Radiosity

- Used when the outgoing angle doesn't matter much (e.g. cotton cloth)

Radiosity is the total power leaving a point on a surface per unit area on the surface

$$B(\boldsymbol{x}) = \int_{\Omega} L(\boldsymbol{x},\theta,\phi)\cos\theta d\omega$$

- For constant radiance:

$$B(\boldsymbol{x}) = \int_{\Omega} L_o(\boldsymbol{x})\cos\theta d\omega = \pi L_o(\boldsymbol{x})$$

Directional Hemispheric Reflectance

- Used for surfaces for which the light exiting does not depend on the exit direction

Directional Hemispheric Reflectance is the fraction of the incident irradiance in a given direction that is reflected by the surface

$$\rho_{dh}(\boldsymbol{x},\theta_i,\phi_i) = \int_{\Omega} \rho_{bd}(\boldsymbol{x},\theta_o,\phi_o,\theta_i,\phi_i)\cos\theta_o d\omega_o$$

Ideal Diffuse Surfaces

- For some surfaces, the amount of light reflected does not depend in the incoming direction (e.g. matte paints)
- Directional hemispheric reflectance is generally denoted ρ_d

$$\rho_{bd} = \frac{\rho_d}{\pi}$$

Exitance

- Measure of the power of light sources

> **Exitance** is the total power leaving a point on a surface per unit area on the surface

$$E(\boldsymbol{x}) = \int_{\Omega} L_e(\boldsymbol{x}, \theta, \phi) \cos\theta \, d\omega$$

- Note the similarity to radiosity
- For constant emitted radiance:

$$E(\boldsymbol{x}) = \int_{\Omega} L_e(\boldsymbol{x}) \cos\theta \, d\omega = \pi L_e(\boldsymbol{x})$$

Qualitative Reasoning

- Much can be deduced by considering what the world looks like from the point receiving the radiance.
- Examples:
  - Overcast sky, plane.
  - Overcast sky, plane, high wall.
  - Overcast sky, plane, half wall.
  - Point light source.
  - Area lights and shadows.

Global Illumination Equation

Emitted radiance

$$L(\boldsymbol{x}, \theta_o, \phi_o) = L_e(\boldsymbol{x}, \theta_o, \phi_o) + \int_{\Omega} \rho_{bd}(\boldsymbol{x}, \theta_o, \phi_o, \theta, \phi) L_i(\boldsymbol{x}, \theta, \phi) \cos\theta \, d\omega$$

Integral over incoming directions

Incoming radiance

Outgoing radiance

BRDF

Foreshortening factor

Assumptions of the Radiosity Method

- All surfaces and emitters are diffuse
  - Can convert Global Illumination Equation to use radiosity instead of radiance
- Outgoing radiosity is constant over a patch
  - Can discretize the integral equation

Radiance to Radiosity

- Recall: $B(\boldsymbol{x}) = \pi L_0(\boldsymbol{x})$
  $E(\boldsymbol{x}) = \pi L_e(\boldsymbol{x})$

$$\rho_{bd}(\boldsymbol{x}, \theta_o, \phi_o, \theta_i, \phi_i) = \frac{\rho_d(\boldsymbol{x})}{\pi}$$

- Simplifying gives:

$$B(\boldsymbol{x}) = E(\boldsymbol{x}) + \rho_d(\boldsymbol{x}) \underbrace{H(\boldsymbol{x})}_{\text{Irradiance}}$$

Diffuse Illumination Equation

- Use the fact that radiance is constant along lines
- Define $V(x, y) = \begin{cases} 1 & \text{if } x \text{ and } y \text{ are mutually visible} \\ 0 & \text{otherwise} \end{cases}$
- Then:

$$B(x) = E(x) + \rho_d(x) \int_{y \in S} B(y) \frac{\cos\theta \cos\theta'}{\pi r^2} V(x, y)\, dy$$

Discrete Formulation

- Assume world is broken into N disjoint patches, P_j, j=1..N, each with area A_j
- Define: $$B_i = \frac{1}{A_i}\int_{x \in P_i} B(\boldsymbol{x})dx$$ $$E_i = \frac{1}{A_i}\int_{x \in P_i} E(\boldsymbol{x})dx$$
- Then: $$B_i = E_i + \rho_i \sum_{j=1}^{N} F_{ij} B_j$$ **Radiosity Equation**

The Form Factor

$$F_{ij} = \frac{1}{A_i}\int_{x \in P_i}\int_{y \in P_j} \frac{\cos\theta \cos\theta'}{\pi r^2} V(x, y)dydx$$

F_{ij} is the proportion of the total power leaving patch P_i that is received by patch P_j

Form Factor Properties

- Depends only on geometry
- Reciprocity: $A_i F_{ij} = A_j F_{ji}$
- Additivity: $F_{i(j \cup k)} = F_{ij} + F_{ik}$
- Reverse additivity is not true
- Sum to unity: $\forall i, \sum_{j=1}^{N} F_{ij} = 1$

For Next Lecture

- Read Goral et. al. “Modeling the Interaction of Light Between Diffuse Surfaces” sections 1-2
- Read Kajiya “The Rendering Equation” sections 1-3
- Compare their formulations to the one presented in class (from Sillion and Puech)