## CS/ECE 252: INTRODUCTION TO COMPUTER ENGINEERING

# UNIVERSITY OF WISCONSIN—MADISON 

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Midterm Examination 2
In Class (50 minutes)
Friday, October 24, 2014
Weight: 17.5\%

NO: BOOK(S), NOTE(S), OR CALCULATORS OF ANY SORT.

The exam has nine pages. Circle your final answers. Plan your time carefully since some problems are longer than others. You must turn in the pages 1-9. Use the blank sides of the exam for scratch work.

LAST NAME: $\qquad$
FIRST NAME: $\qquad$
ID\#: $\qquad$

## Problem 1

Consider the logic equation $Z=B$ OR (A AND C).
a. (1 point) Draw a gate-level circuit for Z using NOT gates and 2-input AND/OR gates.

b. (1 points) Draw a gate-level circuit for $Z$ using only 2-input NAND gates.

c. (2 points) Implement a logic circuit for $Z$ using a $4 \times 1$ multiplexer where $B$ and $C$ are connected to the select lines. Draw your answer using the $4 \times 1$ multiplexer below. Do not use any additional logic gates.


## Problem 2

(6 Points)
The finite state machine (FSM) below (in Figure 1(a)) recognizes a certain bit sequence. The machine takes one input every clock cycle, which can be 1 or 0 . The machine outputs a ' 1 ' when this certain bit sequence is recognized; otherwise it outputs a ' 0 '. Each state is represented as $S_{1} S_{0}$. For example, the state marked as " 01 " has $S_{1}=0$, and $S_{0}=1$. $X$ is the output in each state. $S_{1}$ ' $S_{0}$ ' represents the next state and the labels on each transition is the input value that triggers the transition. Assume that the initial state is 00 .

(a)


| S1 | SO | IN | $\mathbf{S 1}^{\prime}$ | SO' $^{\prime}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 |

Figure 1
a. (2 points) Complete the Next State truth table above for the FSM.
b. (3 Points) Draw the logic circuit which implements the above FSM using combinational logic and flip flops. Use representation shown in figure 1(b) for any 1-bit flip flop required in the circuit (where CLK is the clock). You can use any kind of logic gates for implementing the combinational logic. Note: You should implement both the states $(\mathrm{S} 1, \mathrm{~S} 0)$ as well as the output (X).

c. (1 point) Which bit sequence does the above FSM recognize? Your answer should be a string of bits (e.g, "1001" or "11001").

110

## Problem 3

(3 Points)
Consider the following circuit containing a multiplexer, a single-bit full adder and a decoder. X and Y are inputs to this circuit, and the circuit produces an output Z . Fill in the truth table below for this combinational circuit.


| $X$ | $Y$ | $Z$ |
| :---: | :---: | :---: |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

Consider the logic equation $Z=\operatorname{NOT}(A$ AND NOT(NOT(C) AND NOT(B))). (Hint: You may want to use DeMorgan's law to simplify the equation.)
a. (2 points) Fill out the following truth table for $Z$.

| $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{Z}$ |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |

b. (3 points) Label the inputs on the transistor-level circuit below so that the circuit implements the logic function $Z$.


## Problem 5

Suppose a 32-bit instruction takes the following format:

| OPCODE | SR | DR | UNUSED | IMM |
| :--- | :--- | :--- | :--- | :--- |

Where SR = source register, DR= destination register, $\mathrm{IMM}=$ immediate value

Assuming that there are 400 opcodes and the IMM field is 11-bits wide, answer the following:
a. (1 point) What is the minimum number of bits required to represent the OPCODE?

$$
2^{9}=512>400, \text { so } 9 \text { bits }
$$

b. (1 point) If the source register (SR) and destination register (DR) each have 5 bits, what is the maximum number of registers supported by this instruction set architecture?

```
25}=32\mathrm{ registers
```

c. (1 point) If the UNUSED bits were instead used for the register fields (SR and DR), how many more registers could we have in our computer?

32-9-5-5-11=2 bits in the UNUSED field. This means we can give 2/2=1 bit to each the SR and DR field. So, we could represent $2^{6}-2^{5}=32$ more registers
a. (1 point) What is the addressability (number of bytes per memory location) of a 1024 byte memory which uses 9 bits for each memory address? Show your work.

$$
\begin{gathered}
1024 \text { bytes }=2^{10}=2^{10} \times 2^{3} \text { bits } \\
2^{13} \text { bits } \div 2^{9} \text { bits }=2^{4} \text { bits }=16 \text { bits }
\end{gathered}
$$

b. (2 points) How many different n-input Boolean functions are possible? Show your work.

Given n inputs, $\mathbf{2}^{2^{\mathrm{n}}}$ functions are possible.
c. (1 point) How many select lines does an n-input mux have?

Let $\mathbf{n}=2^{y}$, since we know $\mathbf{n}$ must be a power of 2 .
The number of select lines is $y$, or $\log _{2} n$.

$$
\gg \log _{2} n
$$

d. (1 point) How many outputs does an n-input decoder have?

A decoder has n inputs and $2^{\mathrm{n}}$ outputs
e. (1 point) Mention two important things that happen during the FETCH phase of the instruction cycle.

Increment PC
Read the instruction, store in IR
f. (2 points) How many n-type transistors are present in a 16-bit wide register? Show your work.

Each Gated D latch has 4 NAND gates and 1 inverter. Each NAND gate has 2 n-type transistors, and each inverter has 1 n-type transistor.

Therefore, we have $16 * 4 * 2+16=144$ n-type transistors.
g. (1 Point) Which of the following stages of instruction processing are required for the processing of an ADD instruction which reads value of 2 registers and stores the final value into another register. Circle all that apply:

Fetch
Decode
Evaluate address
Fetch operands from the memory
Execute Operation
Store result

