & 16 & 17 \\
16 & 17 & 18 & 19 & 20 & 21 \\
20 & 21 & 22 & 23 & 24 & 25 \\
24 & 25 & 26 & 27 & 28 & 29 \\
28 & 29 & 30 & 31 & 32 & 1 \\
\end{array}
\]

\[
\begin{array}{cccccc}
P & 16 & 7 & 20 & 21 \\
29 & 12 & 28 & 17 \\
1 & 15 & 23 & 26 \\
5 & 18 & 31 & 10 \\
2 & 8 & 24 & 14 \\
32 & 27 & 3 & 9 \\
19 & 13 & 30 & 6 \\
22 & 11 & 4 & 25 \\
\end{array}
\]
### S-boxes

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**Figure:** The DES S-boxes.
Adversary $A$ knows $E : \{0,1\}^k \times \{0,1\}^\ell \rightarrow \{0,1\}^\ell$

$T \xleftarrow{\$} \{0,1\}^k$ is the target key.

Given: $(M_1, C_1), \ldots, (M_q, C_q)$ where $C_i = E(T, M_i)$ for $i = 1, \ldots, q$

and $M_1, \ldots, M_q$ are distinct.

Find: $T$
Cryptanalysis: Key Recovery Attacks on Block Ciphers

Adversary $A$ knows $E: \{0, 1\}^k \times \{0, 1\}^\ell \rightarrow \{0, 1\}^\ell$

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**Find:** $T$

Certainly $A$ should be given $C_1, \ldots, C_q$. But why does $A$ know $M_1, \ldots, M_q$?

- A posteriori revelation of data
- A priori knowledge of context

Good to be conservative!
A posteriori revelation of data

- $S, R$ share key $K$
- On January 10, $S$ encrypts
  \[ M = \text{Let’s meet tomorrow at 5 pm} \]
  and sends ciphertext $C$ to $R$.
- Adversary captures $C$
- On January 11, adversary observes $S, R$ meeting at 5 pm and deduces that $M$ is as above
- Adversary knows $C$ and its decryption $M$
A priori knowledge of context

- $S, R$ share key $K$
- E-mails always begin with the keyword “From”
- $S$ encrypts an email
- Adversary gets ciphertext $C$
- Since it knows part of the plaintext (“From”) it may have an input-output example of the block cipher under $K$
Adversary $A$ knows $E : \{0, 1\}^k \times \{0, 1\}^\ell \rightarrow \{0, 1\}^\ell$

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Find: $T$
Types of attacks

Given: \((M_1, C_1), \ldots, (M_q, C_q)\) where \(C_i = E(T, M_i)\) for \(i = 1, \ldots, q\) and \(M_1, \ldots, M_q\) are distinct.

Known Message Attack: \(M_1, \ldots, M_q\) arbitrary, not chosen by A.
Types of attacks

Given: \((M_1, C_1), \ldots, (M_q, C_q)\) where \(C_i = E(T, M_i)\) for \(i = 1, \ldots, q\) and \(M_1, \ldots, M_q\) are distinct.

Chosen Message Attack: A can pick \(M_1, \ldots, M_q\), even adaptively, meaning pick \(M_i\) as a function of \((M_1, C_1), \ldots, (M_{i-1}, C_{i-1})\) for \(i = 1, \ldots, q\).

Examples:
- A sends S e-mails which S encrypts and forwards to R
- S is a router encrypting any packet it receives
Adversary $A$ knows $E : \{0, 1\}^k \times \{0, 1\}^\ell \rightarrow \{0, 1\}^\ell$

$T \xleftarrow{\$} \{0, 1\}^k$ is the target key.

Given: $(M_1, C_1), \ldots , (M_q, C_q)$ where $C_i = E(T, M_i)$ for $i = 1, \ldots , q$
and $M_1, \ldots , M_q$ are distinct.

Find: $T$
Exhaustive Key Search

Let \( T_1, \ldots, T_{2^k} \) be a list of all \( k \) bit keys. Let \( T \leftarrow \{0, 1\}^k \) be the target key and let \((M_1, C_1)\) satisfy \( E_T(M_1) = C_1 \).

algorithm \( EKS_E(M_1, C_1) \)
  for \( i = 1, \ldots, 2^k \) do
    if \( E(T_i, M_1) = C_1 \) then return \( T_i \)
Exhaustive Key Search

Let $T_1, \ldots, T_{2^k}$ be a list of all $k$ bit keys. Let $T \leftarrow \{0, 1\}^k$ be the target key and let $(M_1, C_1)$ satisfy $E_T(M_1) = C_1$.

algorithm $EKS_E(M_1, C_1)$
    for $i = 1, \ldots, 2^k$ do
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Does this find the target key $T$?
Exhaustive Key Search

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**Definition:** A key $K$ is consistent with $(M_1, C_1)$ if $C_1 = E(K, M_1)$
Exhaustive Key Search

Let $T_1, \ldots, T_{2^k}$ be a list of all $k$ bit keys. Let $T \leftarrow \{0, 1\}^k$ be the target key and let $(M_1, C_1)$ satisfy $E_T(M_1) = C_1$.

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Does this find the target key $T$?

Definition: A key $K$ is consistent with $(M_1, C_1)$ if $C_1 = E(K, M_1)$

Let $S$ be the set of all keys consistent with $(M_1, C_1)$. Then $EKS_E$ finds some key in $S$. 
Let $T_1, \ldots, T_{2^k}$ be a list of all $k$ bit keys. Let $T \leftarrow \{0, 1\}^k$ be the target key and let $(M_1, C_1)$ satisfy $E_T(M_1) = C_1$.

algorithm $EKS_E(M_1, C_1)$
   for $i = 1, \ldots, 2^k$ do
      if $E(T_i, M_1) = C_1$ then return $T_i$

Does this find the target key $T$?

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Let $S$ be the set of all keys consistent with $(M_1, C_1)$. Then $EKS_E$ finds some key in $S$.

**Fact:** If $\ell \geq k$ then $T$ is “usually” the only key in $S$. 
Exhaustive Key Search

Let $T_1, \ldots, T_{2^k}$ be a list of all $k$ bit keys. Let $T \leftarrow \{0, 1\}^k$ be the target key and let $(M_1, C_1)$ satisfy $E_T(M_1) = C_1$.

algorithm $EKS_E(M_1, C_1)$

for $i = 1, \ldots, 2^k$ do
if $E(T_i, M_1) = C_1$ then return $T_i$

Does this find the target key $T$? Yes, usually.
Let $T_1, \ldots, T_{2^k}$ be a list of all $k$ bit keys. Let $T \leftarrow \{0, 1\}^k$ be the target key and let $(M_1, C_1), \ldots, (M_q, C_q)$ satisfy $E_T(M_i) = C_i$ for all $1 \leq i \leq q$.

algorithm $EKS_E(((M_1, C_1), \ldots, (M_q, C_q))$

for $i = 1, \ldots, 2^k$ do

if ($E(T_i, M_1) = C_1$ and $\cdots$ and $E(T_i, M_q) = C_q$) then

return $T_i$
Let $T_1, \ldots, T_{2^k}$ be a list of all $k$ bit keys. Let $T \leftarrow \{0, 1\}^k$ be the target key and let $(M_1, C_1)$ satisfy $E_T(M_1) = C_1$.

algorithm $EKS_E(M_1, C_1)$
for $i = 1, \ldots, 2^k$ do
    if $E(T_i, M_1) = C_1$ then return $T_i$
How long does exhaustive key search take?

DES can be computed at 1.6 Gbits/sec in hardware.

DES plaintext = 64 bits

Chip can perform \((1.6 \times 10^9)/64 = 2.5 \times 10^7\) DES computations per second

Expect EKS to succeed in \(2^{55}\) DES computations, so it takes time

\[
\frac{2^{55}}{2.5 \times 10^7} \approx 1.4 \times 10^9 \text{ seconds}
\]

\[
\approx 45 \text{ years!}
\]

Key Complementation ⇒ 22.5 years

But this is prohibitive.

Does this mean DES is secure?
Exhaustive key search is a generic attack: Did not attempt to “look inside” DES and find/exploit weaknesses.

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But merely storing $2^{44}$ input-output pairs requires 281 Tera-bytes.

In practice these attacks are prohibitively expensive.
Let $T_1, \ldots, T_{2^k}$ be a list of all $k$ bit keys. Let $T \leftarrow \{0, 1\}^k$ be the target key and let $(M_1, C_1)$ satisfy $E_T(M_1) = C_1$.

algorithm $EKS_E(M_1, C_1)$
  for $i = 1, \ldots, 2^k$ do
    if $E(T_i, M_1) = C_1$ then return $T_i$
EKS revisited

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- Wiener 1993:
  - $1$ million
  - $57$ chips
  - Finds key in $3.5$ hours

- EFF
  - $250,000$
  - Finds key in $56$ hours
DES is considered broken because its short key size permits rapid key-search.

But DES is a very strong design as evidenced by the fact that there are no practical attacks that exploit its structure.
Block cipher $2DES : \{0, 1\}^{112} \times \{0, 1\}^{64} \rightarrow \{0, 1\}^{64}$ is defined by

$$2DES_{K_1K_2}(M) = DES_{K_2}(DES_{K_1}(M))$$

- Exhaustive key search takes $2^{112}$ $DES$ computations, which is too much even for machines
- Resistant to differential and linear cryptanalysis.
Suppose $K_1K_2$ is a target 2DES key and adversary has $M, C$ such that

$$C = 2DES_{K_1K_2}(M) = DES_{K_2}(DES_{K_1}(M))$$

Then

$$DES_{K_2}^{-1}(C) = DES_{K_1}(M)$$
Meet-in-the-middle attack on 2DES

Suppose $\text{DES}^{-1}_{K_2}(C) = \text{DES}_{K_1}(M)$ and $T_1, \ldots, T_N$ are all possible DES keys, where $N = 2^{56}$.

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<th>$T_1$</th>
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<td>$T_i$</td>
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<tr>
<td>$T_N$</td>
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Table $L$

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<th>$\text{DES}^{-1}(T_1, C)$</th>
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<td>$\text{DES}^{-1}(T_j, C)$</td>
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Table $R$

Attack idea:

- Build L,R tables
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Table $R$

Attack idea:

- Build L,R tables
- Find $i,j$ s.t. $L[i] = R[j]$
- Guess that $K_1K_2 = T_i T_j$
Meet-in-the-middle attack on 2DES

Let $T_1, \ldots, T_{2^{56}}$ denote an enumeration of DES keys.

$MinM_{2\text{DES}}(M_1, C_1)$

- for $i = 1, \ldots, 2^{56}$ do $L[i] \leftarrow \text{DES}(T_i, M_1)$
- for $j = 1, \ldots, 2^{56}$ do $R[j] \leftarrow \text{DES}^{-1}(T_j, C_1)$
- $S \leftarrow \{ (i, j) : L[i] = R[j] \}$
- Pick some $(l, r) \in S$ and return $T_l \parallel T_r$

Attack takes about $2^{57}$ DES/DES$^{-1}$ computations.

Interesting, but not practical.
3DES

Block ciphers

3DES3 : \( \{0, 1\}^{168} \times \{0, 1\}^{64} \rightarrow \{0, 1\}^{64} \)

3DES2 : \( \{0, 1\}^{112} \times \{0, 1\}^{64} \rightarrow \{0, 1\}^{64} \)

are defined by

\[
3DES3_{K_1 || K_2 || K_3}(M) = DES_{K_3}(DES_{K_2}^{-1}(DES_{K_1}(M)))
\]

\[
3DES2_{K_1 || K_2}(M) = DES_{K_2}(DES_{K_1}^{-1}(DES_{K_2}(M)))
\]

Meet-in-the-middle attack on 3DES3 reduces its “effective” key length to 112.
DESX

\[ DES_{KK_1K_2}(M) = K_2 \oplus DES_K(K_1 \oplus M) \]

- Key length = 56 + 64 + 64 = 184
- “effective” key length = 120 due to a $2^{120}$ time meet-in-middle attack
- No more resistant than DES to linear or differential cryptanalysis

Good practical replacement for DES that has lower computational cost than 2DES or 3DES.
Later we will see “birthday” attacks that “break” a block cipher 
\[ E : \{0, 1\}^k \times \{0, 1\}^\ell \rightarrow \{0, 1\}^\ell \] in time \( 2^{\ell/2} \)

For DES this is \( 2^{64/2} = 2^{32} \) which is small, and this is unchanged for 2DES and 3DES.

Would like a larger block size.
1998: NIST announces competition for a new block cipher

- key length **128**
- block length **128**
- faster than **DES** in software

Submissions from all over the world: MARS, Rijndael, Two-Fish, RC6, Serpent, Loki97, Cast-256, Frog, DFC, Magenta, E2, Crypton, HPC, Safer+, Deal
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2001: NIST selects Rijndael to be AES.
function $\text{AES}_K(M)$

$(K_0, \ldots, K_{10}) \leftarrow \text{expand}(K)$

$s \leftarrow M \oplus K_0$

for $r = 1$ to $10$ do

$s \leftarrow S(s)$

$s \leftarrow \text{shift-rows}(s)$

if $r \leq 9$ then $s \leftarrow \text{mix-cols}(s)$ fi

$s \leftarrow s \oplus K_r$

end for

return $s$

- Fewer tables than DES
- Finite field operations
No key-recovery attack better than EKS is known, and latter is prohibitive for 128 bit keys.
Adversary $A$ knows $E: \{0, 1\}^k \times \{0, 1\}^\ell \to \{0, 1\}^\ell$

$T \leftarrow \{0, 1\}^k$ is the target key.

Given: $(M_1, C_1), \ldots, (M_q, C_q)$ where $C_i = E(T, M_i)$ for $i = 1, \ldots, q$ and $M_1, \ldots, M_q$ are distinct.

Find: $T$

So far, a block cipher has been viewed as secure if it resists key recovery, namely if there is no efficient way to solve the above problem.
Limitations of security against key recovery

Is security against key recovery enough?
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Aliens from planet Crypton have a (new) cipher

\[ A : \{0, 1\}^{128} \times \{0, 1\}^{128} \rightarrow \{0, 1\}^{128} \]

that is guaranteed to resist key recovery. Would you use it encrypt?
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The cipher is:

\[ A_K(M) = M \]

- Impossible to find key from input-output examples, but
- Encryption is insecure because given ciphertext I know plaintext.
Possible reaction: But DES, AES are not designed like A, so why does this matter?
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**Answer:** It tells us that security against key recovery is not, as a block-cipher property, sufficient for security of uses of the block cipher.
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Answer: It tells us that security against key recovery is not, as a block-cipher property, sufficient for security of uses of the block cipher.

As designers and users we want to know what properties of a block cipher give us security when the block cipher is used.
So what is a “good” block cipher?

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We can’t define or understand security well via some such (indeterminable) list.

We want a single “master” property of a block cipher that is sufficient to ensure security of common usages of the block cipher.