Effectively Propositional Interpolants

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Effectively Propositional Logic (EPR)

$$\exists x_1 \ldots x_n \forall y_1 \ldots y_m \; \varphi$$

Quantifier-free

No function symbols

EPR

Decidable satisfiability!

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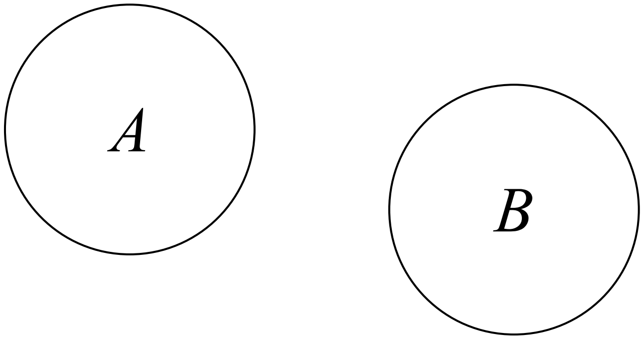
Expressive:

- Linked lists [Itzhaky et al. 2014]
- Software-defined networks [Ball et al. 2014]
- Parameterized distributed protocols [Padon et al. 2016]
- …

Interpolants

Given A and B such that

$$A \wedge B \text{ is unsatisfiable}$$



Interpolants

Given A and B such that

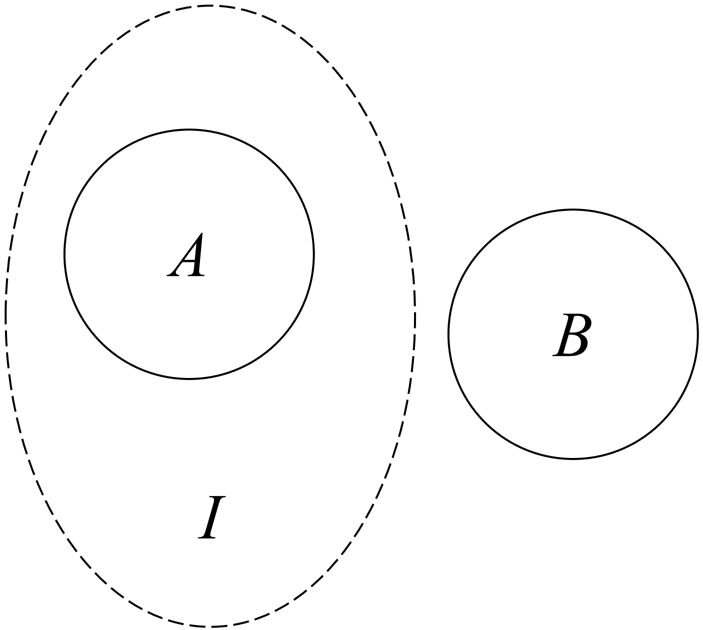
$A \wedge B$ is unsatisfiable

Find I such that

$A \rightarrow I$ is valid

$I \wedge B$ is unsatisfiable

I is in shared vocabulary (A, B)



Restricted Logics for Invariants

$$I(\vec{x}) \land T(\vec{x}, \vec{x}') \to I(\vec{x}')$$

is valid, or

$$I(\vec{x}) \land T(\vec{x}, \vec{x}') \land \neg I(\vec{x}')$$

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Bummer

1. ∃-logic: $\exists^* \varphi$
2. ∀-logic: $\forall^* \varphi$
3. AF-logic:

   boolean combinations of ∃-logic and ∀-logic

   ex: $(\exists^* \varphi_1 \wedge \forall^* \varphi_2) \vee \forall^* \varphi_3$

Models and Diagrams

$$\varphi = \exists a \forall b.\, p(a, b)$$

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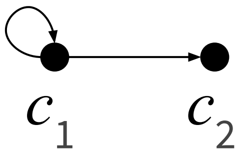
Model $m \models \varphi$



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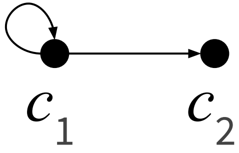

Diagram

$$diag(m) = \exists c_1, c_2 . c_1 \neq c_2 \\ \wedge\ p(c_1, c_1) \wedge \neg p(c_2, c_2) \\ \wedge\ p(c_1, c_2) \wedge \neg p(c_2, c_1)$$

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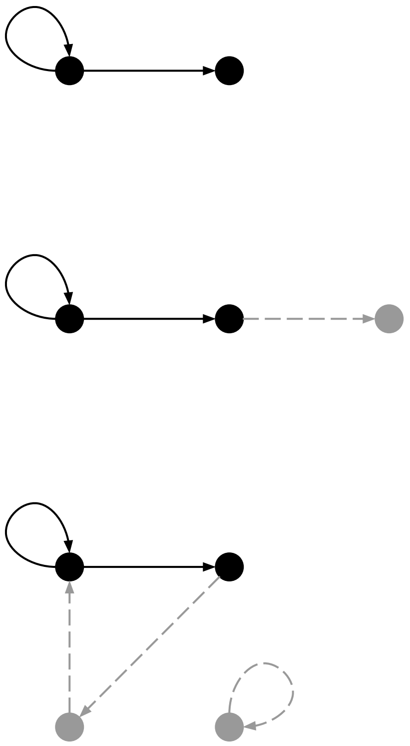

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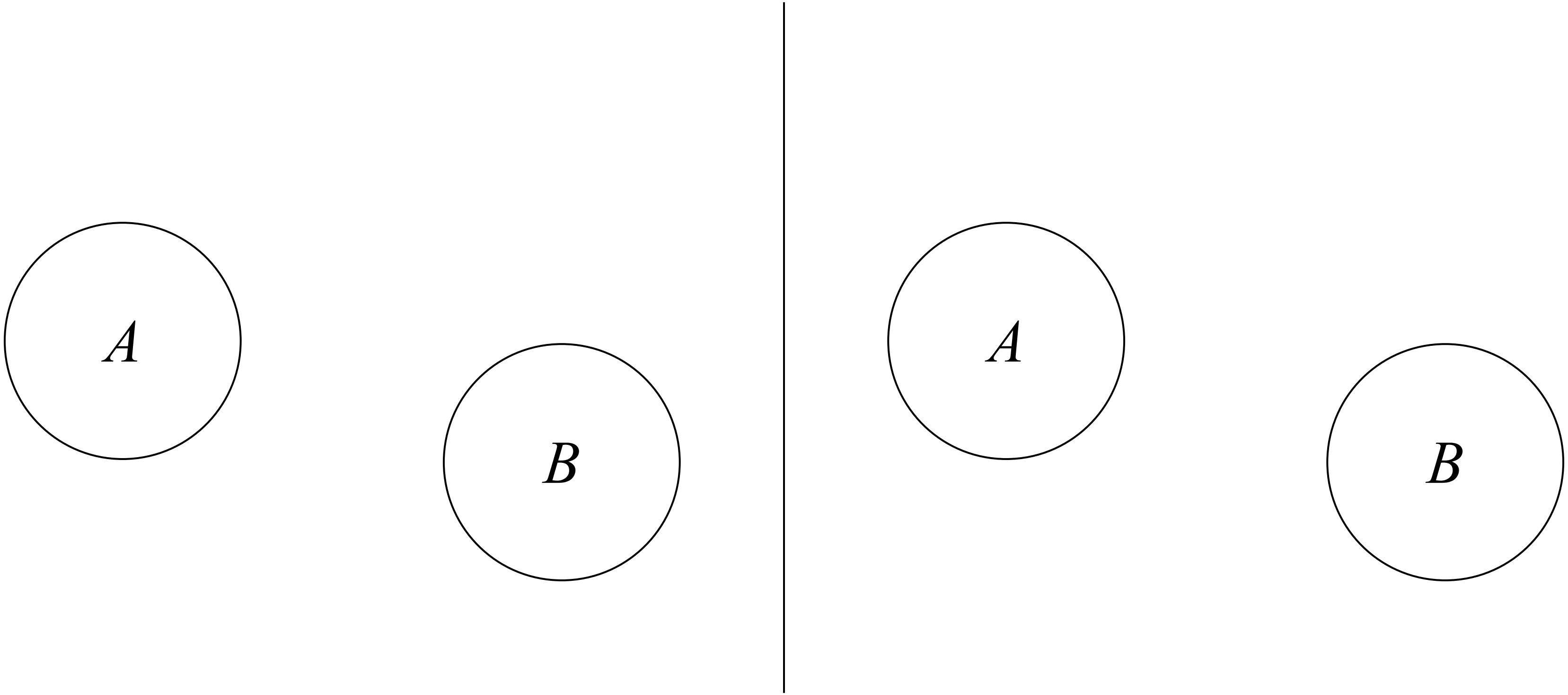

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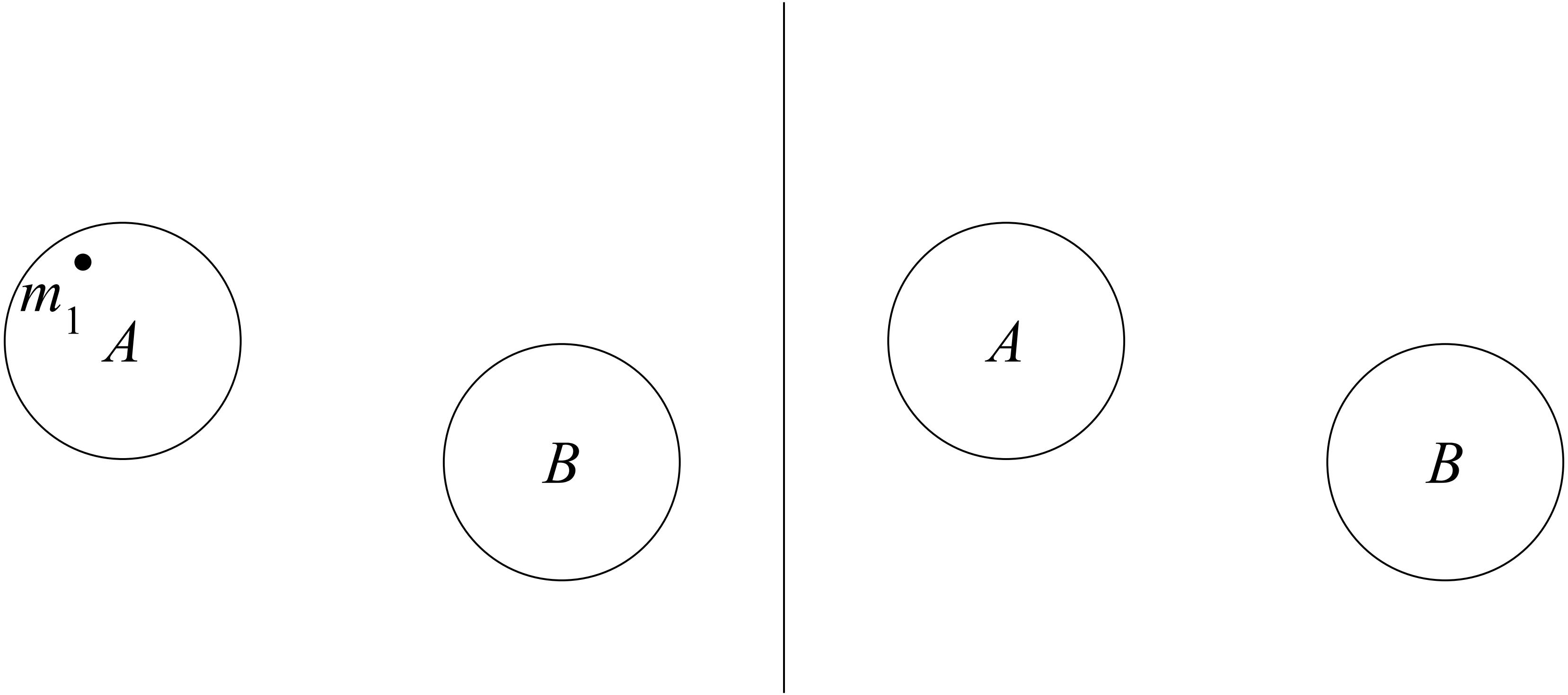

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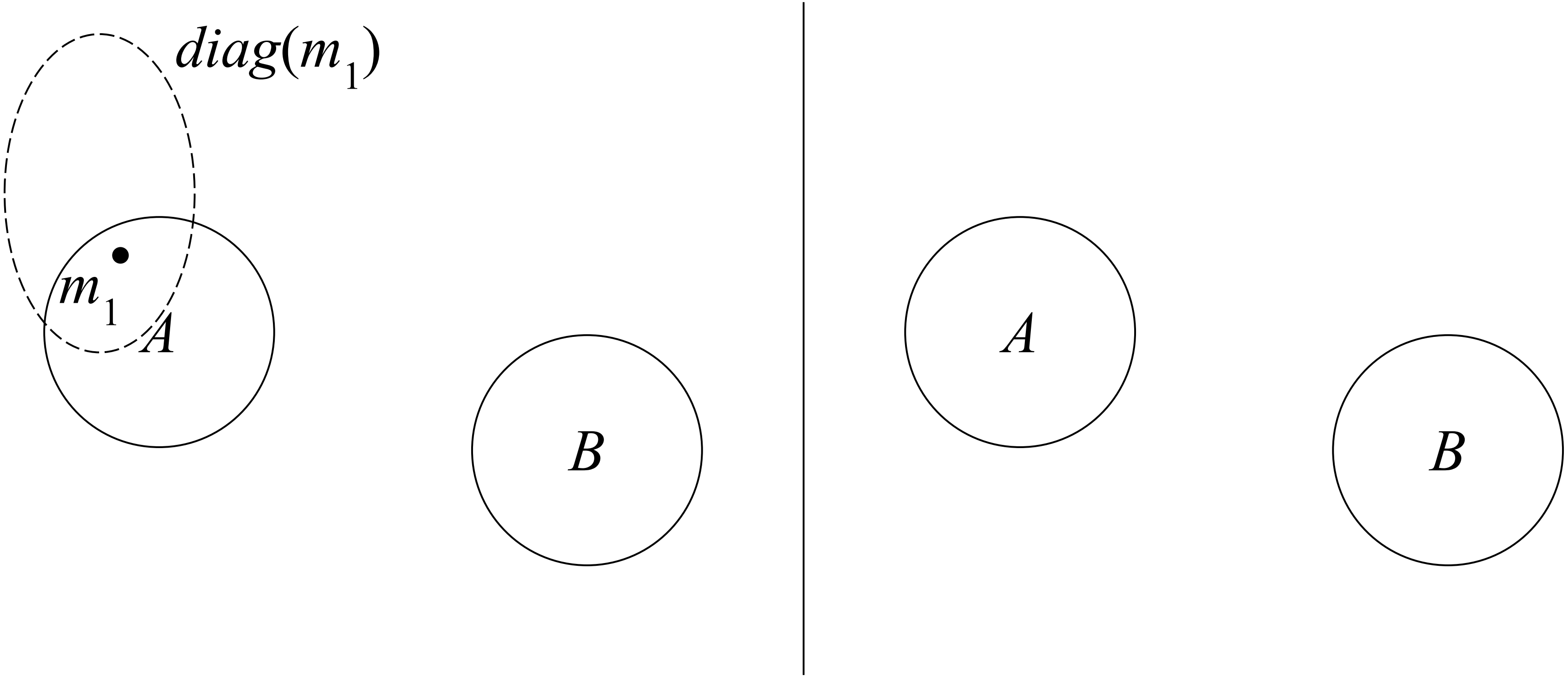

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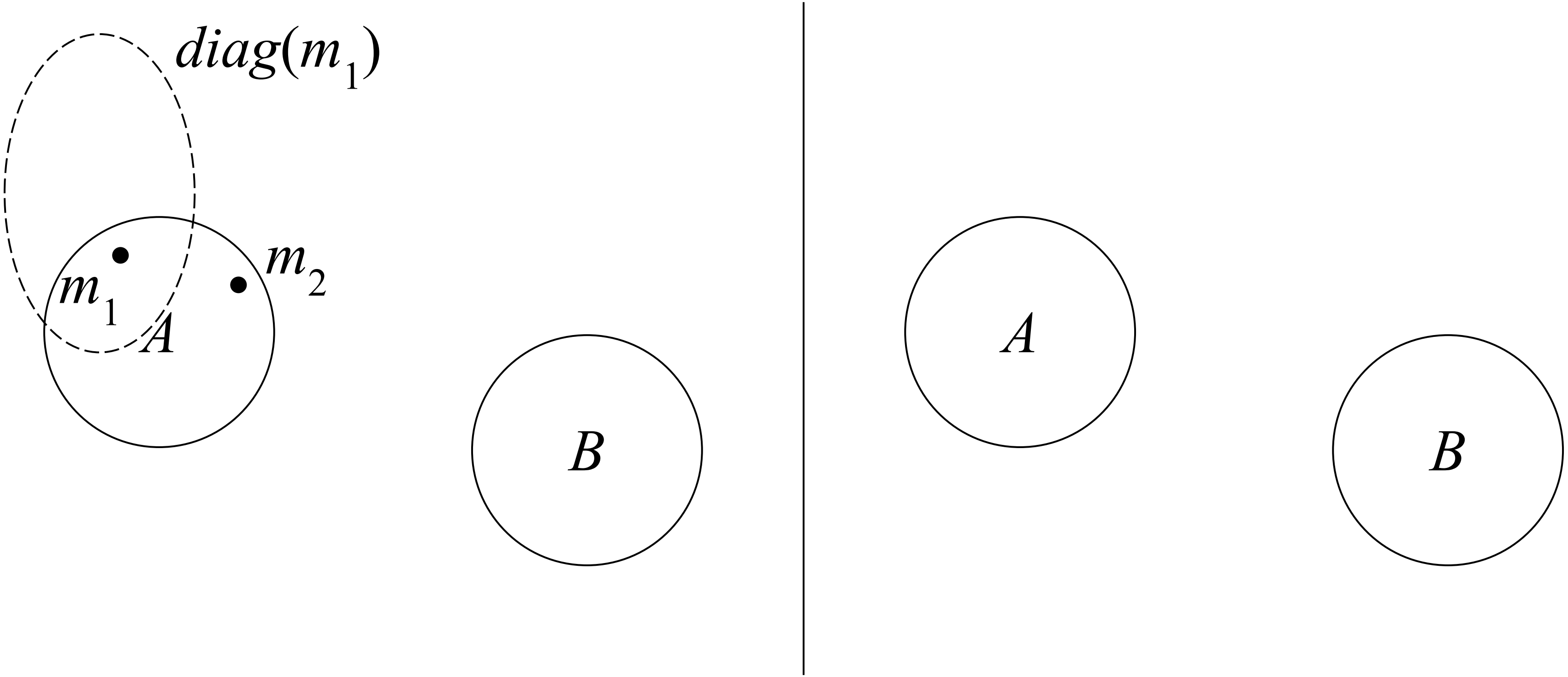

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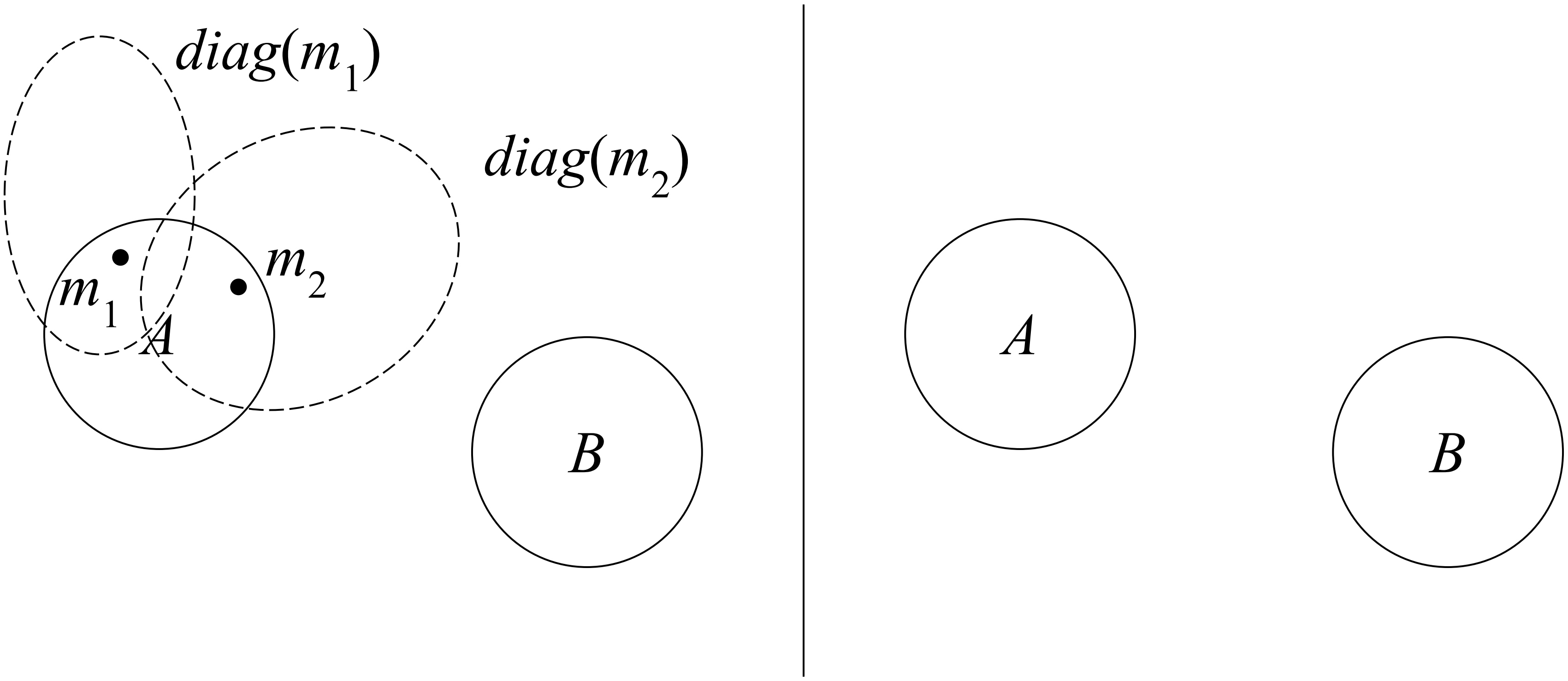

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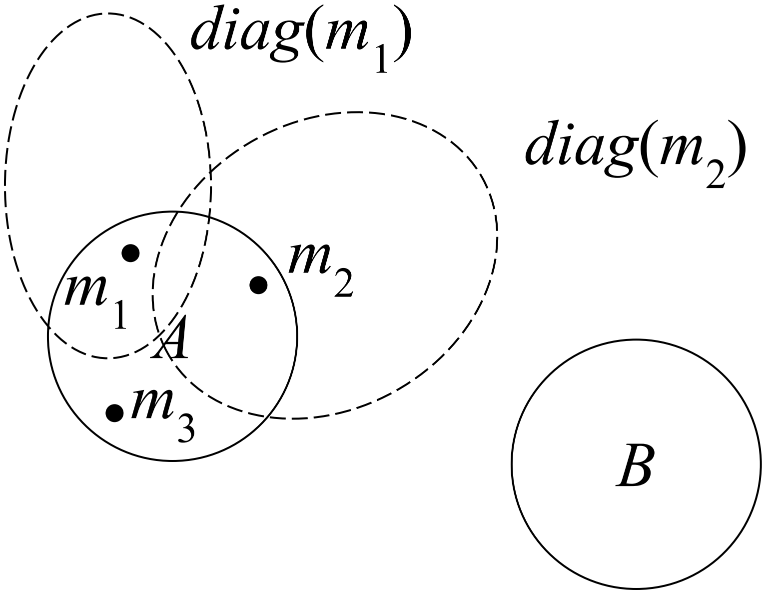

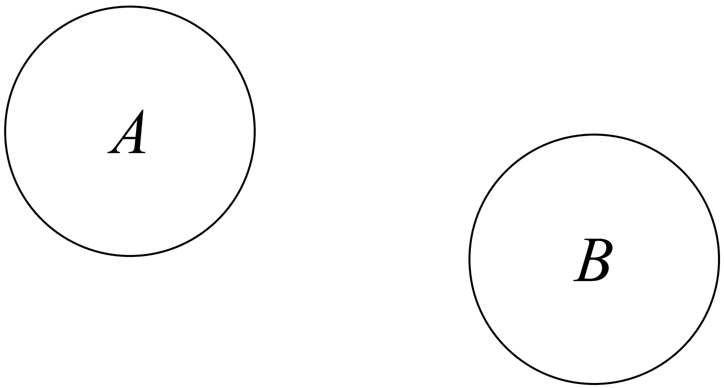

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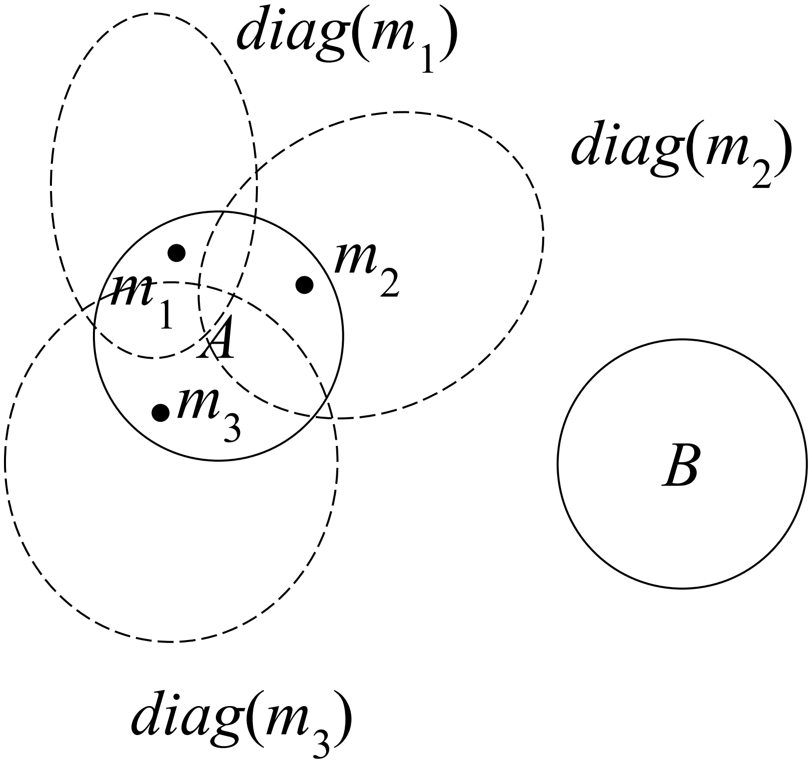

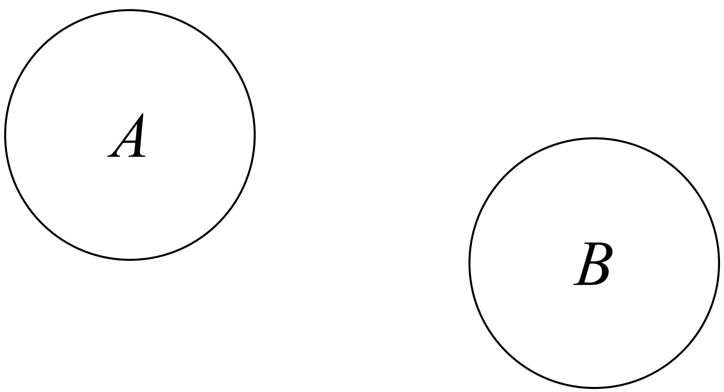

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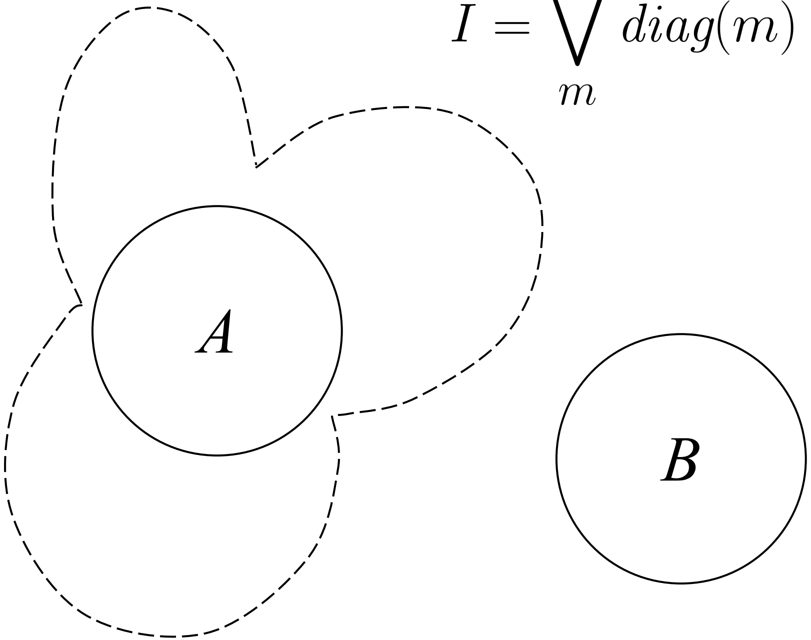

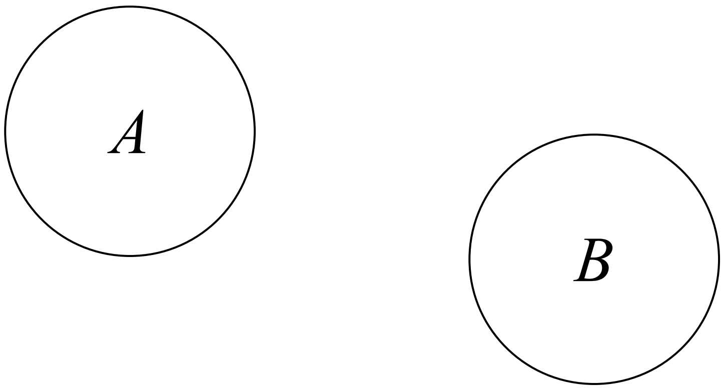

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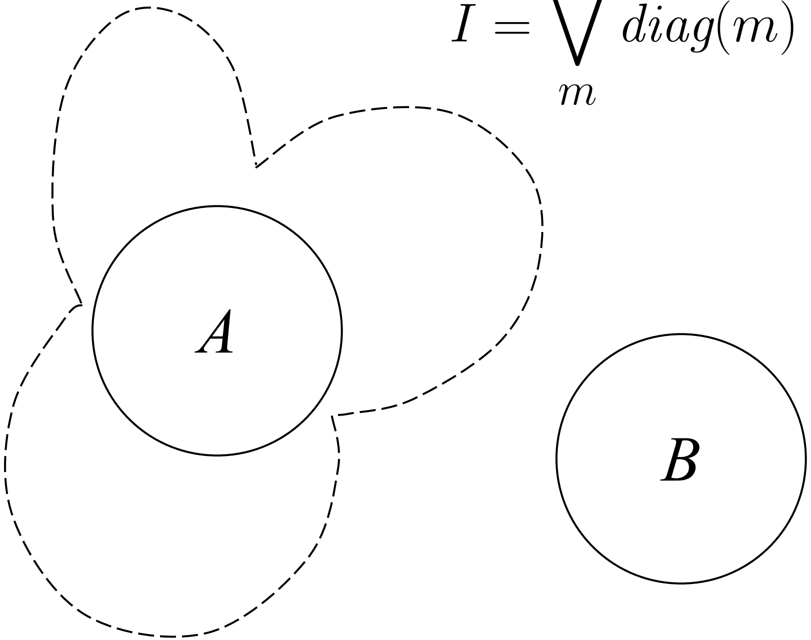

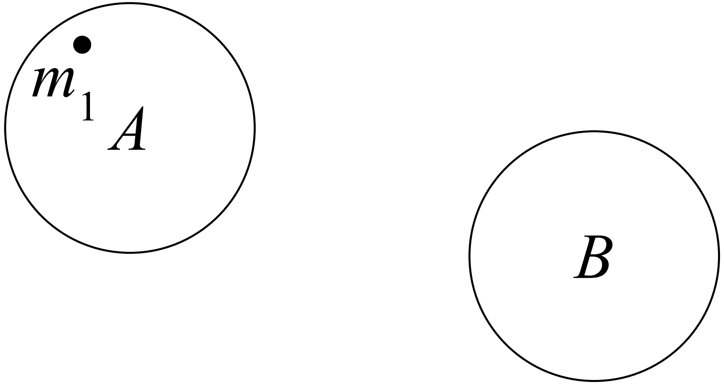

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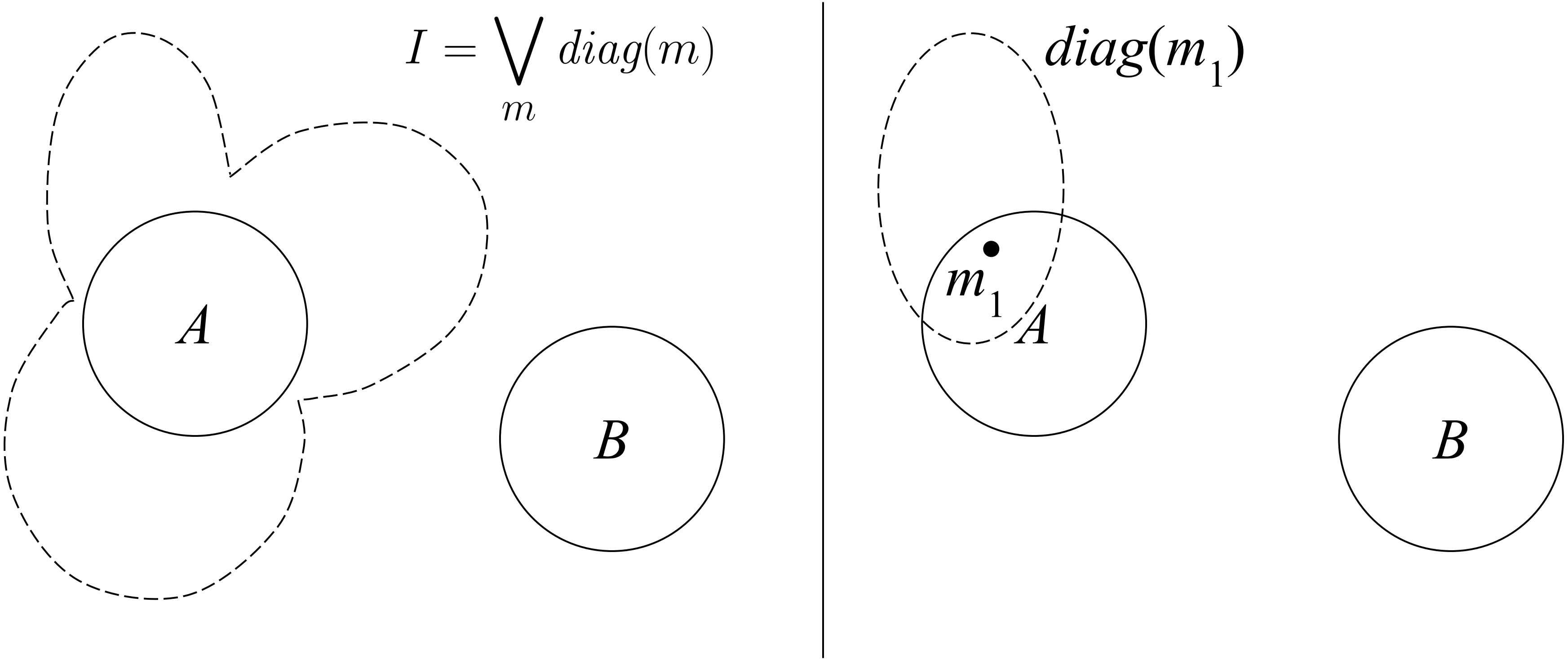

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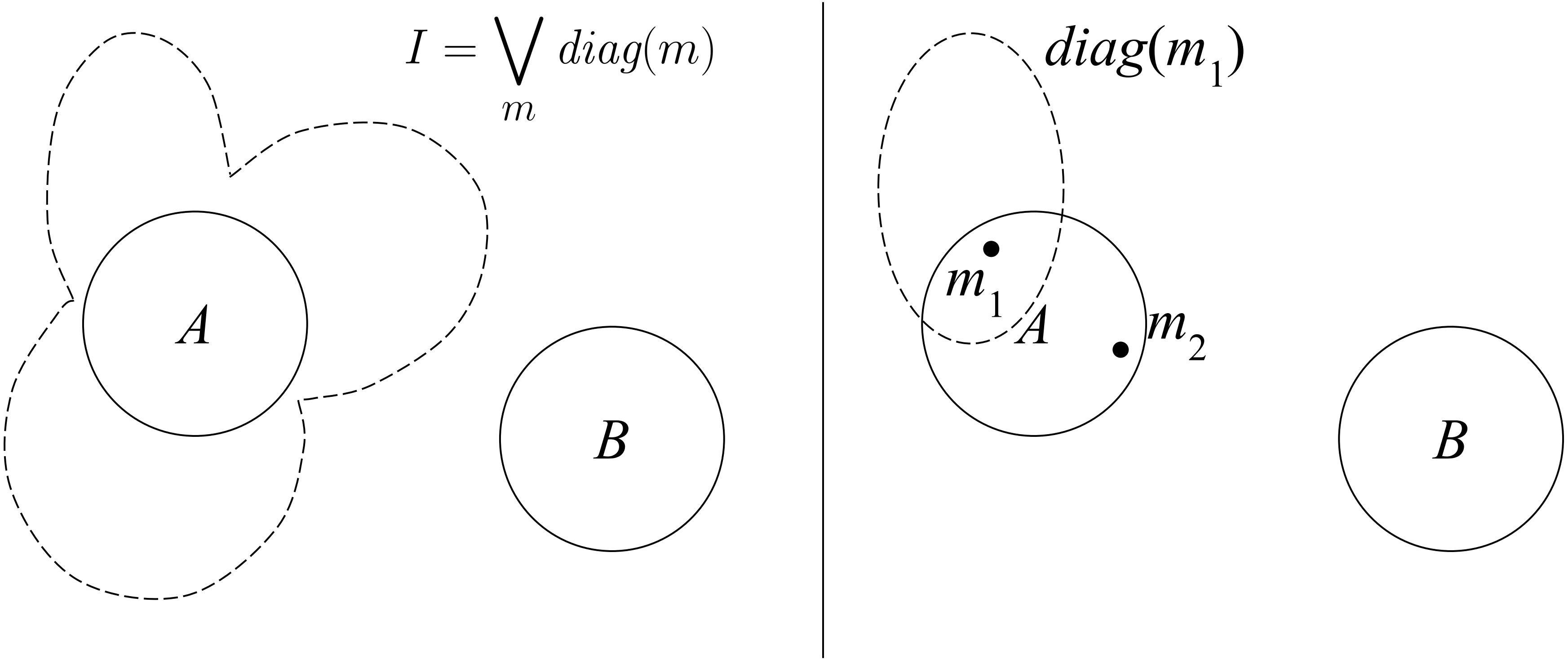

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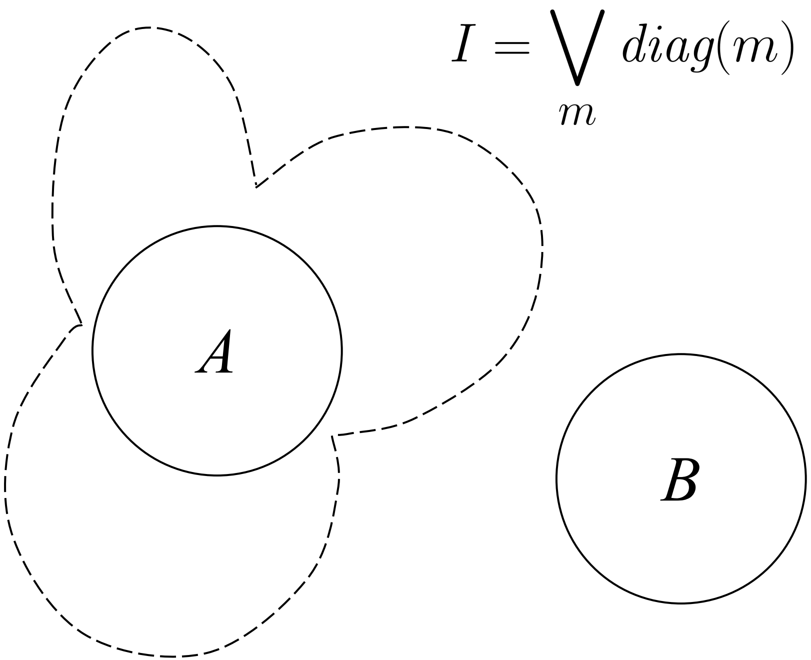

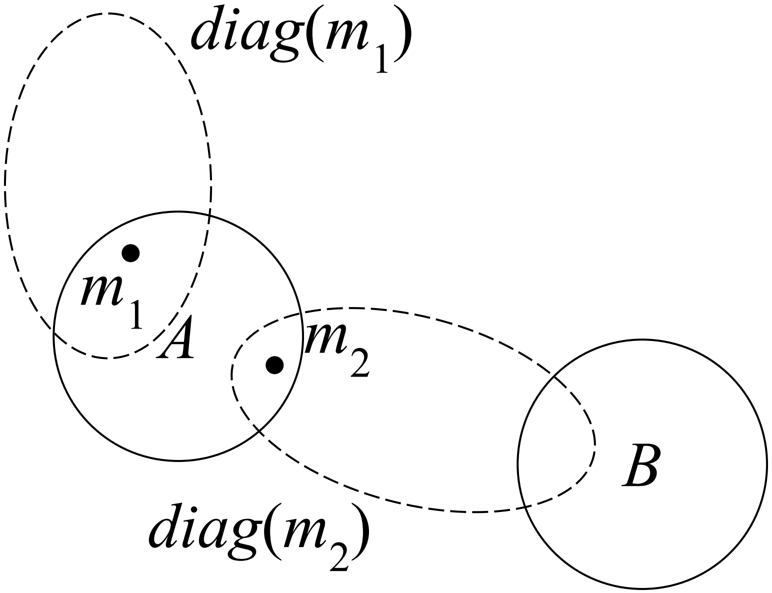

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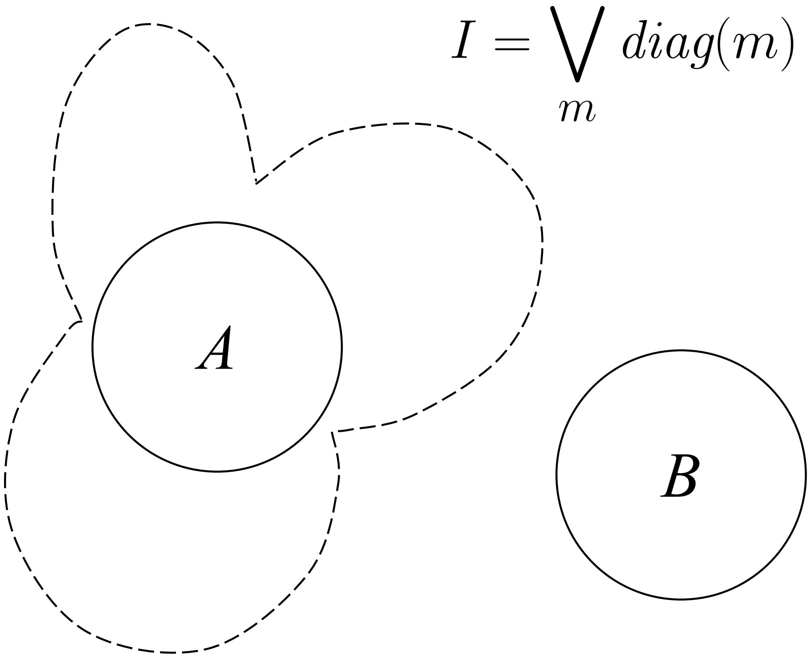

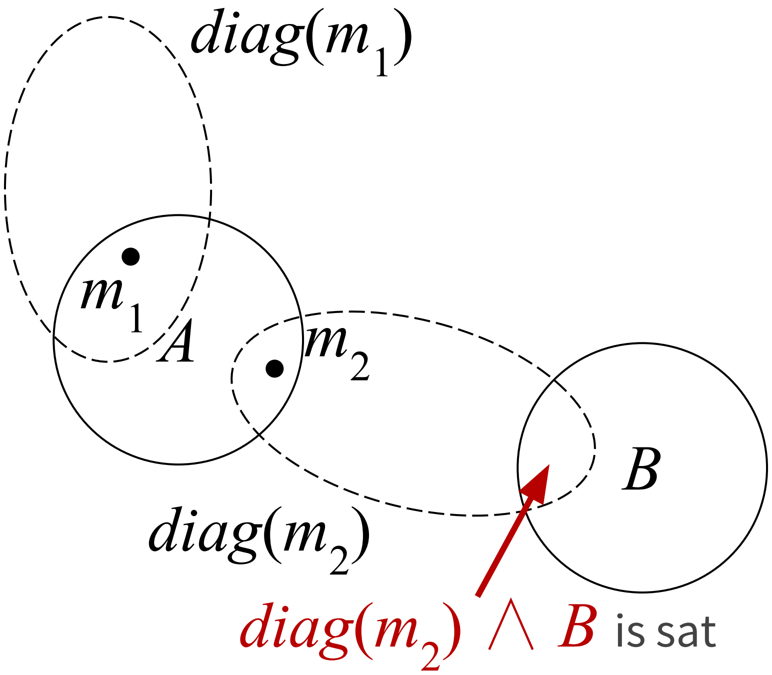

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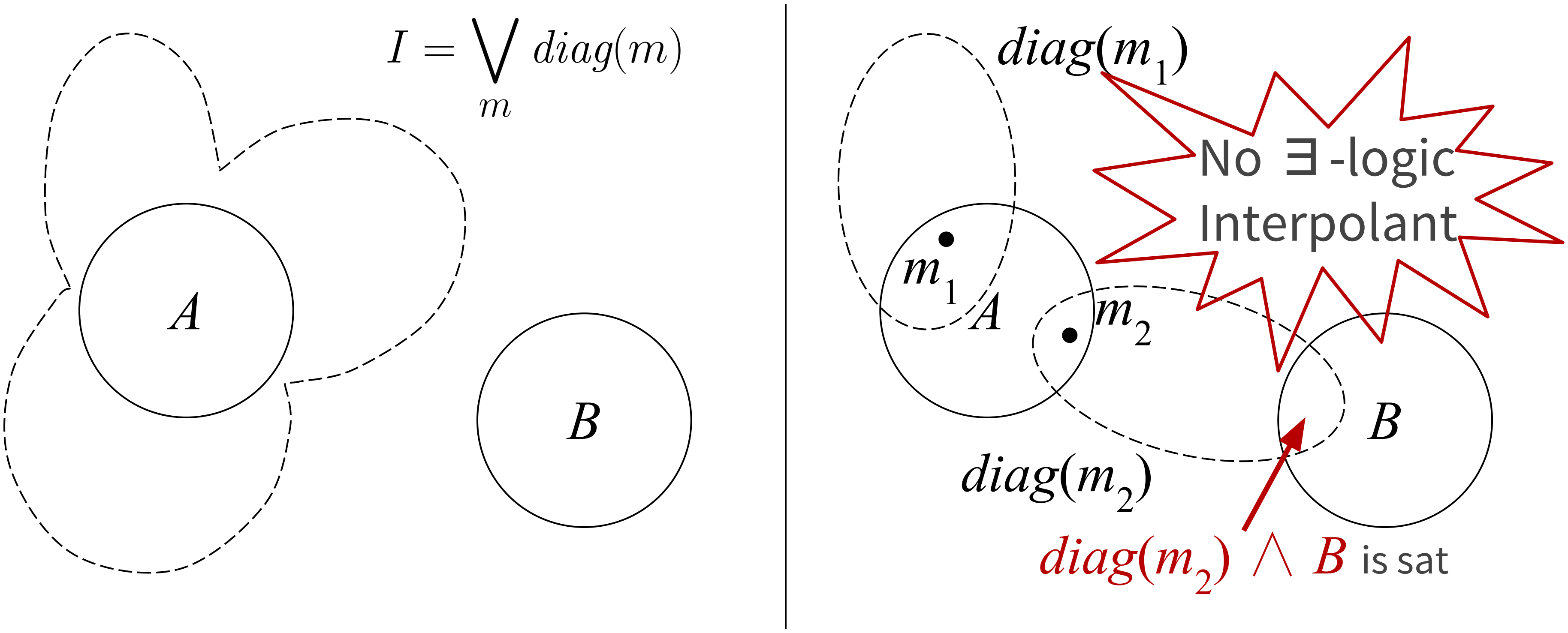

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- Returning I : interpolant by construction
- Returning $none$ is sound:
  $diag(m)$ is the strongest $\exists$ -logic formula that m models

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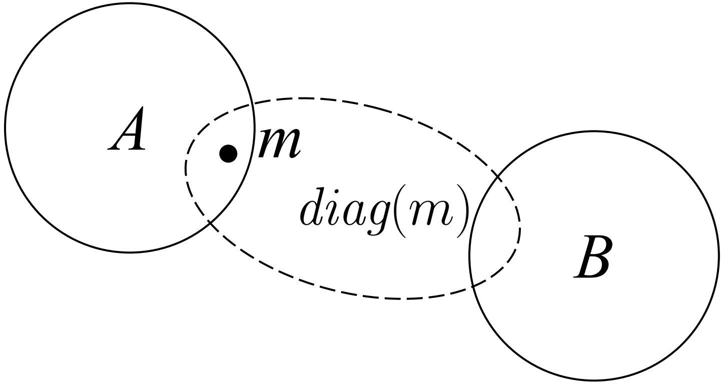

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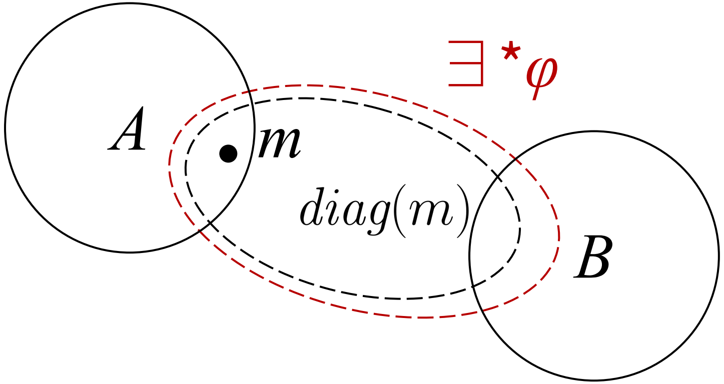

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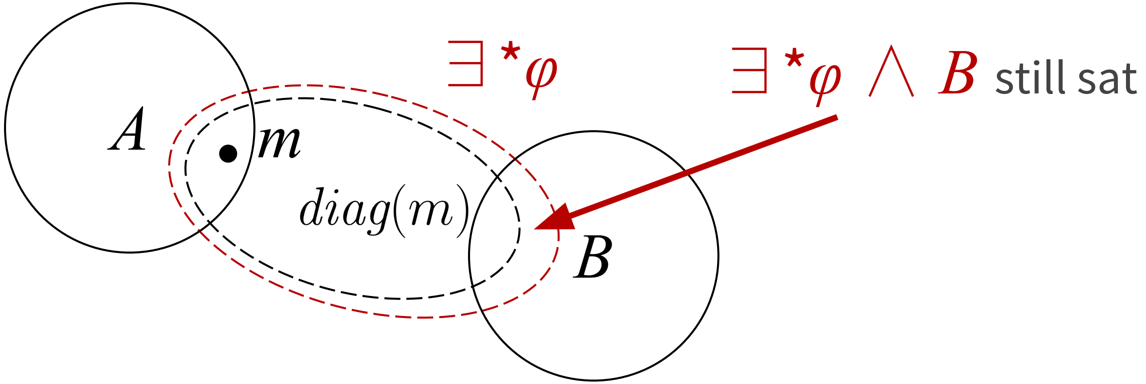

UITP Termination (and Completeness)

EPR small model property:

All EPR A have a bound k
such that $m \models A \rightarrow \exists m_{small}$:

- $m_{small} \models A$
- $m_{small} \subseteq m$
- $|m_{small}| \leq k$

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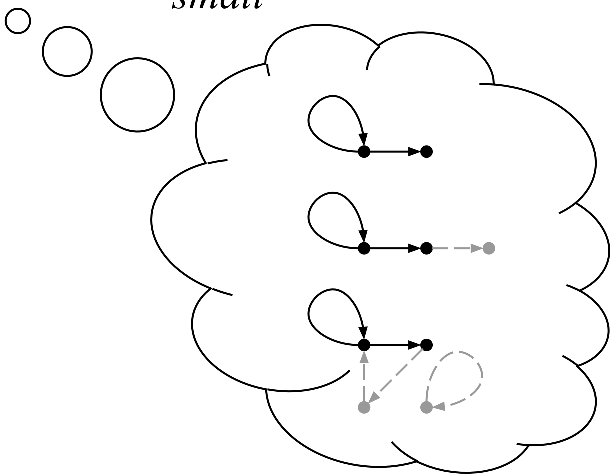

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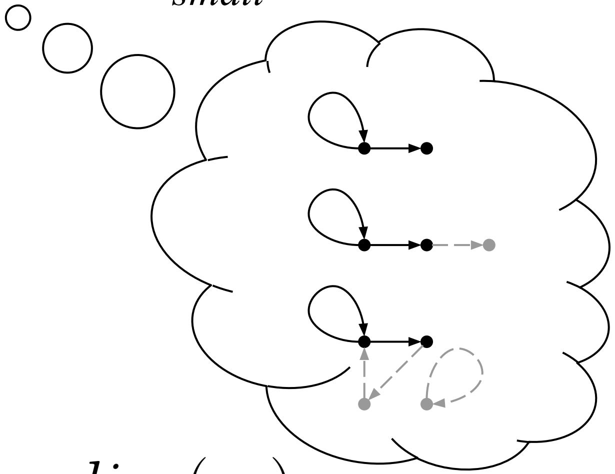

$$A \rightarrow \bigvee_{m \models A : |m| \leq k} diag(m)$$

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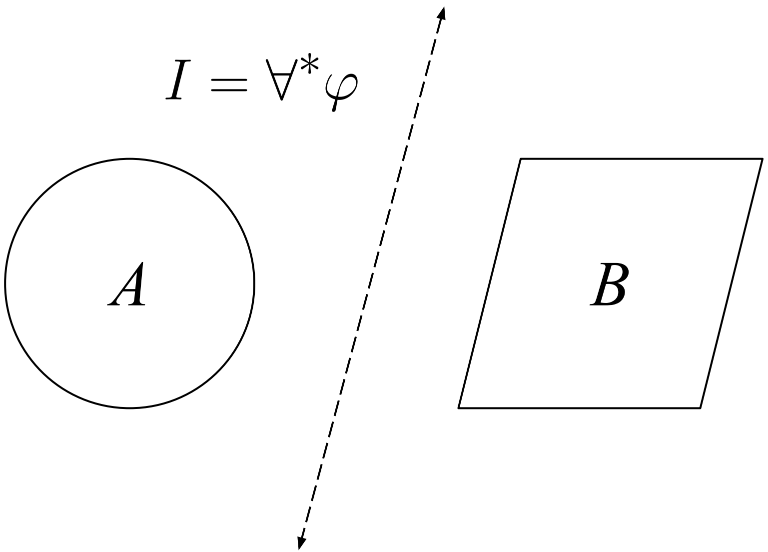

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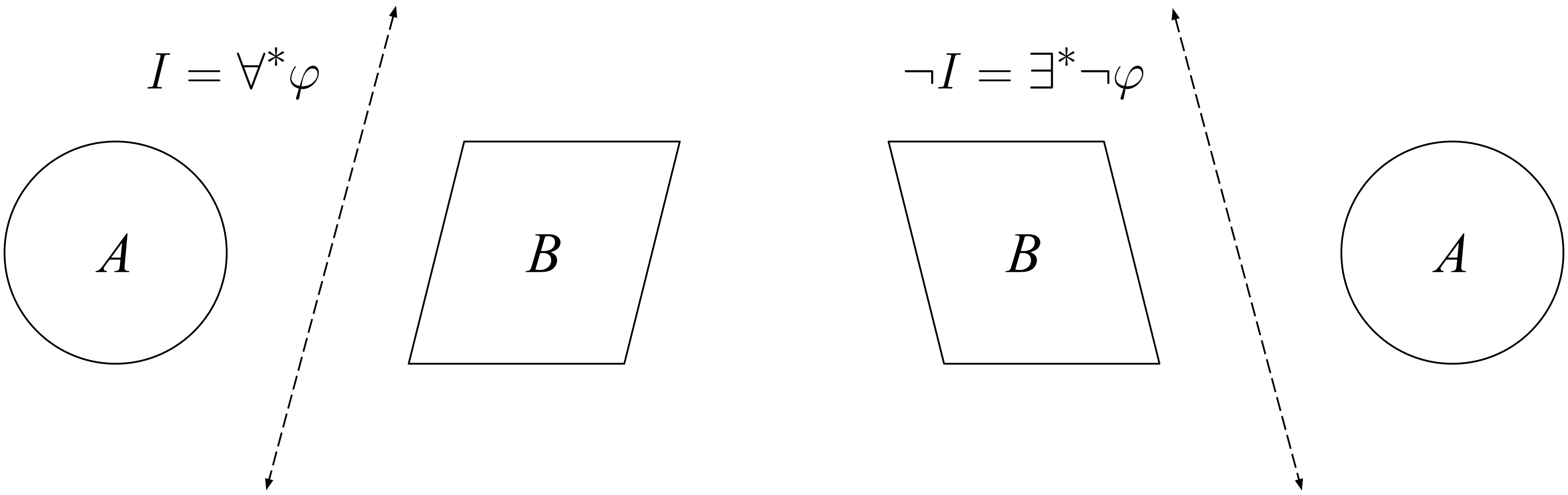
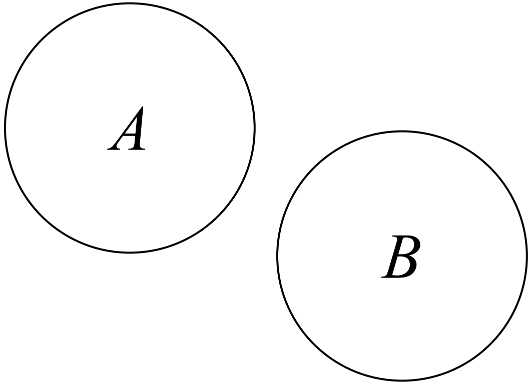


BITP: for AF-Logic Interpolants



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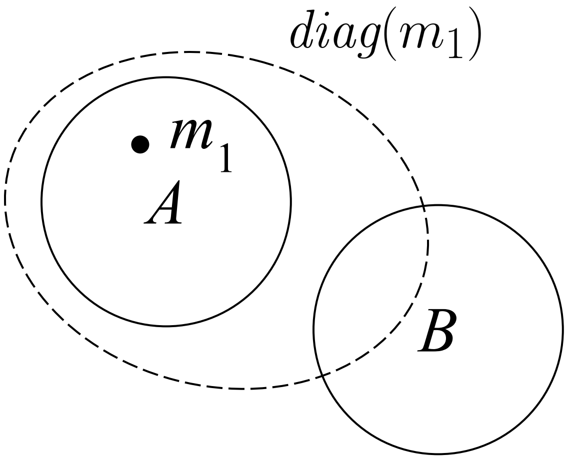

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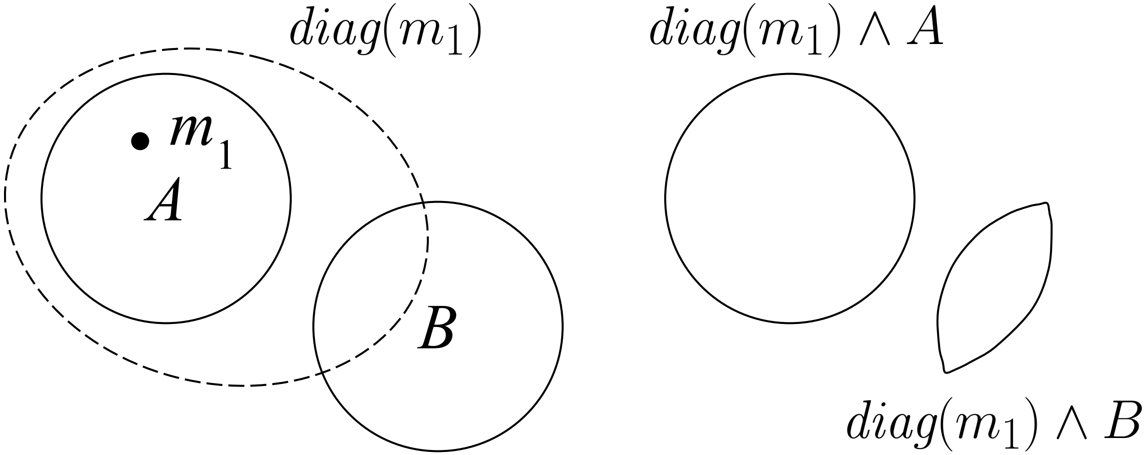

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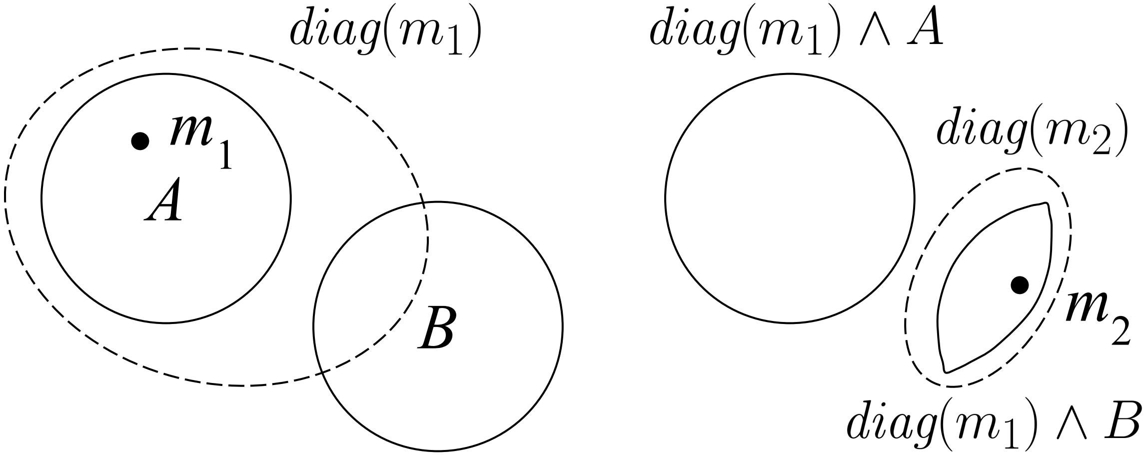

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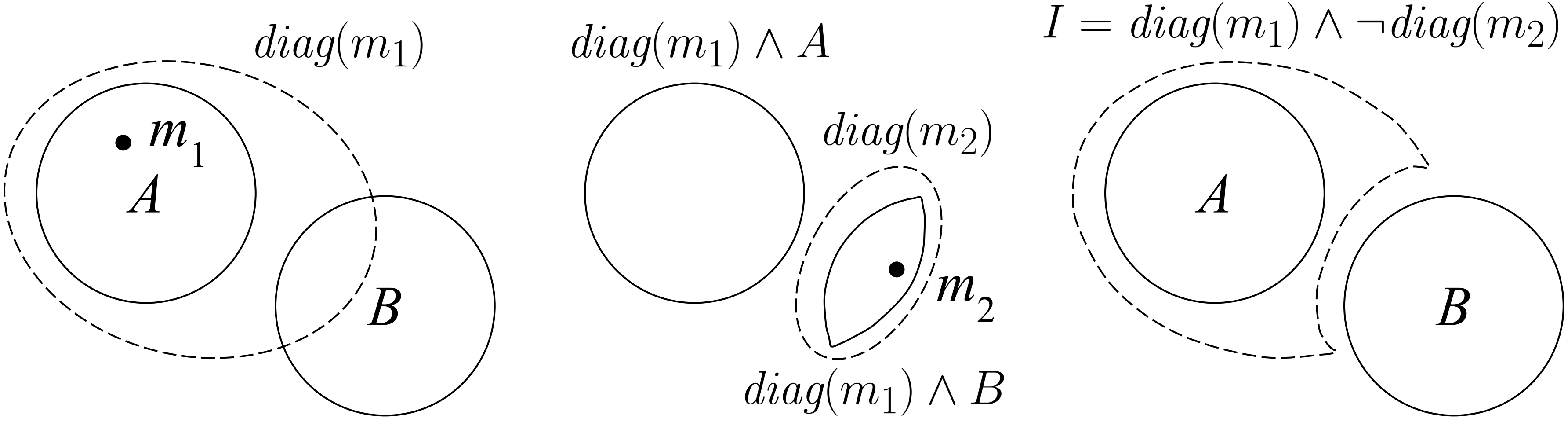

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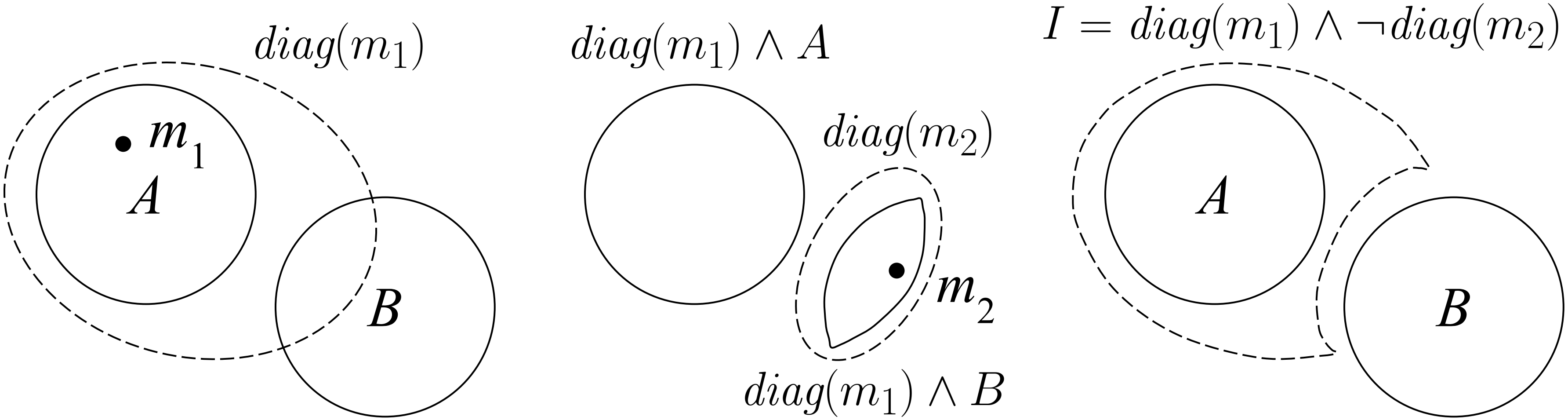
$\exists^{*}\varphi_1 \wedge \forall^{*}\varphi_2$



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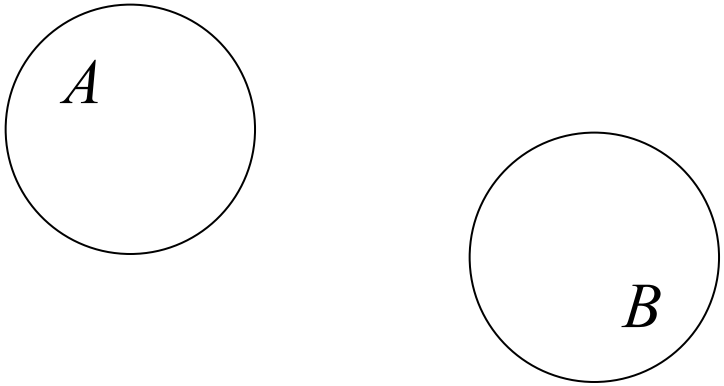
Soundness: returned I is interpolant by construction

Rel. Compl.: Existence of AF-logic interpolant → termination



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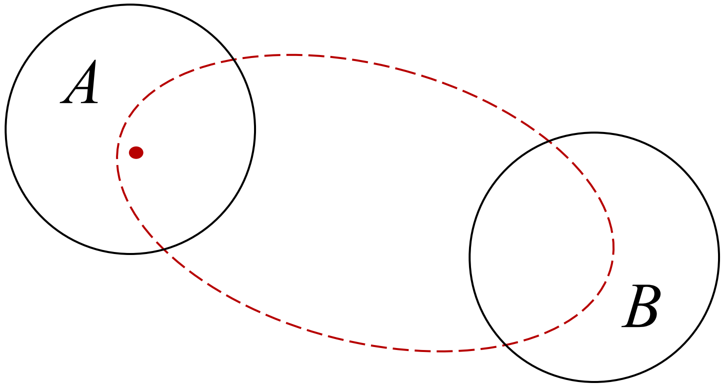

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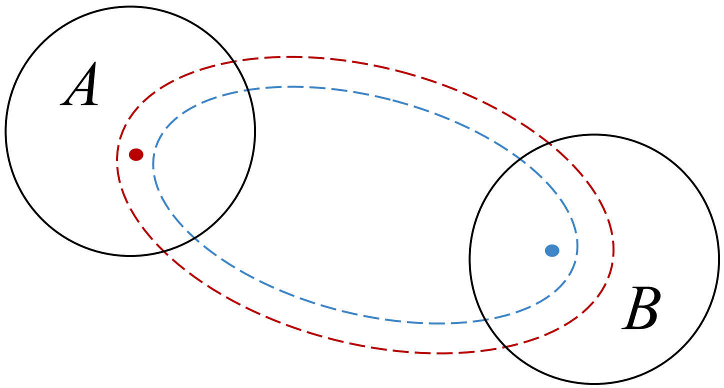

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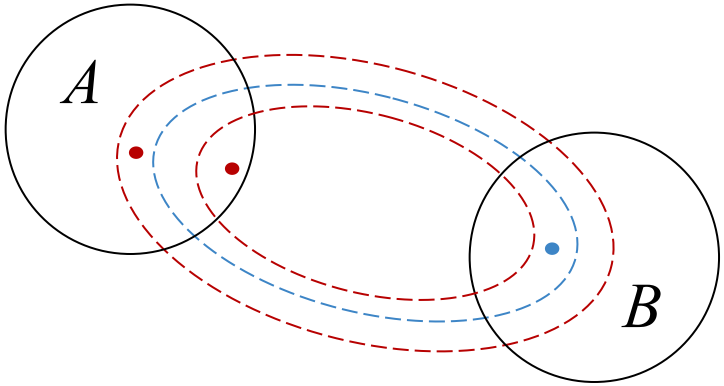

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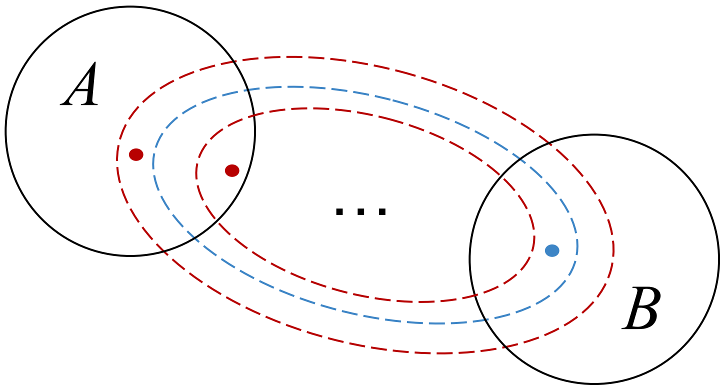

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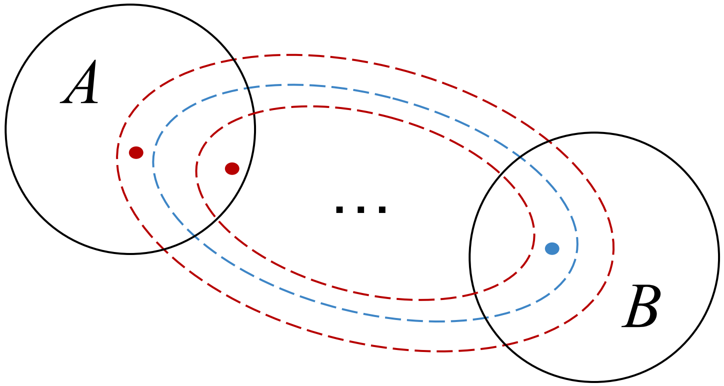

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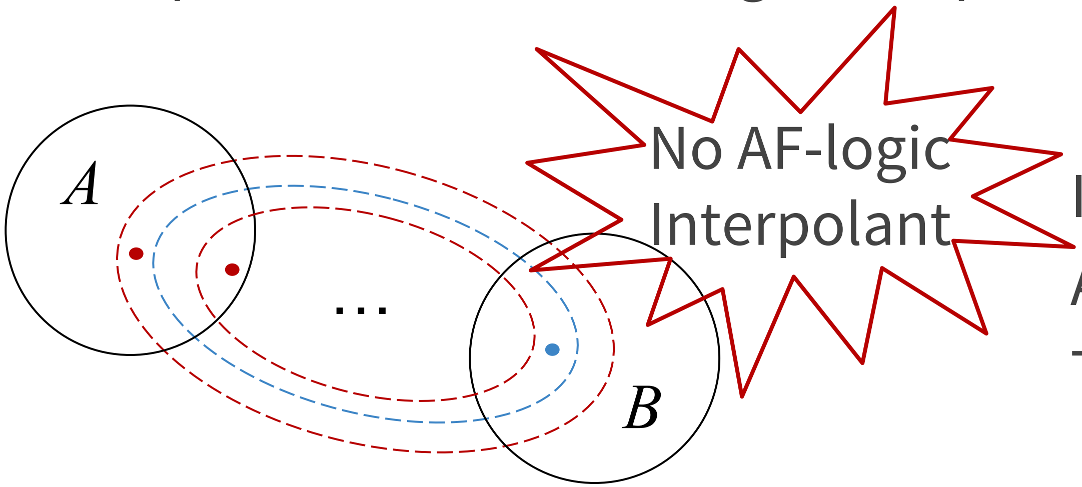

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Experiments

ITPV: an interpolation-based verifier

Compared to $PDR_\forall$ [Itzhaky et al., 2014] on linked-list programs

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Conclusion

UITP and BITP interpolate EPR formulae

UITP: sound/complete finding interpolants in ∃- and ∀-logic

BITP: sound/rel.comp. finding interpolants in AF-logic