Secretary Problems with Convex Cost

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Problem Definition

- Given U, pick A to maximize profit

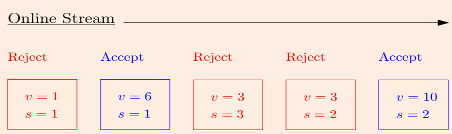
$$\max_{A \subset U} \sum_{e \in A} v(e) \; - \; c\left(\sum_{e \in A} s(e)\right)$$

$$\text{s.t.} \qquad A \in \mathcal{F}$$

- v: value function, s: size function; c: convex cost function
- $\mathcal{F}$ is a known feasibility constraint; v and s are adversarial
- Elements arrive one by one in random order
- Algorithm is online

Challenge: Objective function can take negative values; breaks all previous techniques

Example



Cost function: $c = s^2$
Total Profit $= 16 - 3^2 = 7$

Applications

Resource allocation problems:

- Wireless access point accepting connection requests
- Cloud computing services accepting jobs
- Sponsored search auctions accepting bids for ad slots

Extension – Multi-Dimensional Costs

- ℓ cost functions $c_1, \ldots, c_\ell$; ℓ sizes $s_1, \ldots, s_\ell$
- Given U, pick A to solve

$$\max_{A \subset U} \sum_{e \in A} v(e) \; - \; \sum_{i=1}^{\ell} c_i \left(\sum_{e \in A} s_i(e)\right)$$

$$\text{s.t.} \qquad A \in \mathcal{F}$$

Results – Competitive Ratios

Single-Dimensional Cost

- Unconstrained $O(1)$
- Matroid/knapsack constraints $O(\alpha)$ *

Multi-Dimensional Cost

- Unconstrained $O(\ell)$ ← Tight to within a constant factor
- Matroid/knapsack constraints $O(\alpha \ell^5)$*

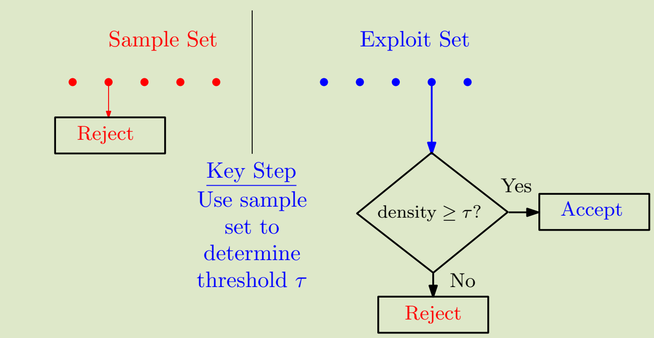
Extends work on submodular matroid secretary [Gupta et al. 10, Bateni et al. 10]

* α is the competitive ratio for the corresponding matroid/knapsack secretary problem

Unconstrained Setting

Challenge:
Initial bad decisions may drive cost too high to accept even high-value elements later on

High-Level Approach



Intuition

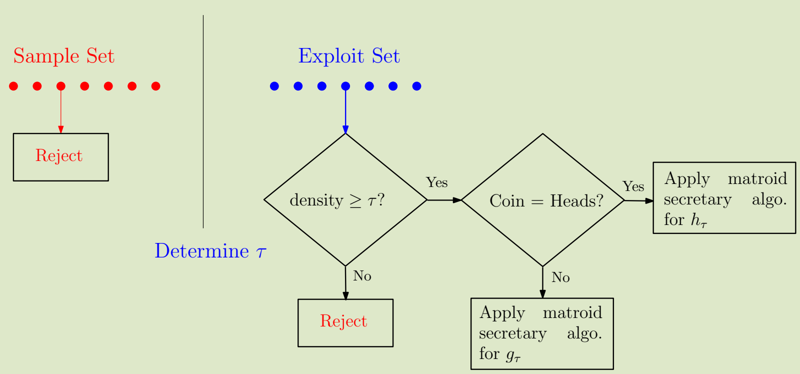
- Define density = value/size, as in the greedy knapsack algorithm
- Greedy by decreasing density is near-optimal
- With high probability, we do not accept any element with density below cut-off for greedy by density

Constrained Setting – Matroids

Challenge:
Greedy by density is no longer near-optimal

High-Level Approach

At the beginning of the algorithm, we toss a fair coin



Intuition

- We can decompose profit $\pi(A) = g_\tau(A) + h_\tau(A)$; both h_τ and g_τ are sum-of-values type functions
- If τ is "good", then optimizing both h_τ and g_τ are non-negative value matroid secretary problems

The full version can be found at **http://pages.cs.wisc.edu/~shuchi/papers/convex-cost-secretary.pdf**