NCNA 2017 Solution Slides

NCNA Judges

Problem Set Developers

- Dr. Larry Pyeatt (Chief Judge)
- Bryce Sandlund (Associate Chief Judge)
- Robert Hochberg
- Bowen Yu
- Bruce Elenbogen
- Ivor Page
- Antonio Molina
- Menghui Wang
- Andrew Morgan
- ECNA 2017 Developers (IsaHasa and Sheba's Amoeba's), specifically John Bonomo and Bob Roos
- The Kattis Team, specifically Greg Hamerly and Fredrik Niemela
- NWERC and SWERC, to which these slides were modeled off of

H - Zebras and Ocelots

Problem

Given a vertical stack of Zebras (Z's) and Ocelots (O's), determine how many steps until they all turn into Z's, given that at each step, the lowest O turns into a Z, and all Z's below it turn into O's.

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- Simulation gets TLE.

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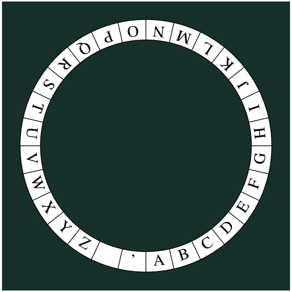
Statistics: 824 submissions, 112 accepted.

I - Racing Around the Alphabet

Problem

Find the total time to pick up all letters of a phrase, running around a circular disk.



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Statistics: 170 submissions, 114 accepted.

G - Sheba's Amoebas

Problem

Count the number of amoebas contained entirely within one another in a 2D grid.

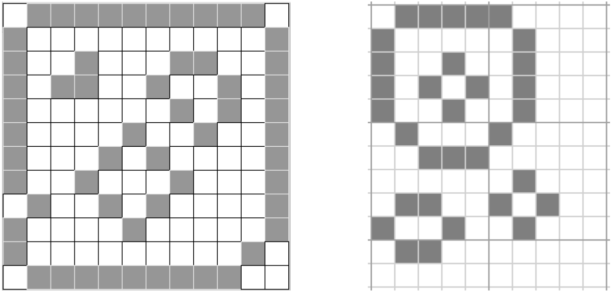

Figure: Two Petri dishes, each with four amoebas.

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Run a modified flood fill:

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Statistics: 143 submissions, 77 accepted.

C - Urban Design

Problem

Given a set of infinite lines in the 2D plane and queries that consist of two regions defined by points within these regions, determine if these regions should get different or same designations, given that regions immediately across a line from one another get different designations.

Solution

- Think of the regions as lines are added into the plane one-by-one.

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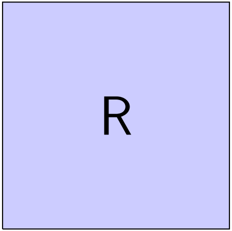

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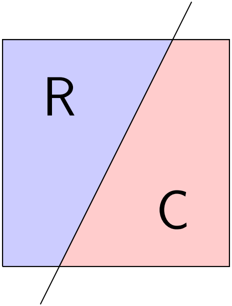

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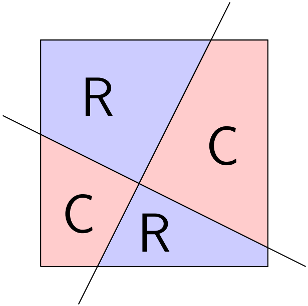

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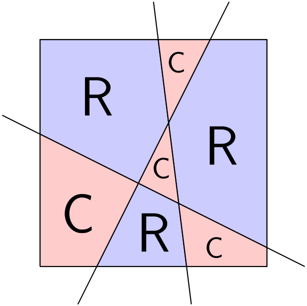

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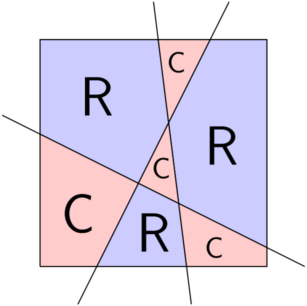

As you pass through a line, the designation of the region changes.

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Statistics: 128 submissions, 25 accepted.

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Problem

Given the distance between every pair of nodes in a weighted tree, recover the tree.

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Statistics: 131 submissions, 17 accepted.

B - Pokemon Go Go

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- Time complexity: $O(n^2 2^D)$, where D is the number of distinct pokemon.

B - Pokemon Go Go

Statistics: 73 submissions, 7 accepted.

E - Is-A? Has-A? Who Knowz-A?

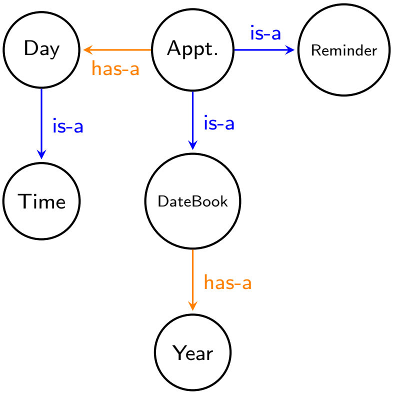
Problem

Given a set of is-a and has-a relationships, answer is-a and has-a queries, defined as follows:

1. A is-a B if and only if there is a path of is-a relationships from A to B
2. A has-a B if and only if there is a path of is-a and has-a relationships from A to B that includes at least one has-a relationship.

E - Is-A? Has-A? Who Knowz-A?

Can be modeled as a graph with two types of edges. Ex, Sample Input 1:



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- Alternatively, can preprocess all relationships via clever application of Floyd-Warshall. The algorithm is as follows:

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for (int k = 0; k < D; ++k) {
    for (int i = 0; i < D; ++i) {
        for (int j = 0; j < D; ++j) {
            is_a[i][j] = is_a[i][j] || (is_a[i][k] && is_a[k][j]);
            has_a[i][j] = has_a[i][j] || (has_a[i][k] && has_a[k][j]);
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Statistics: 215 submissions, 4 accepted.

A - Stoichiometry

Problem

Balance a chemical equation.

A - Stoichiometry

Solution

- Make a system of equations. Each coefficient is an unknown and for every unique atom, we get an equation, since the number of atoms of each type is preserved through the chemical reaction.

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- Define matrix A where A_{ij} = number of atoms of type i in molecule j.

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- Ex, Sample Input 1:

$$\_H_2O + \_CO_2 \rightarrow \_O_2 + \_C_6H_{12}O_6$$

yields

$$\begin{matrix} H \\ O \\ C \end{matrix} \begin{bmatrix} 2 & 0 & 0 & -12 \\ 1 & 2 & -2 & -6 \\ 0 & 1 & 0 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

A - Stoichiometry

Solution

- Put A in reduced row-echelon form, ex:

$$A_{rref} = \begin{matrix} H \\ O \\ C \end{matrix} \begin{bmatrix} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & -6 \end{bmatrix}$$

This requires us to work over a field. Input is small so floating point error is negligible, therefore we can use doubles.

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Statistics: 5 submissions, 2 accepted.

F - Atlantis

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- Observation: Given a set of stores to visit, the best order in which to visit them is in increasing order of h_i.
- Observation: If we prefer stores with smaller t_i, we leave more room for other stores to be visited.

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- Greedy algorithm: Sort stores by increasing t_i. Maintain a feasible solution F. Add store i to F if doing so does not destroy feasibility.

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- Greedy algorithm: Sort stores by increasing t_i. Maintain a feasible solution F. Add store i to F if doing so does not destroy feasibility.
- Checking feasibility in $O(\log n)$ time may require a lazy segment tree or balanced binary search tree.
- Advanced data structures can be avoided if you are clever.

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$O(n \log n)$ Solution with c++ set

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$O(n \log n)$ Solution with c++ set

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2. To see if store i can be added to F, we iterate down the tree starting at the first store scheduled to end before h_i, removing the pairs and keeping track of the sum of t_j's of the removed intervals.

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3. If for any j, $\sum_j t_j + t_i \leq h_i$, we can add store i. Insert $(h_i, \sum_j t_j + t_i)$ back into the tree.

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If we add store i, inserting $(h_i, \sum_j t_j + t_i)$ into the tree is equivalent to scheduling store i right before h_i and pushing everything else earlier to make room.

If we do not add store i, we will not be able to add any store i', $i' > i$ before time h_i, so where stores before h_i are scheduled in F is no longer relevant.

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$O(n \log n)$ Solution with c++ set

- Time complexity of checking feasibility in this approach: $O(\log n \cdot (\#\text{ of intervals removed from the tree} + 1))$.

F - Atlantis

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- Time complexity of checking feasibility in this approach: $O(\log n \cdot (\#\text{ of intervals removed from the tree} + 1))$.
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- Overall time complexity: $O(n \log n)$.

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Without loss of generality assume each t_i is distinct. Let $add(i)$ be a true or false value denoting whether store i is added in this strategy. Then

$$add(i) = \sum_{\substack{j \text{ s.t. } t_j < t_i \\ \text{and } add(j)}} t_j + t_i \leq h_i.$$

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Statistics: 72 submissions, 0 accepted.

D - Smooth Array

Problem

Given an array A of N integers, determine the minimum number of changes in A to make every contiguous subarray of length K sum to S.

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Solution

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- And let $cost(i, v) :=$ number of changes in A to make $A_i, A_{i+K}, A_{i+2K}, \ldots$ equal to v.

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- There are $O(KS)$ states and each takes $O(S)$ time to evaluate, so the complexity is $O(KS^2)$. # of iterations: $5000^3 = 125 * 10^9 \Rightarrow$ TLE!

An $O(NS)$ Solution

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Statistics: 37 submissions, 0 accepted.

Questions? Comments? Concerns? Email Bryce Sandlund: bcsandlund@uwaterloo.ca.