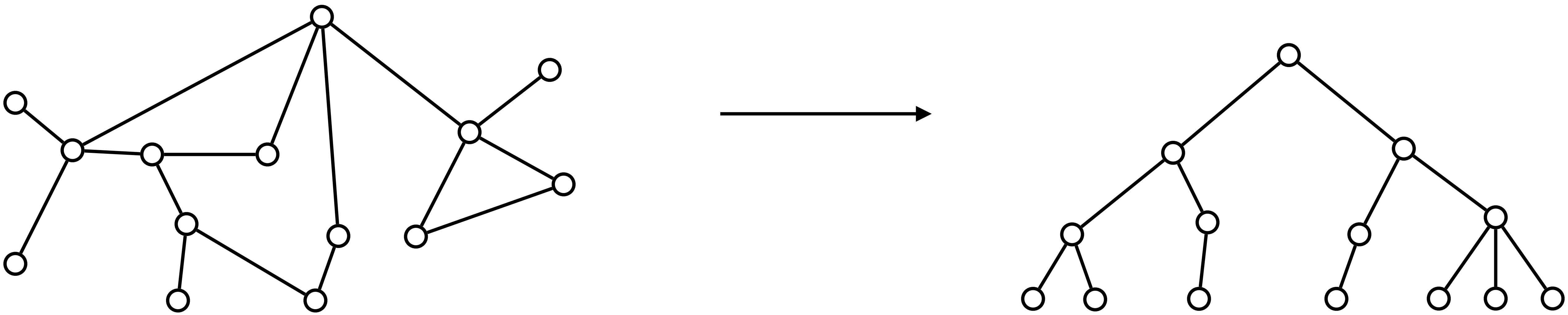
Online Network Design Algorithms via Hierarchical Decompositions

Appeared in SODA 2015

Seeun William Umboh

Eindhoven University of Technology

Overview



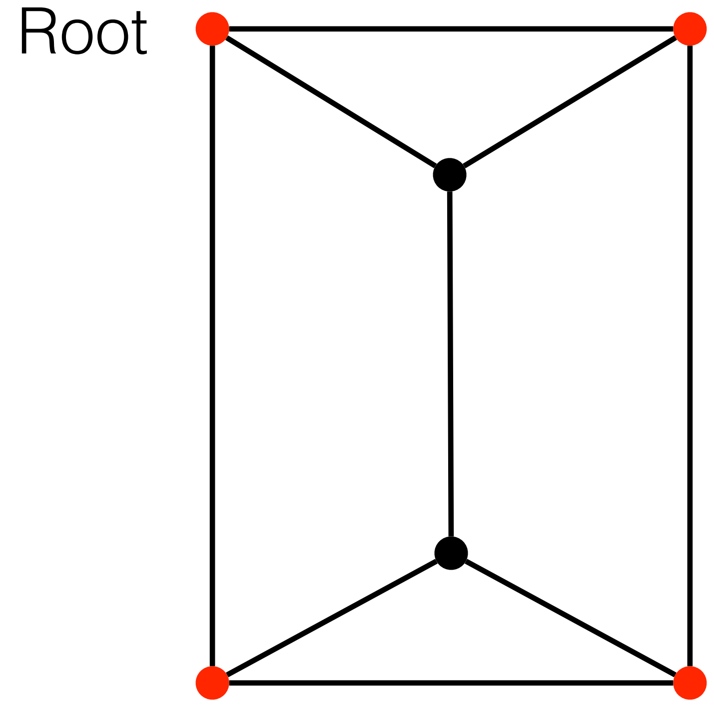
- Analysis framework based on tree embeddings
- Improved algorithms for online network design

Network Design

Fig. 7. INFONET communications networks.

Given a graph G with edge costs and requirements, find a min-cost network in G

Steiner Tree



- Given graph and terminals
- Find min-cost subgraph connecting terminals

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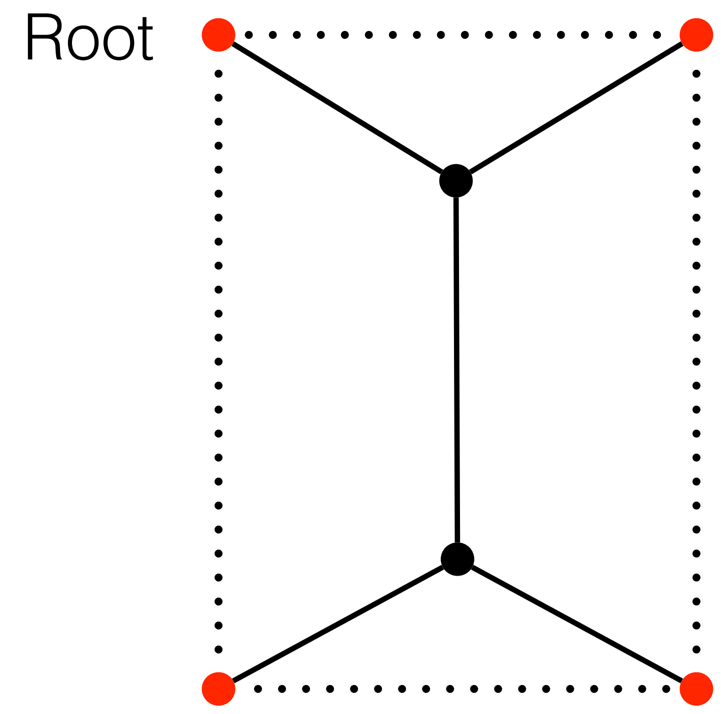

- Given graph and terminals
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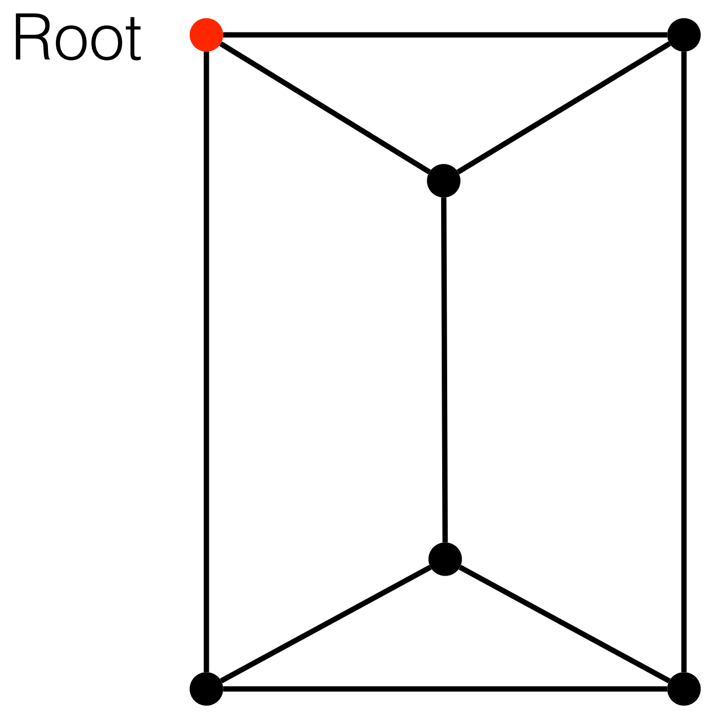
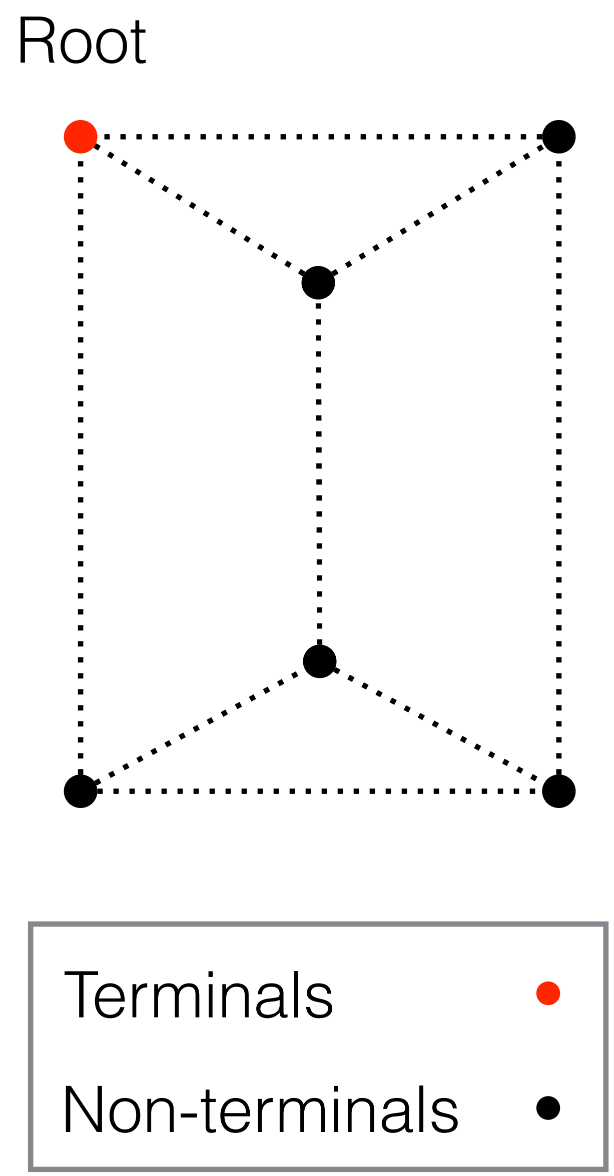
Online Steiner Tree



- Initially, only given graph and root
- At each time step, we are given a terminal i to connect
- Maintain min-cost subgraph connecting terminals to root
- Chosen edges cannot be removed later

Example



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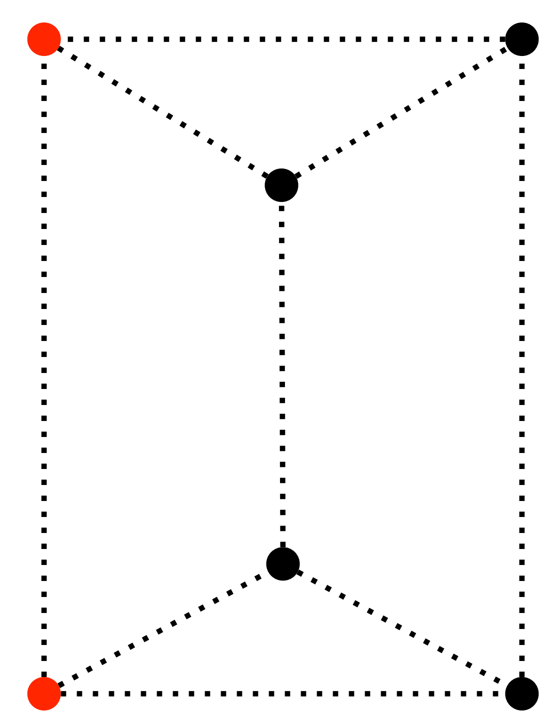

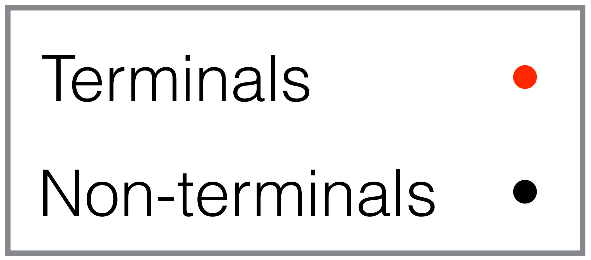

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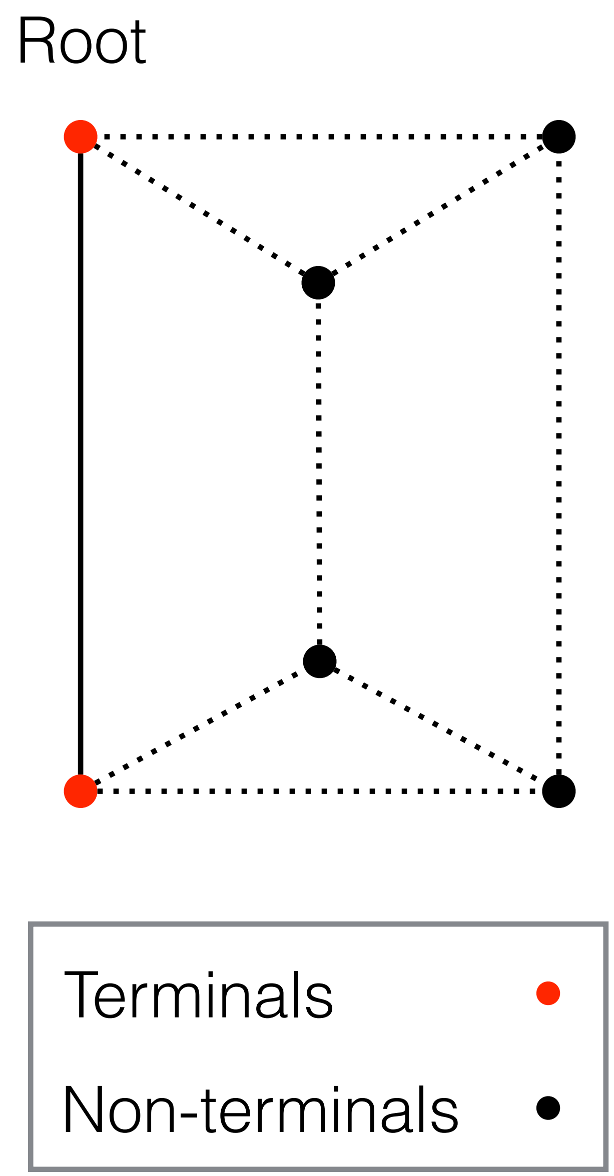

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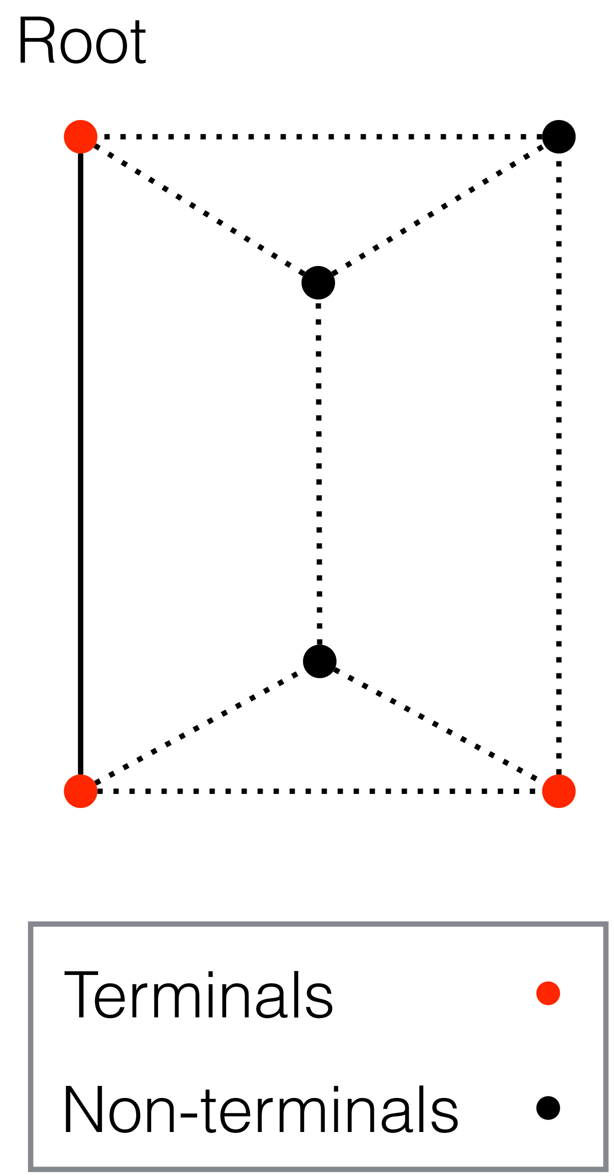

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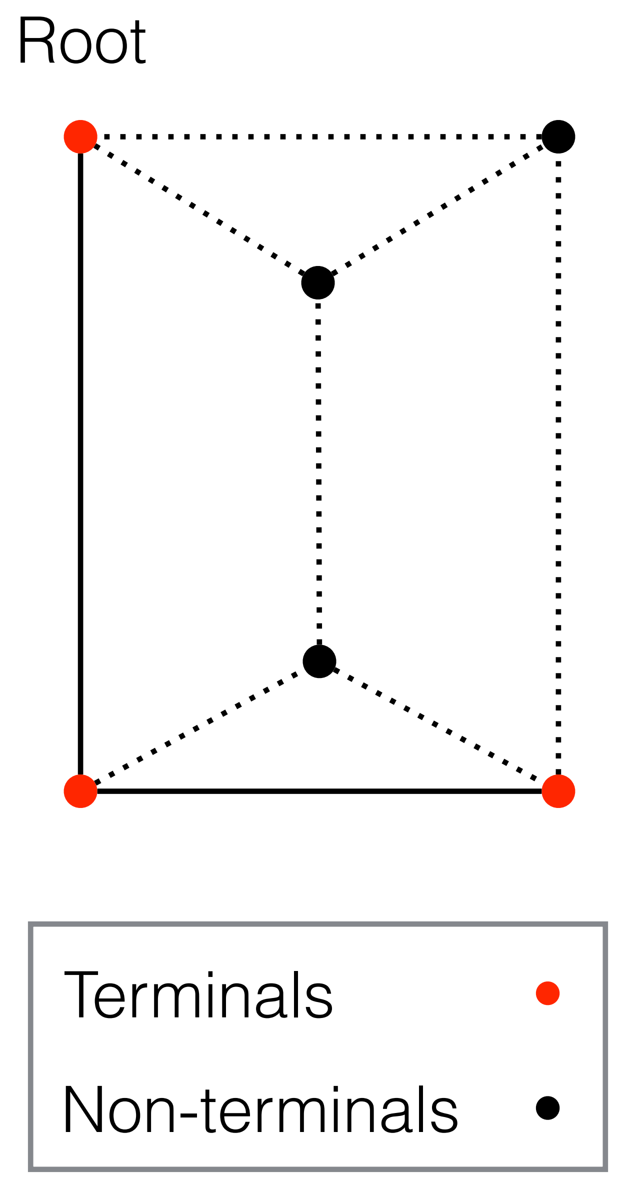

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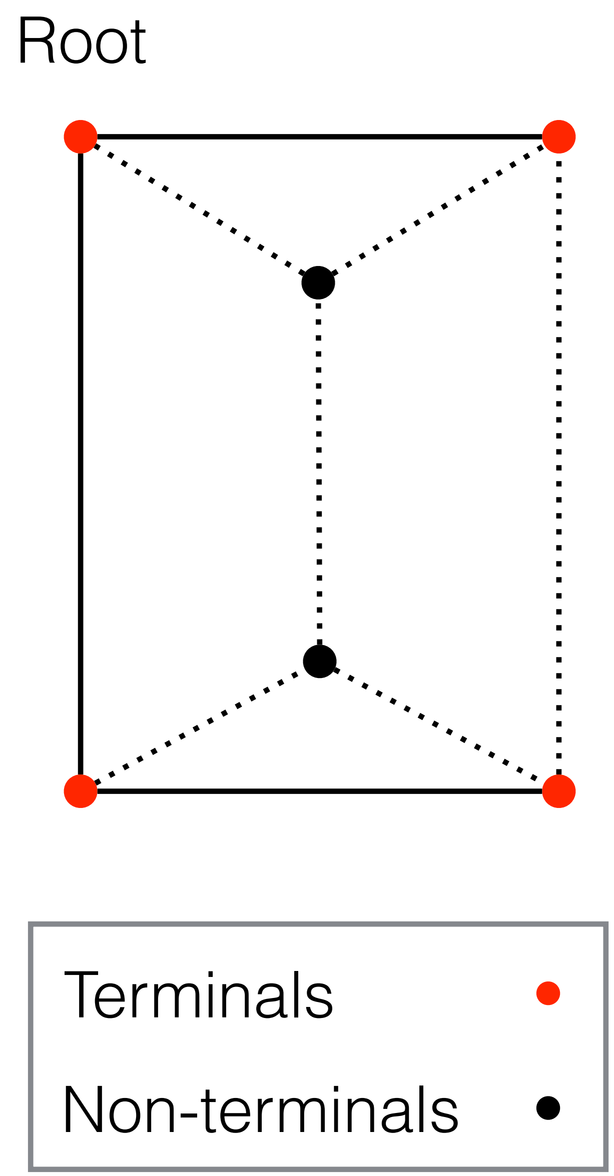

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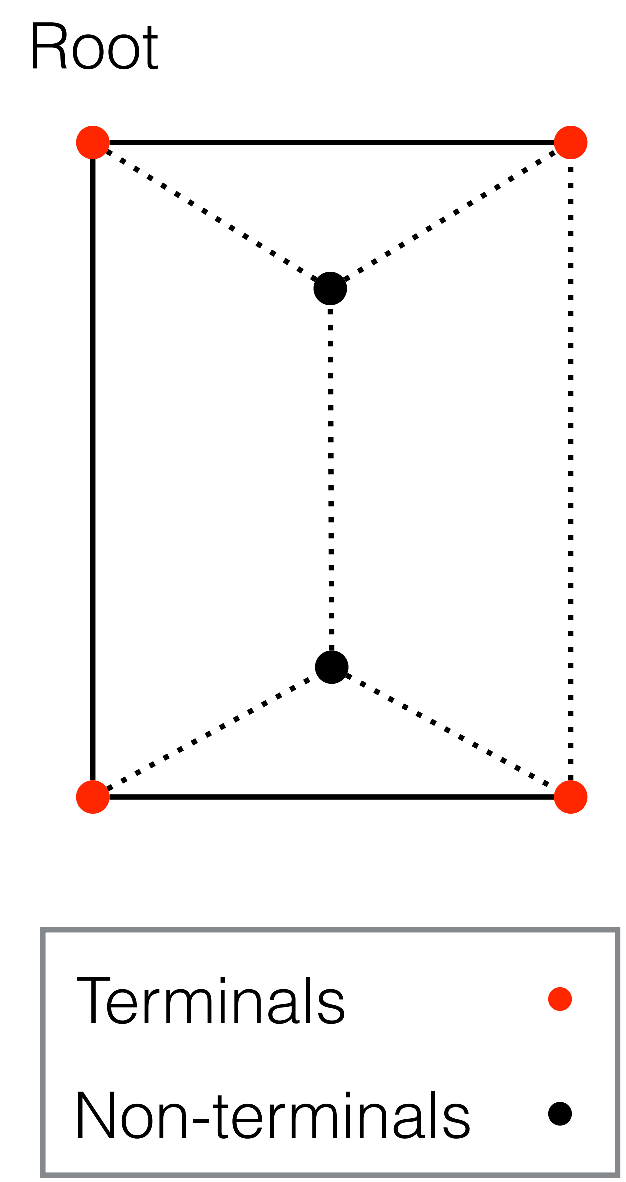

Competitive Ratio: $\displaystyle \max_{\substack{\text{requirements } R \\ \text{sequence } \sigma}} \frac{\text{ALG}(R, \sigma)}{\text{OPT}(R)}$

Problems

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- Connected Facility Location

Results

Problem	Approximation Ratio
Steiner tree Steiner forest	1.39 [Byrka-Grandoni-Rothvoss-Sanita 10] 2 [Agrawal-Klein-Ravi 91; Goemans-Williamson 92]
PC Steiner tree PC Steiner forest	2 [Goemans-Williamson 95] 2.54 [Hajiaghayi-Jain 06]
Steiner network	2 [Jain 98]
Rent-or-buy	3.19 [Grandoni-Rothvoss 11]
Connected facility location	3.19 [Grandoni-Rothvoss 11]

Results

Problem	Approx. Ratio	Previous Competitive Ratios
Steiner tree Steiner forest	1.39 2	O(log k), Ω(log k) [Imase-Waxman 91] O(log k) [Berman-Coulston 97]
PC Steiner tree PC Steiner forest	2 2.54	O(log k) [Qian-Williamson 11]
Steiner network	2	O(log n) randomized [folklore]
Rent-or-buy	3,19	O($\log^2$ k) deterministic, O(log k) randomized [Awerbuch-Azar-Bartal 96]
Connected facility location	3,19	O($\log^2$ k) randomized [San Felice-Williamson-Lee 14]

k = # of terminals; n = # of vertices

Results

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(*Independently improved to $O(\log k)$ randomized)

k = # of terminals; n = # of vertices

Previous Approaches

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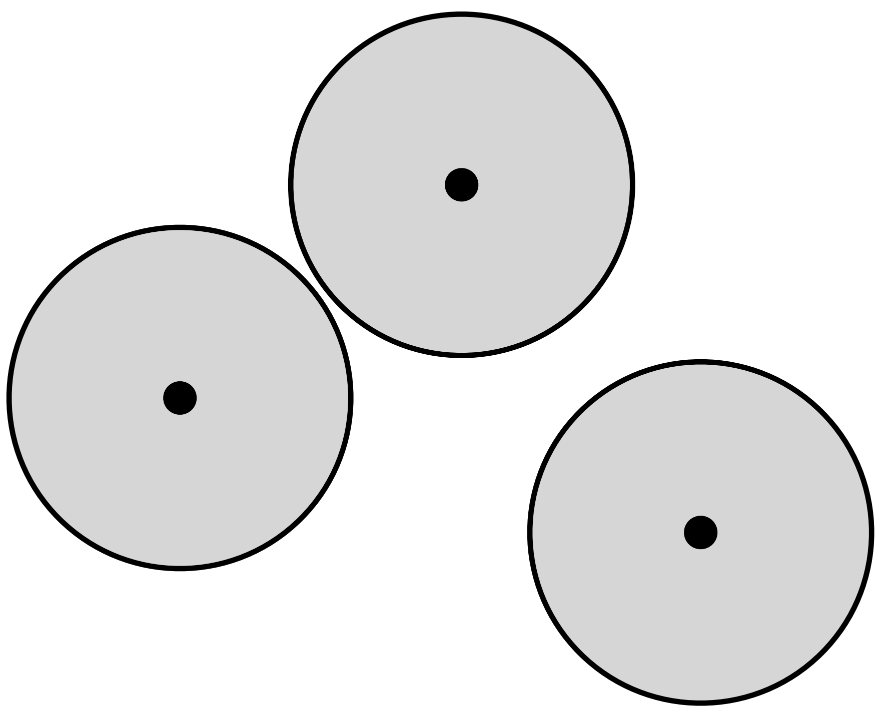
Greedy / Primal-Dual



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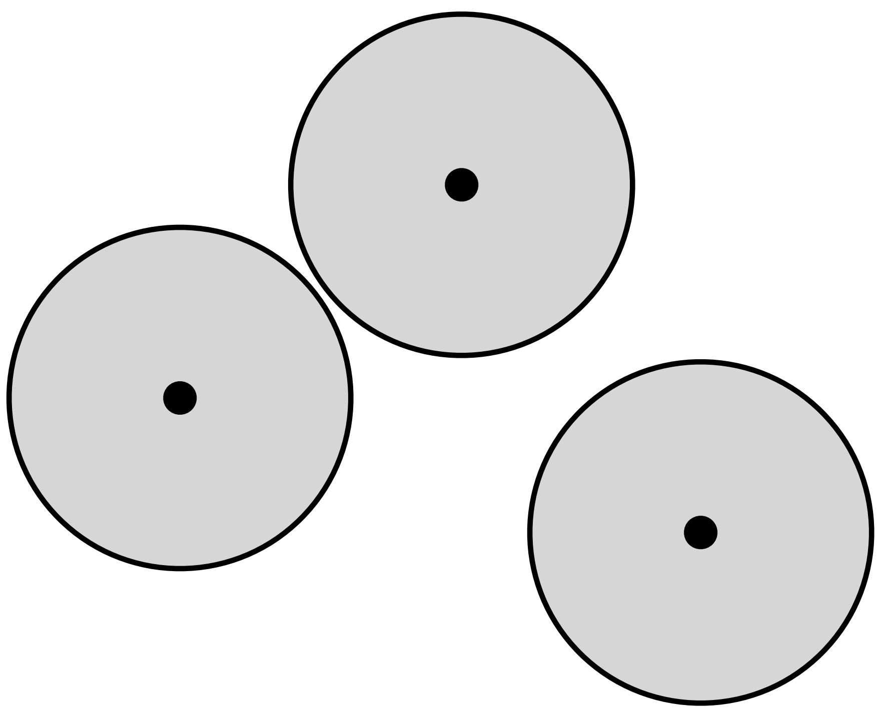
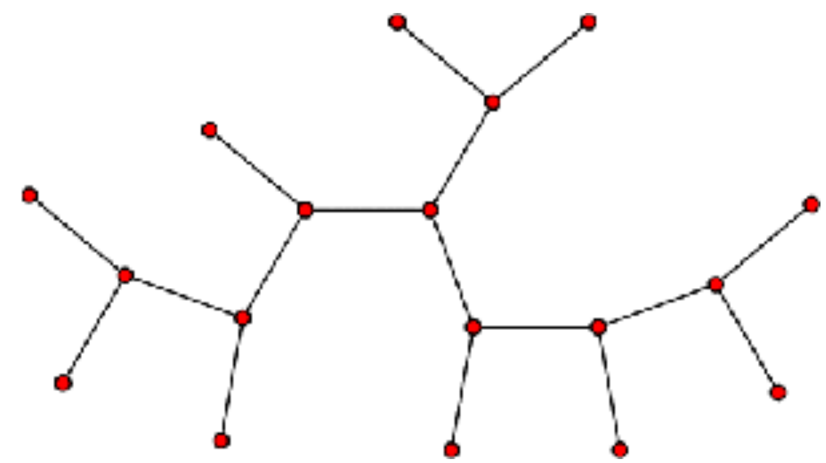


Tree Embeddings



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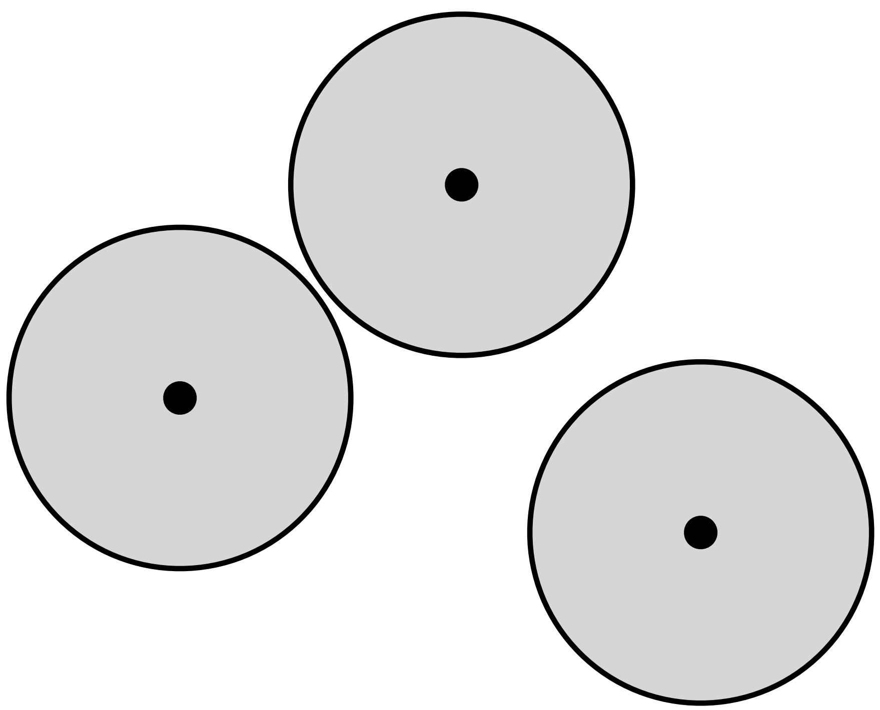
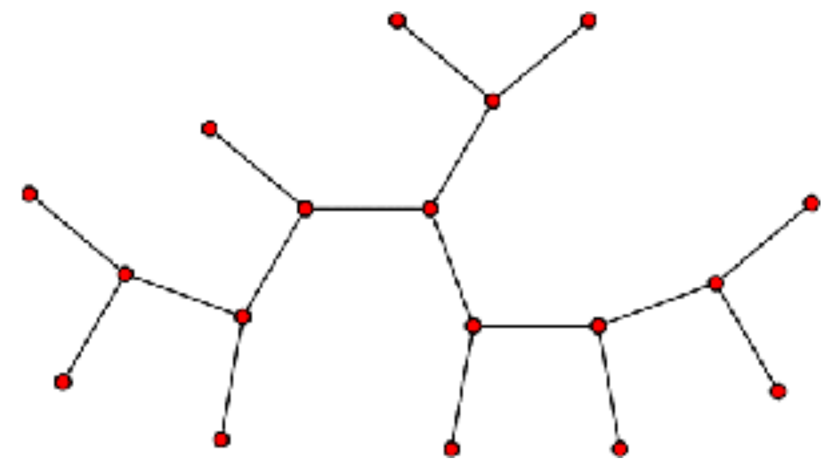


- Deterministic algorithms

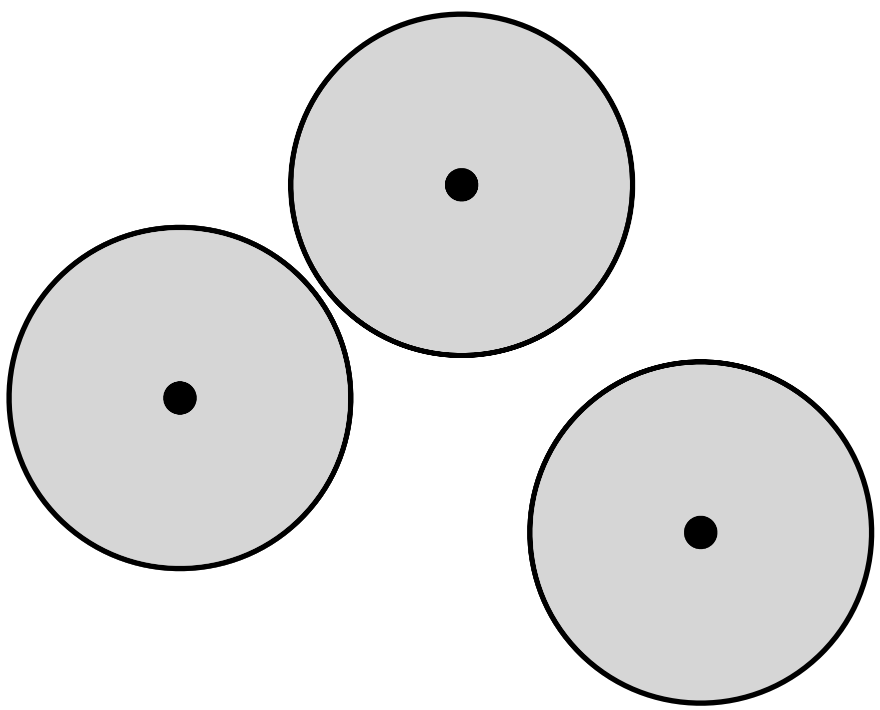
Tree Embeddings



- Randomized algorithms

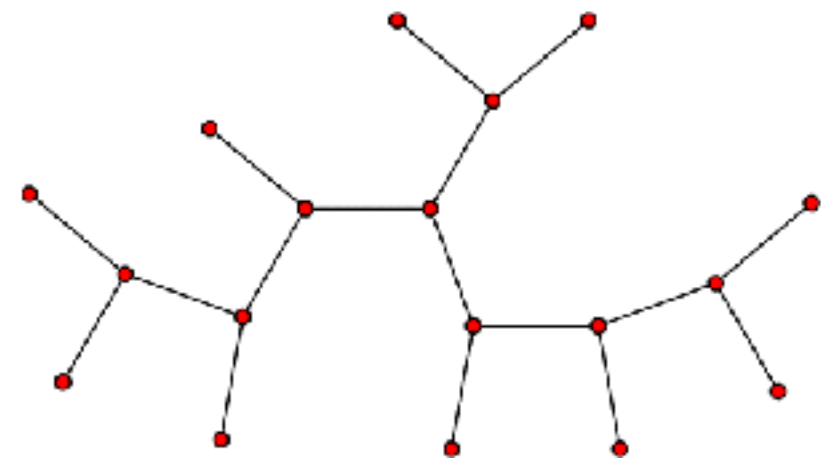
Previous Approaches

Greedy / Primal-Dual



- Deterministic algorithms
- Intricate dual construction

Tree Embeddings



- Randomized algorithms
- Simple analysis

Our Approach

- Greedy algorithms but analyze using tree embeddings
- Deterministic algorithms and simple analyses
- Unified approach to online network design

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Takeaway: Tree embeddings and metric decompositions are useful for designing and analyzing greedy algorithms

Outline

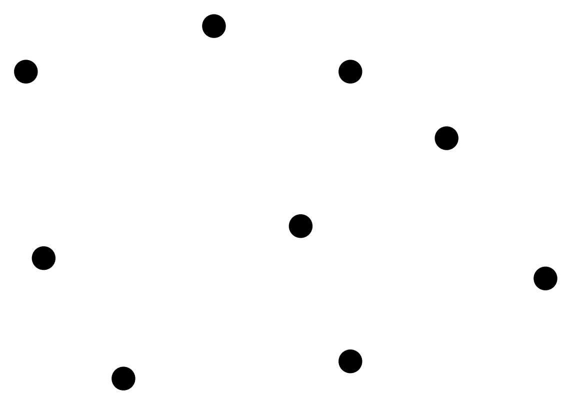
1. Overview of Analysis Framework
2. Warm-Up: Steiner Tree
3. Rent-or-Buy
4. Steiner Network

Metric Problems

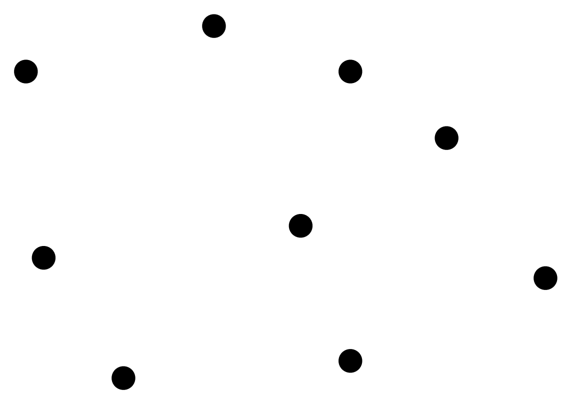
- Up to constants, can assume input graph G is complete and edge costs form a metric
- Henceforth, input is a metric

Tree Embeddings

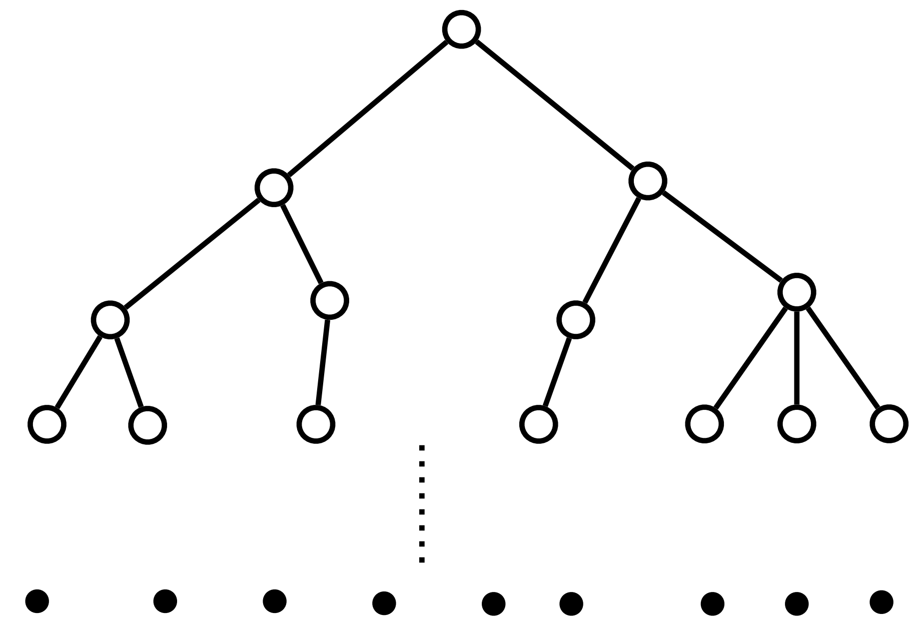
(V, d)



(V, d)



Embedding (V’, T)



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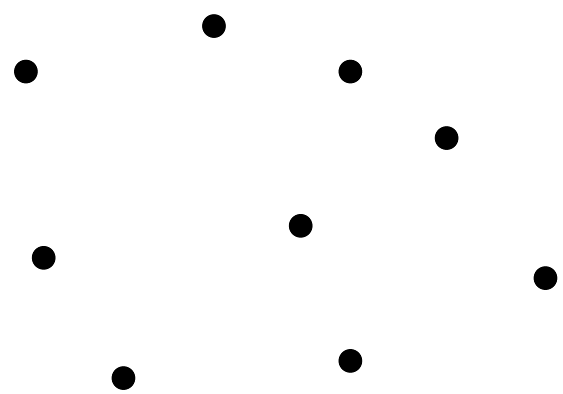

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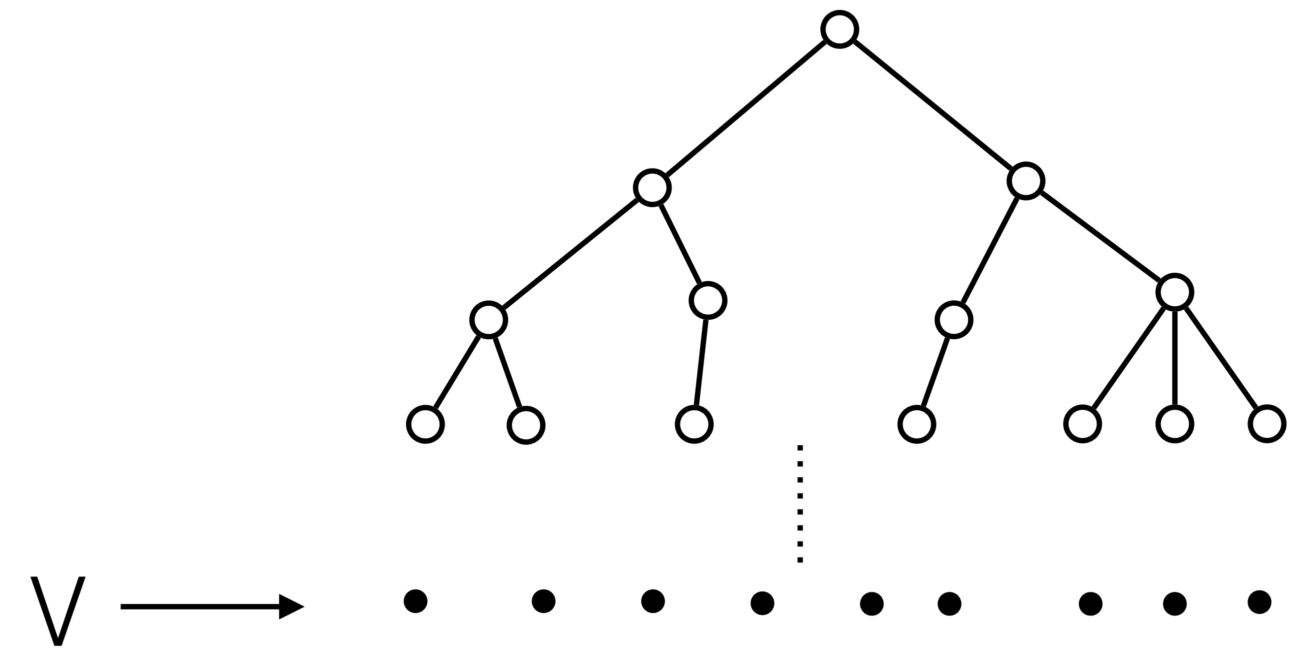

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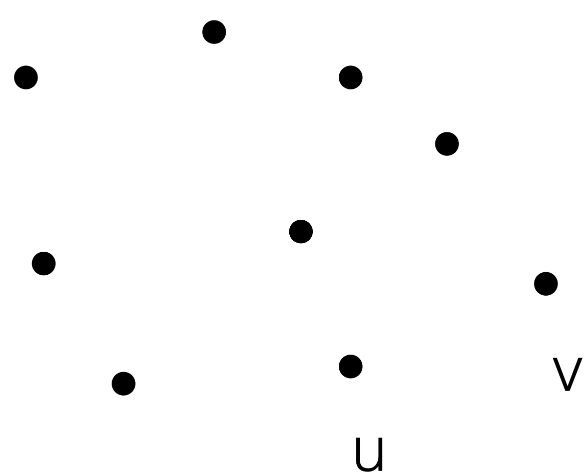

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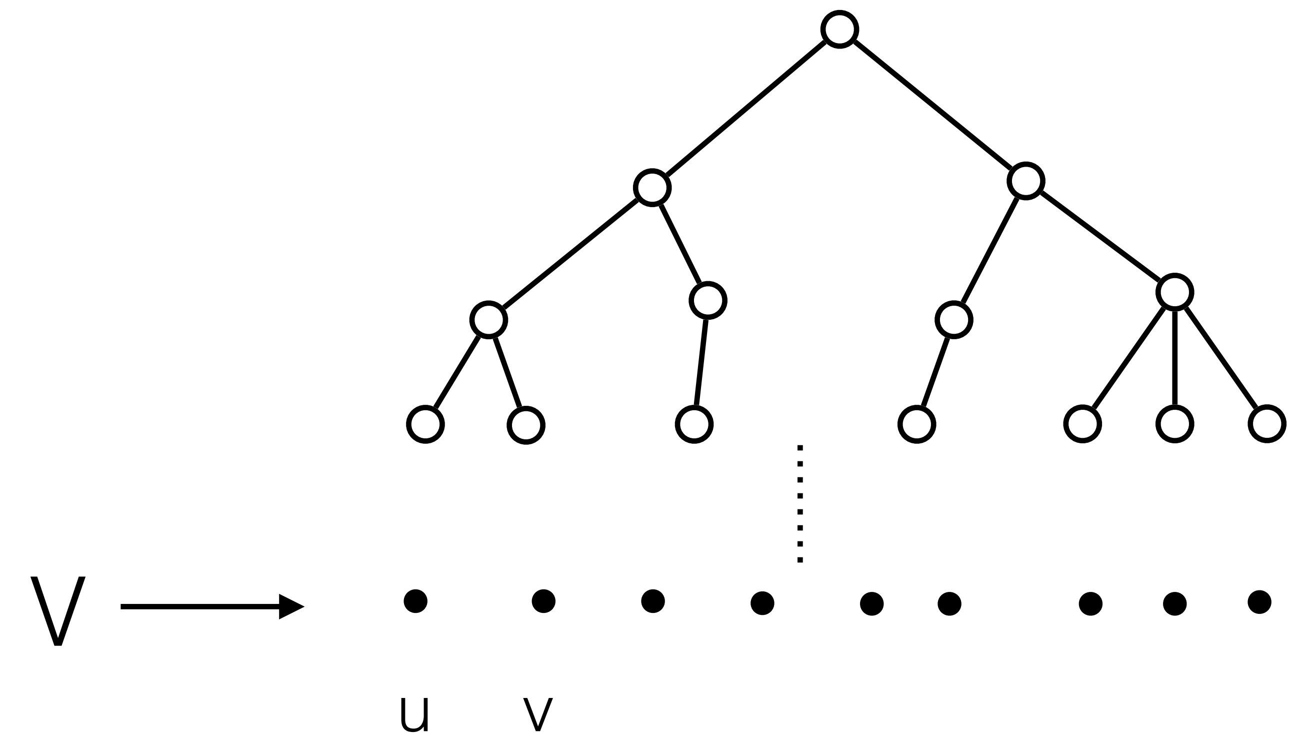

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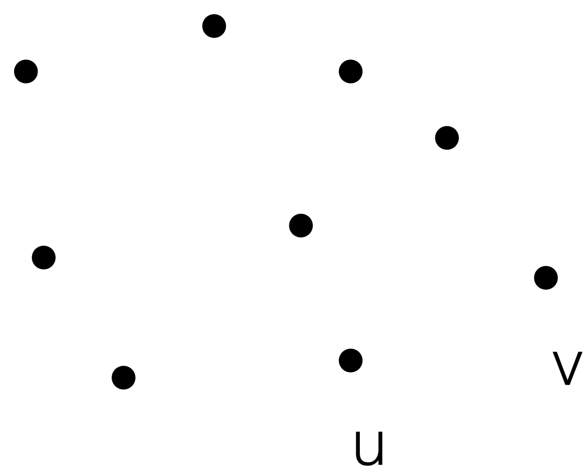

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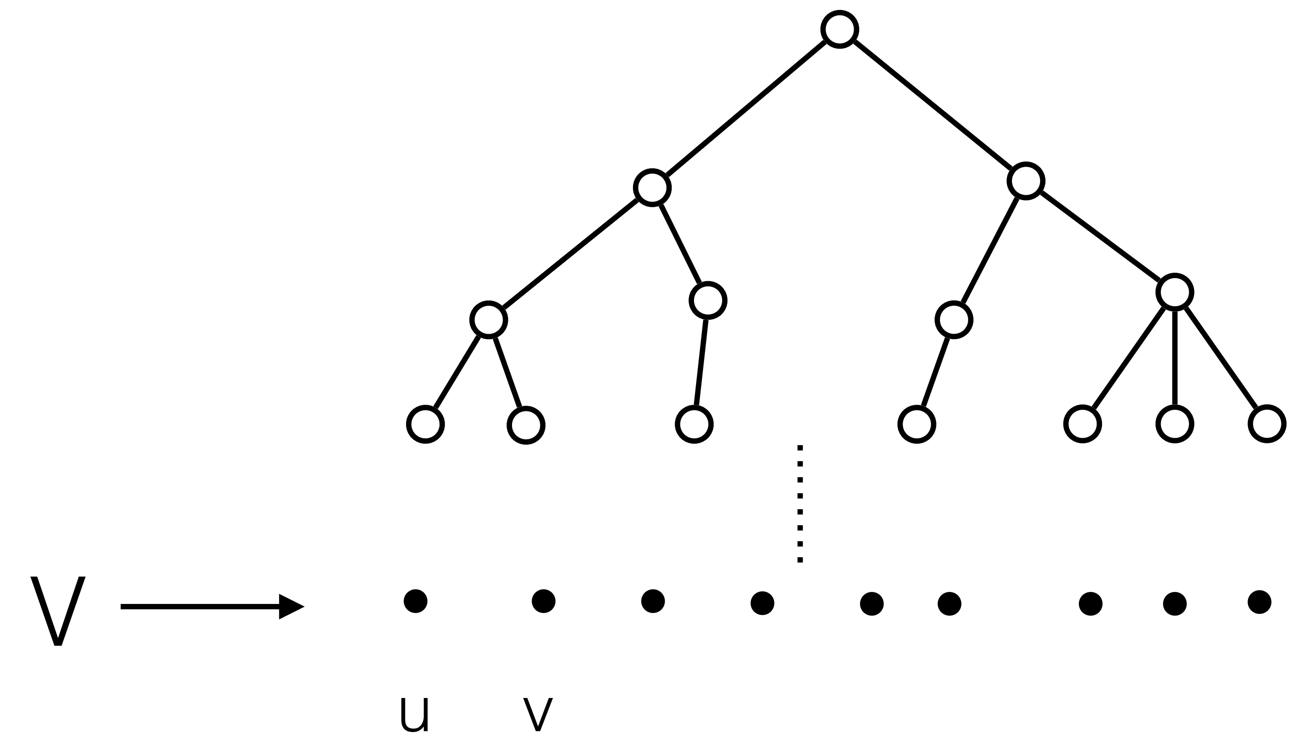
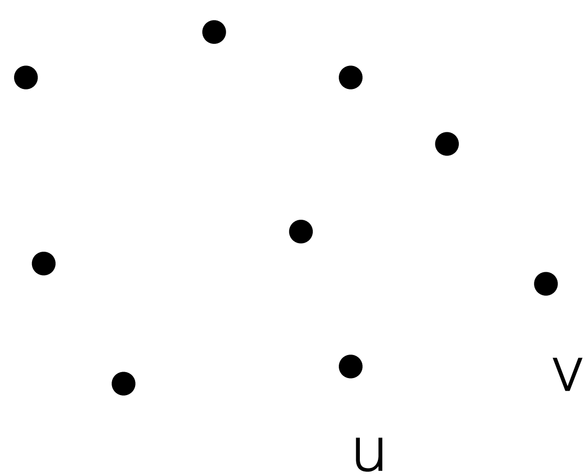


Theorem [Fakcharoenphol-Rao-Talwar 04]:
There exists a randomized embedding into HSTs satisfying:

For all u,v in V

$$T(u,v) \geq d(u,v)$$
$$E[T(u,v)] \leq O(\log n)\, d(u,v)$$

(V, d)



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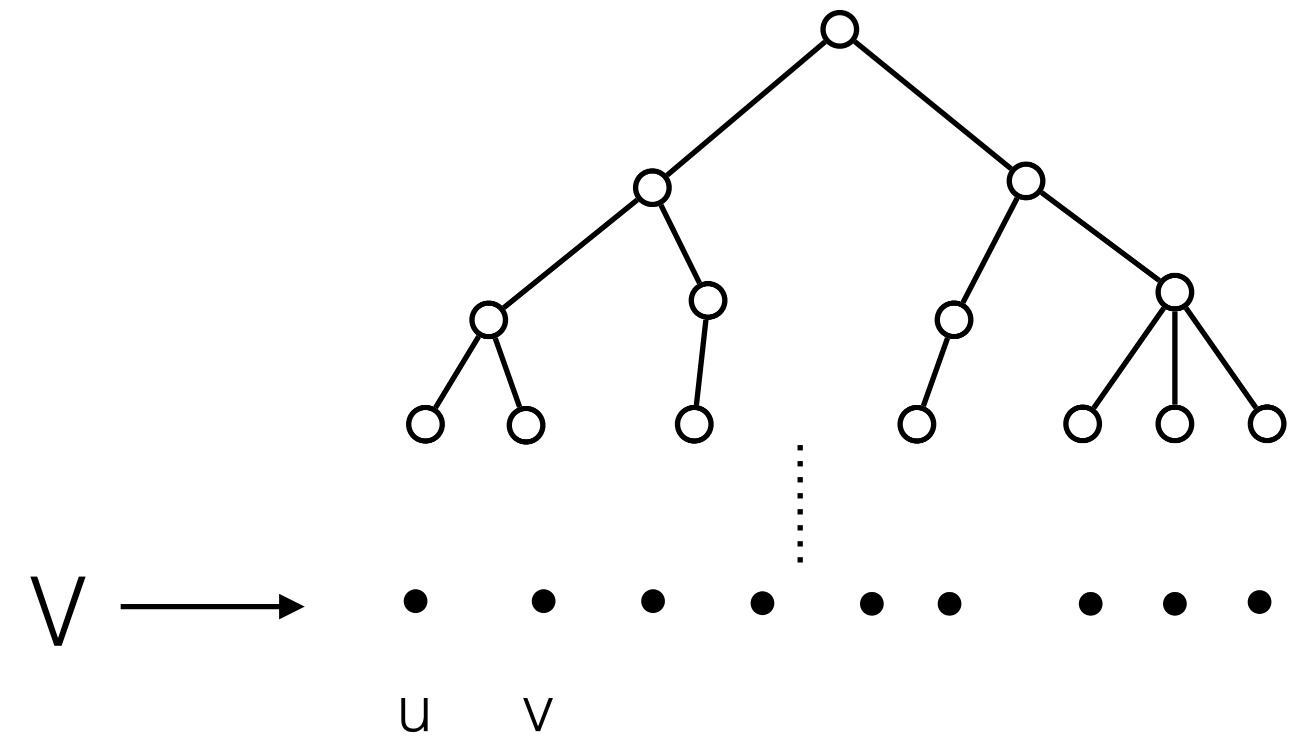

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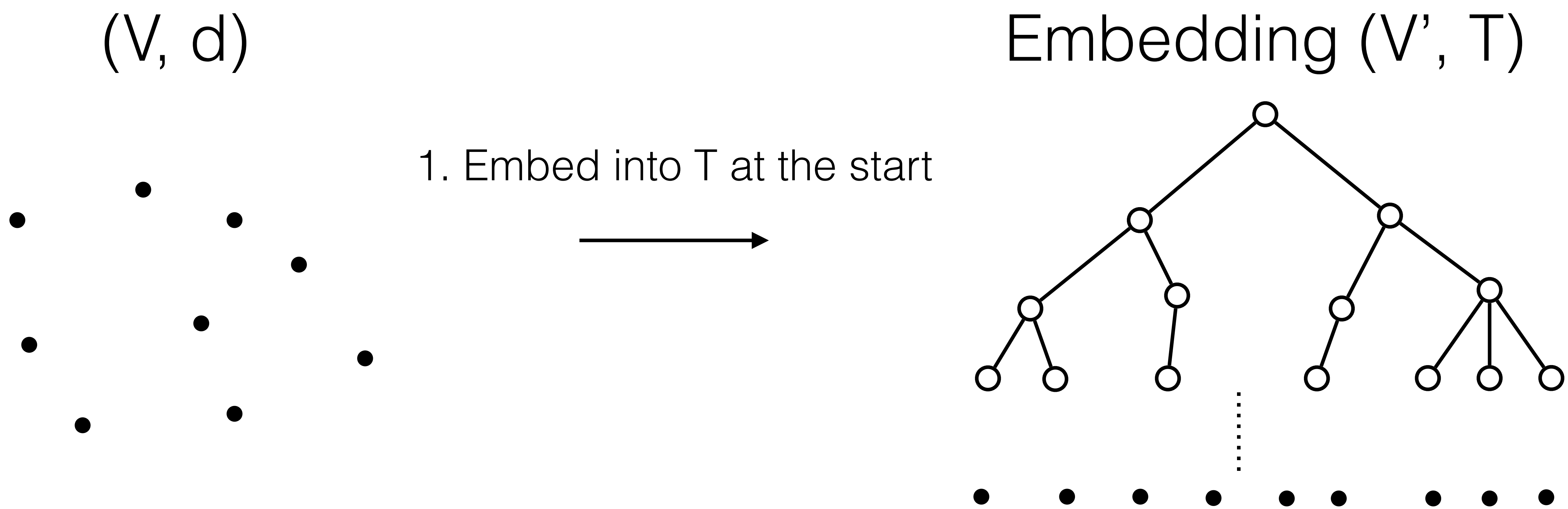
Corollary For many network design problems,

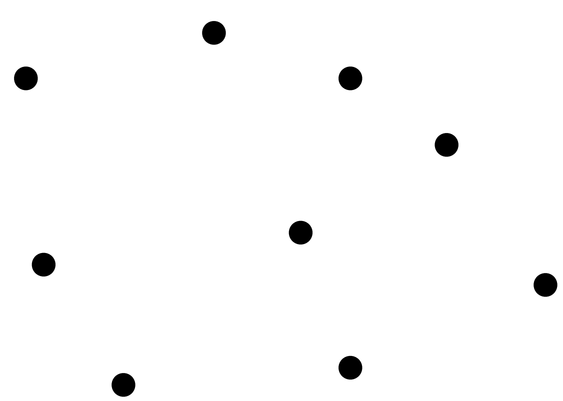
$$OPT \leq E[OPT(T)] \leq O(\log n)\, OPT$$

Previous Online Application



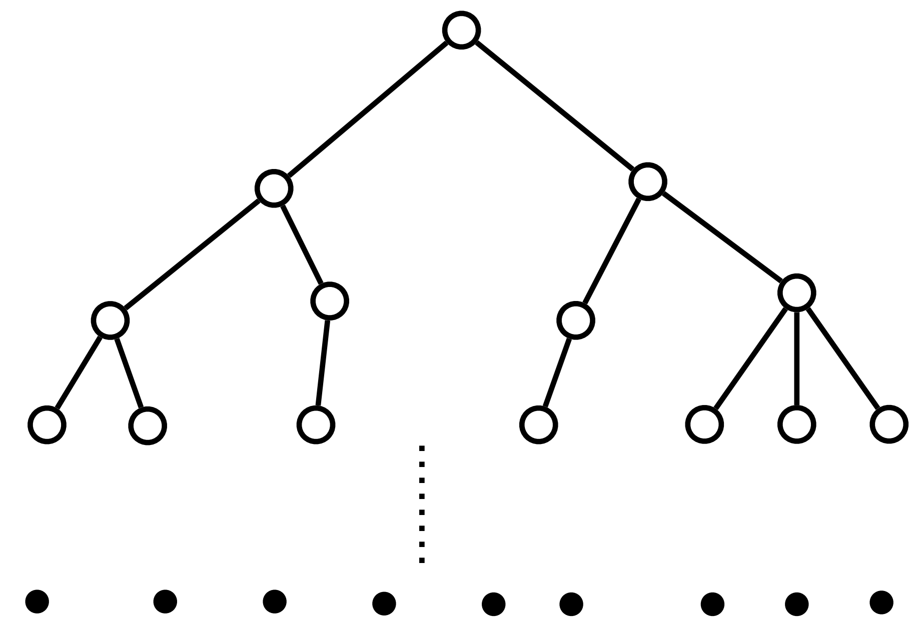
Previous Online Application

(V, d)



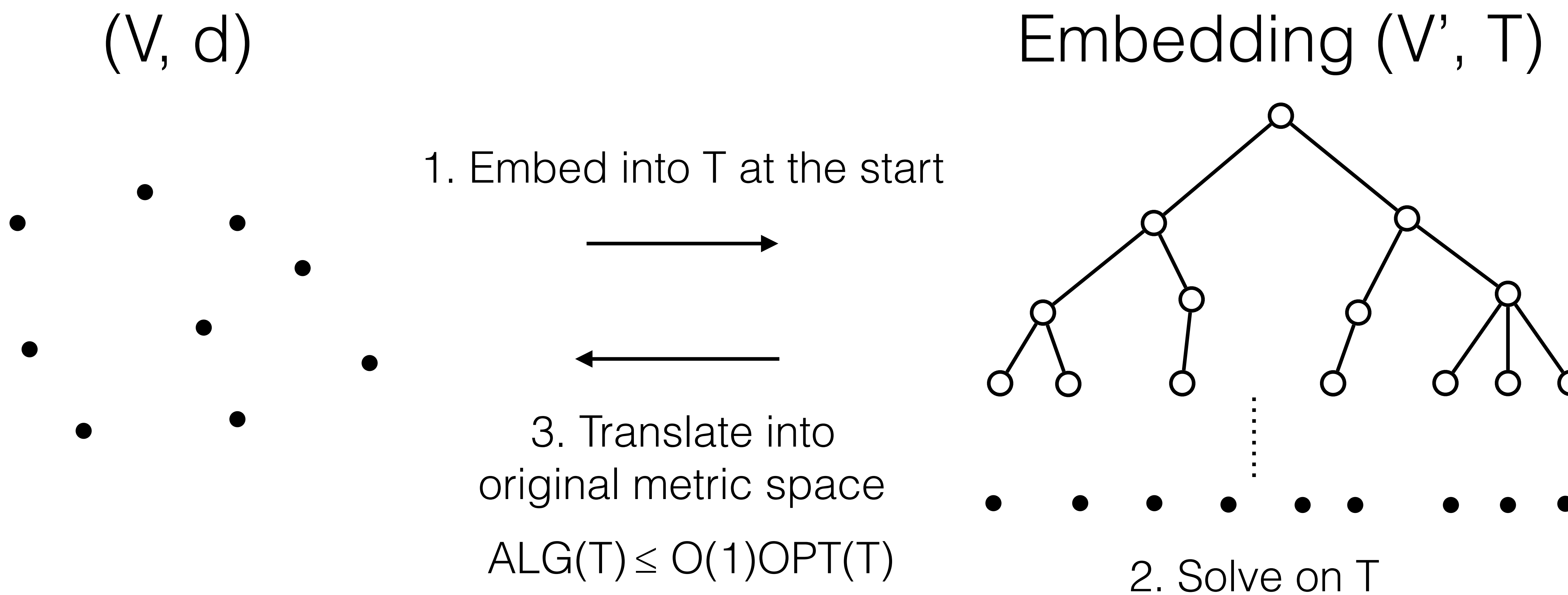
1. Embed into T at the start

Embedding (V', T)



2. Solve on T

Previous Online Application



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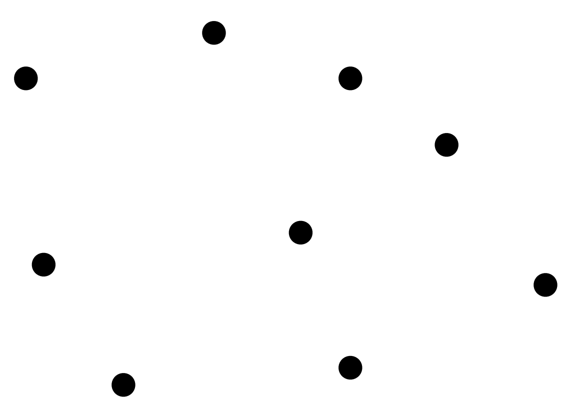

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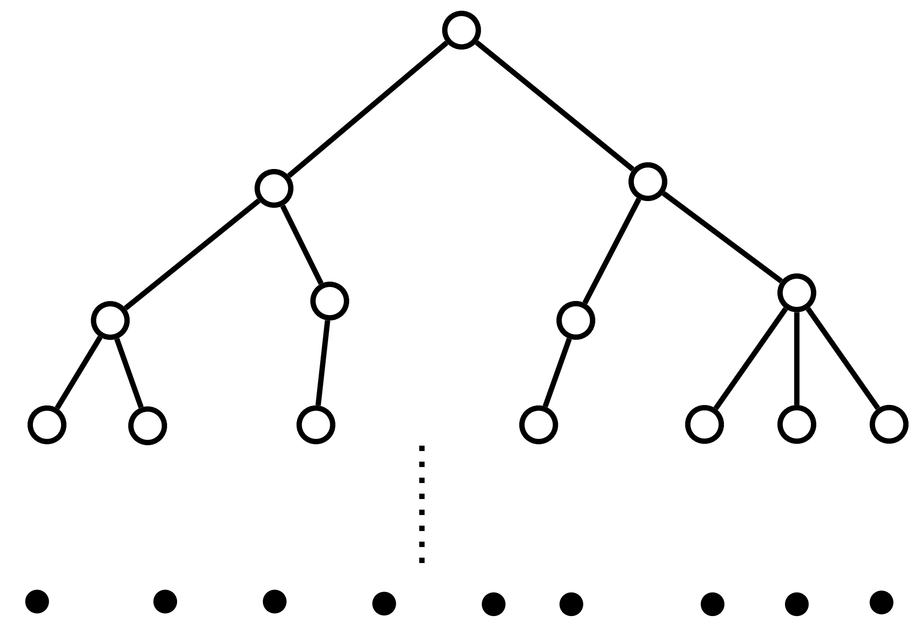


1. Embed into T at the start

2. Solve on T

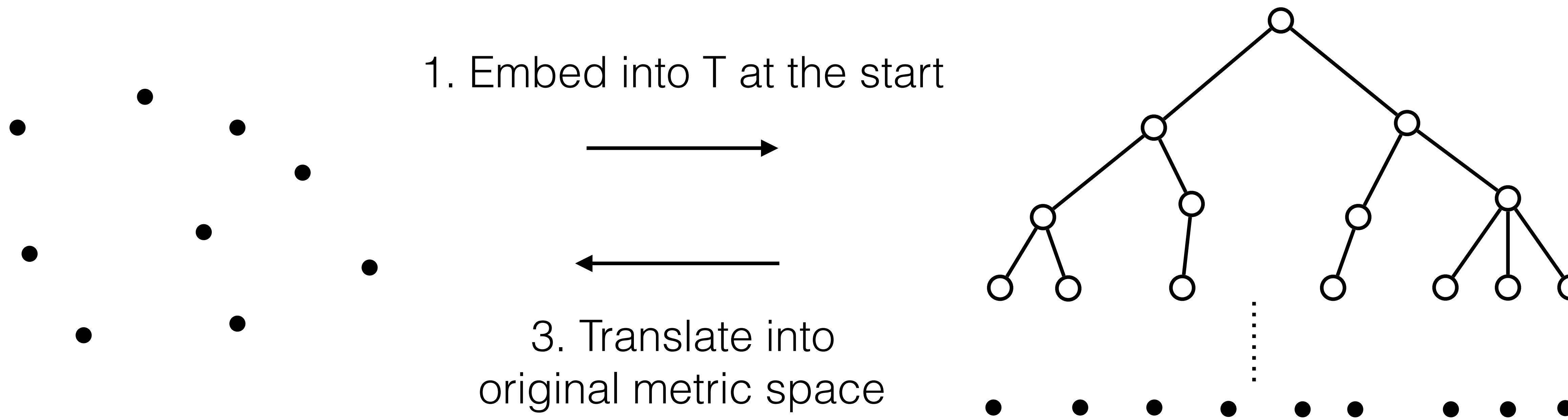
3. Translate into original metric space

ALG(T) ≤ O(1)OPT(T)

E[ALG(T)] ≤ O(log n) OPT

Previous Online Application

(V, d) Embedding (V', T)



ALG(T) ≤ O(1)OPT(T)

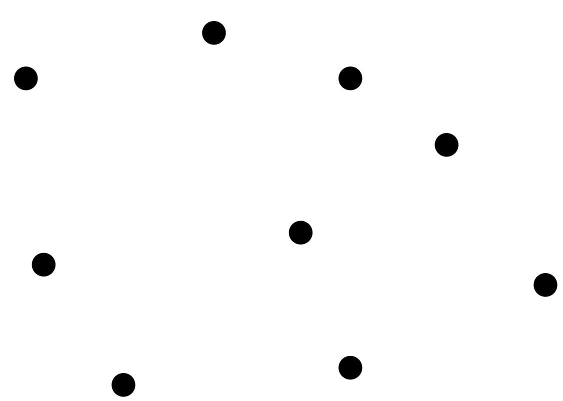
E[ALG(T)] ≤ O(log n) OPT

Drawbacks

- O(log n) competitive ratio even with k ≪ n requests
- Requires randomness

Our Application

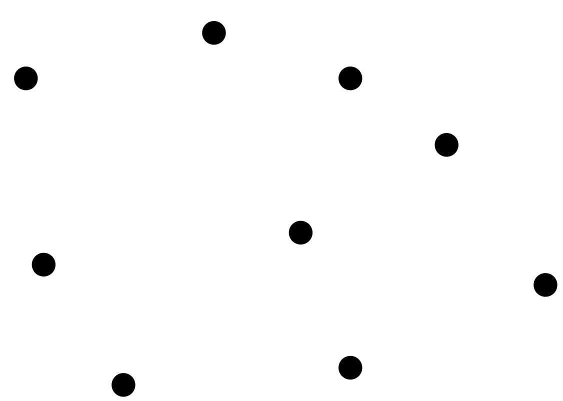
(V, d)



1. Run greedy algorithm ALG

Our Application

(V, d)



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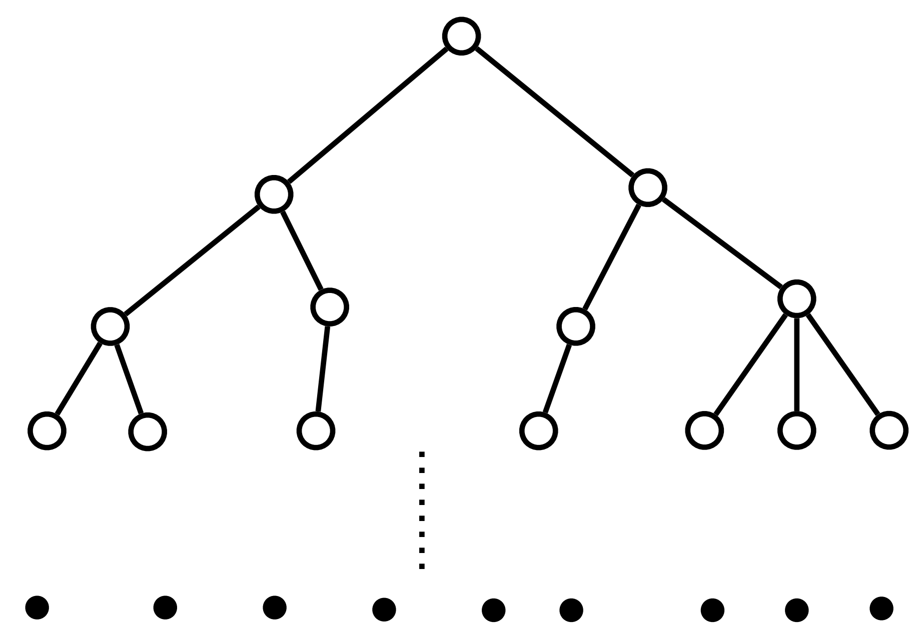


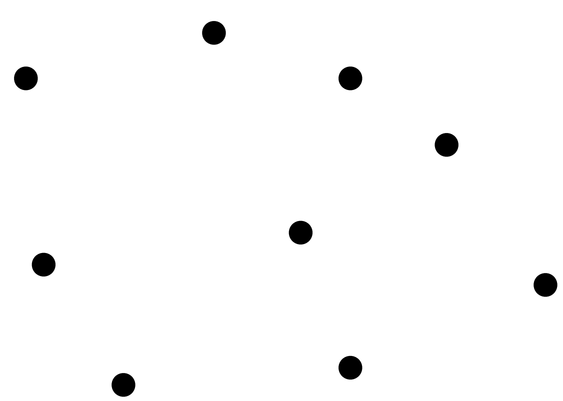
2. Bound ALG against OPT(T)

<u>Main Lemma:</u>

ALG $\leq$ O(1) OPT(T) for *any* HST embedding T

Our Application

(V, d)



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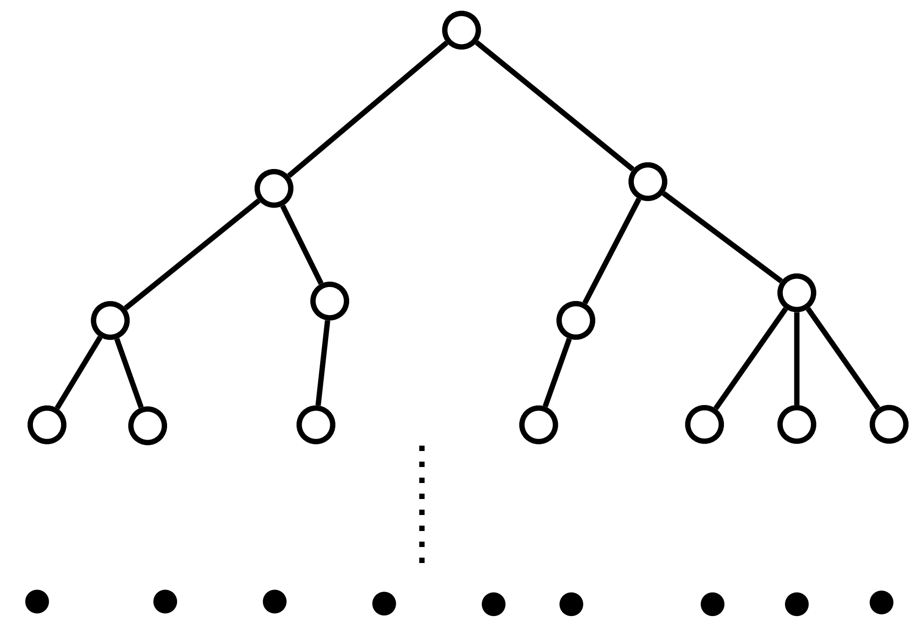

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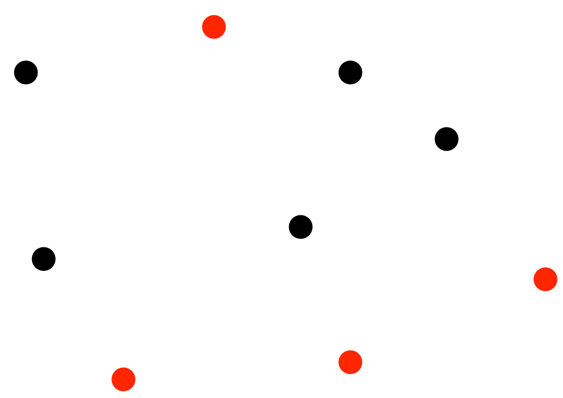
(V, d)



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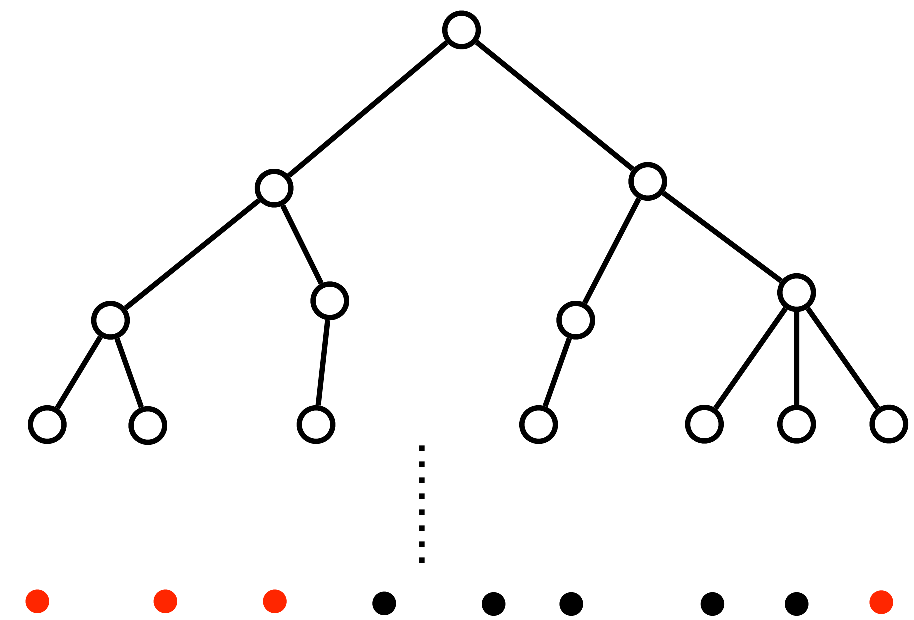
Embedding (V’, T)



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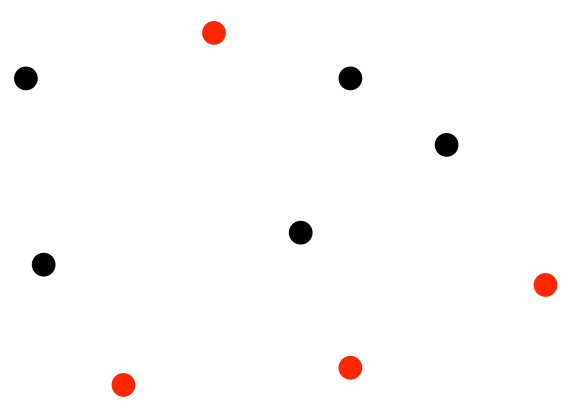

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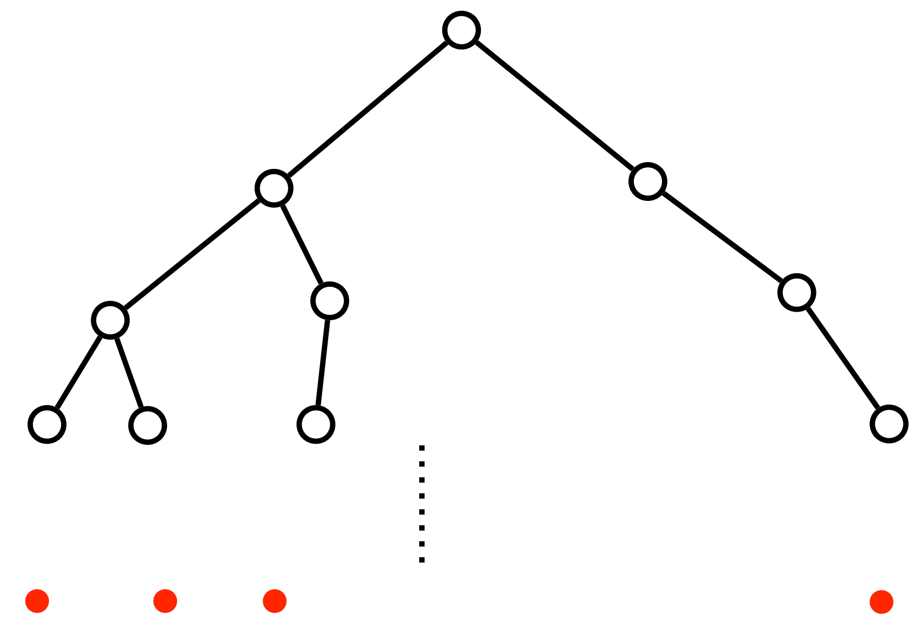

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ALG $\leq$ O(1) OPT(T) for *any* HST embedding T of terminals

HST Embeddings

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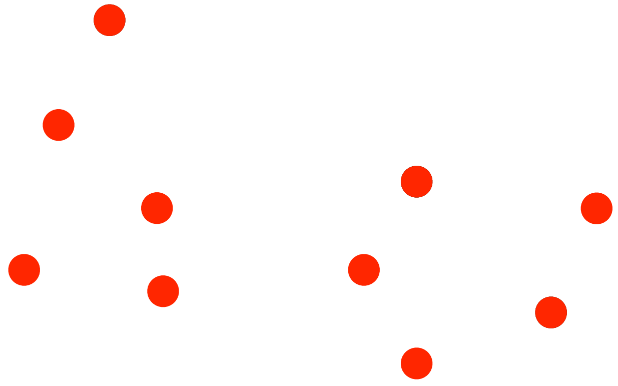
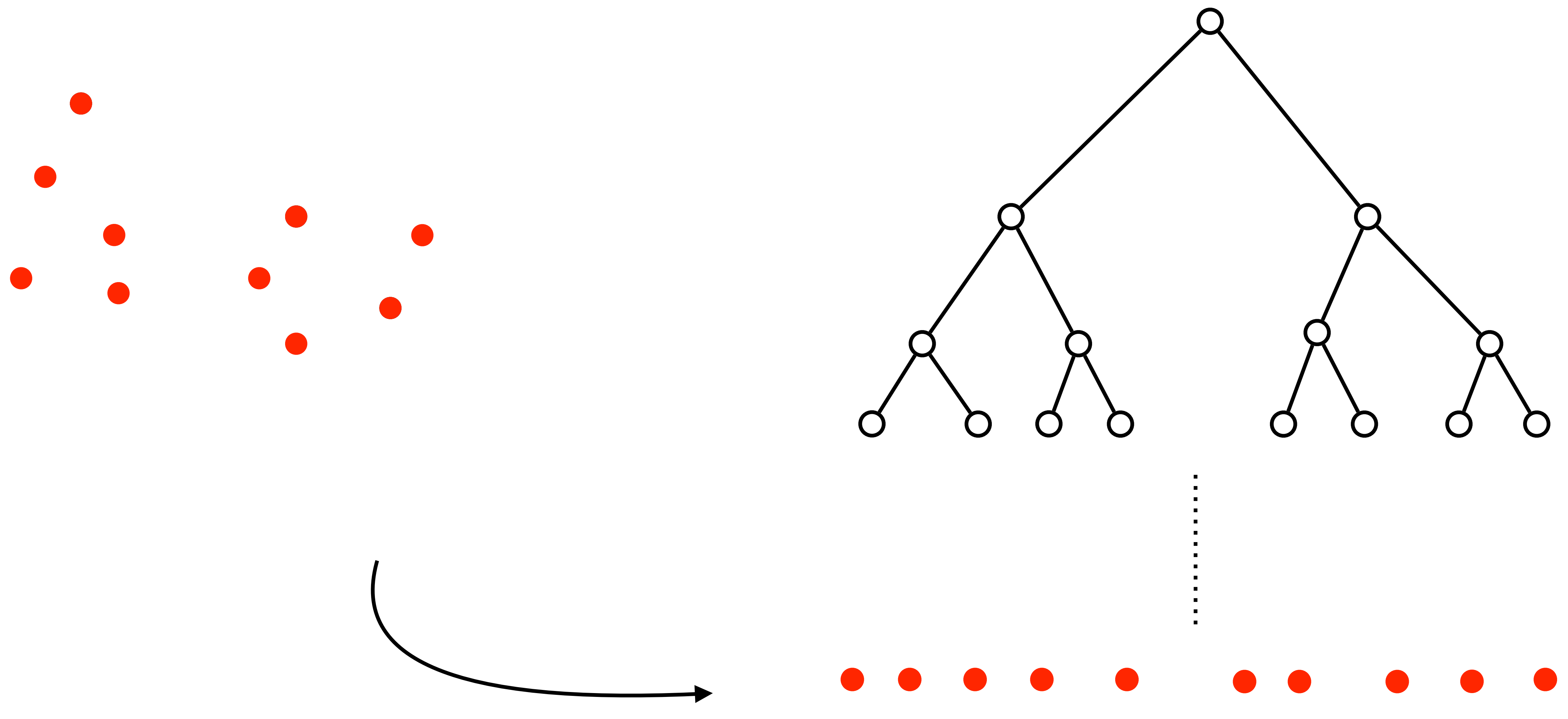
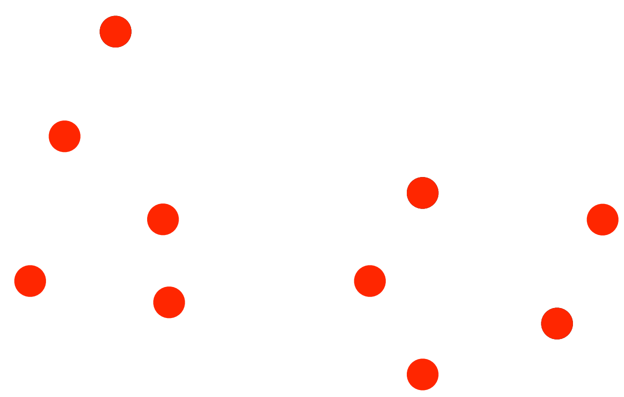


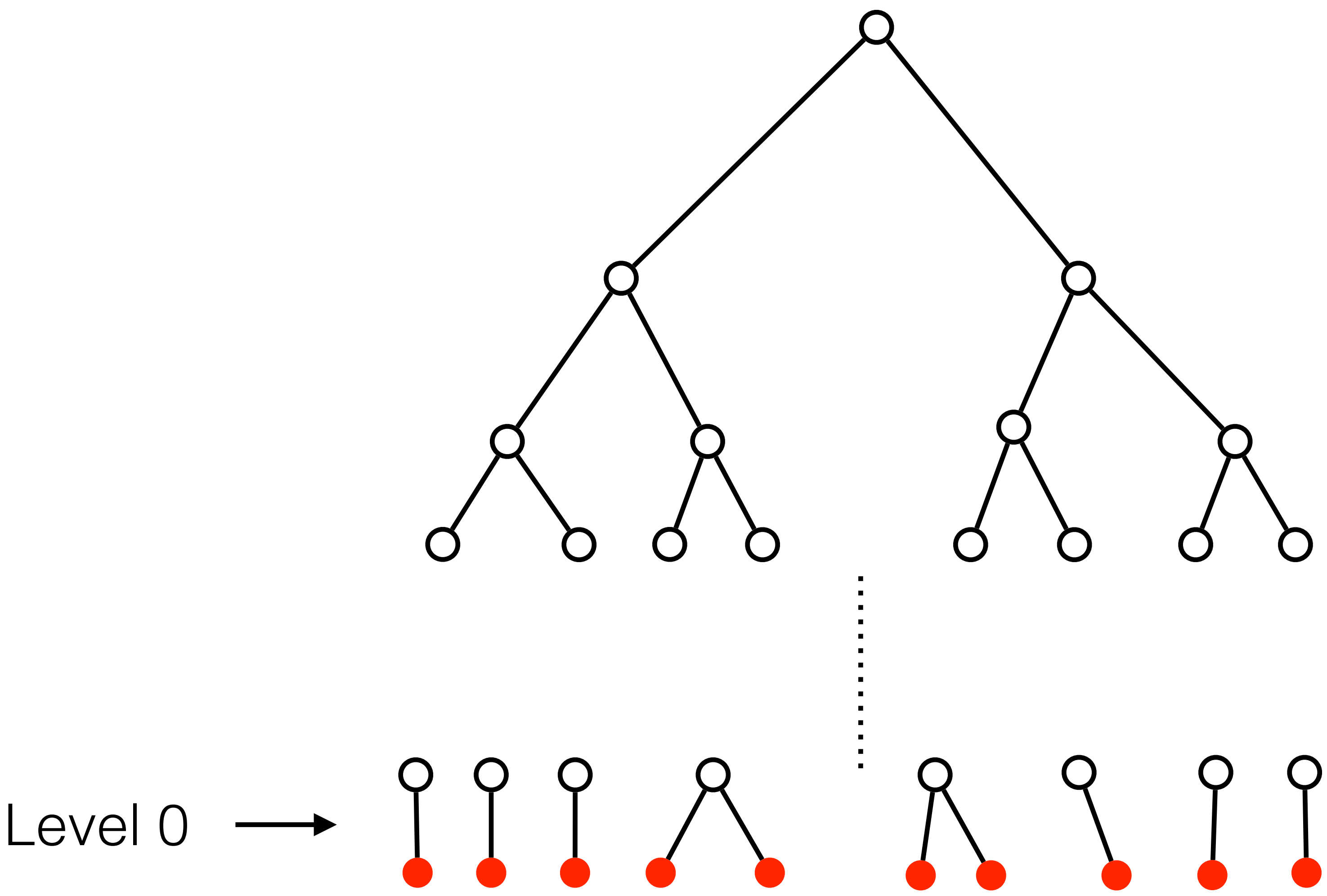
HST Embeddings



Terminals → leaves of T

HST Embeddings





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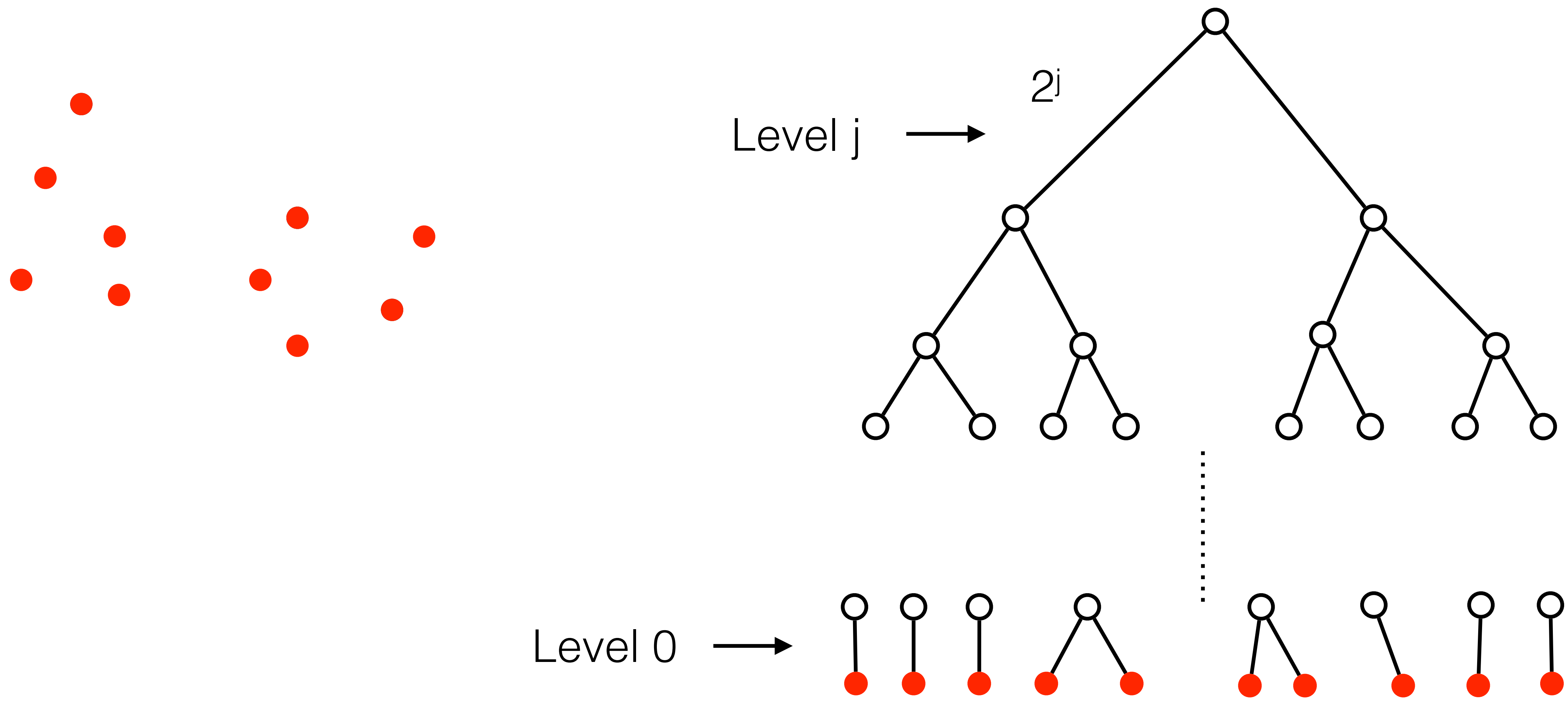

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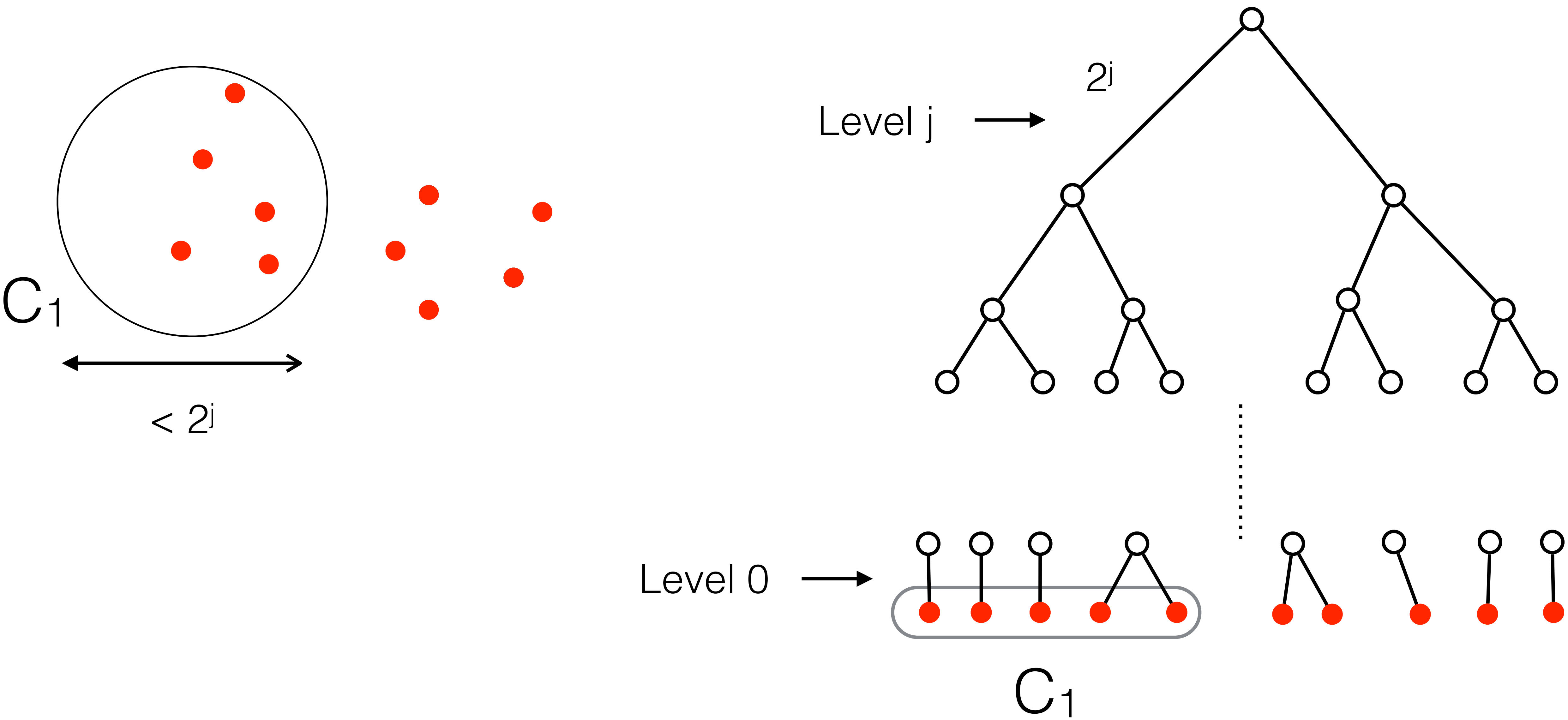
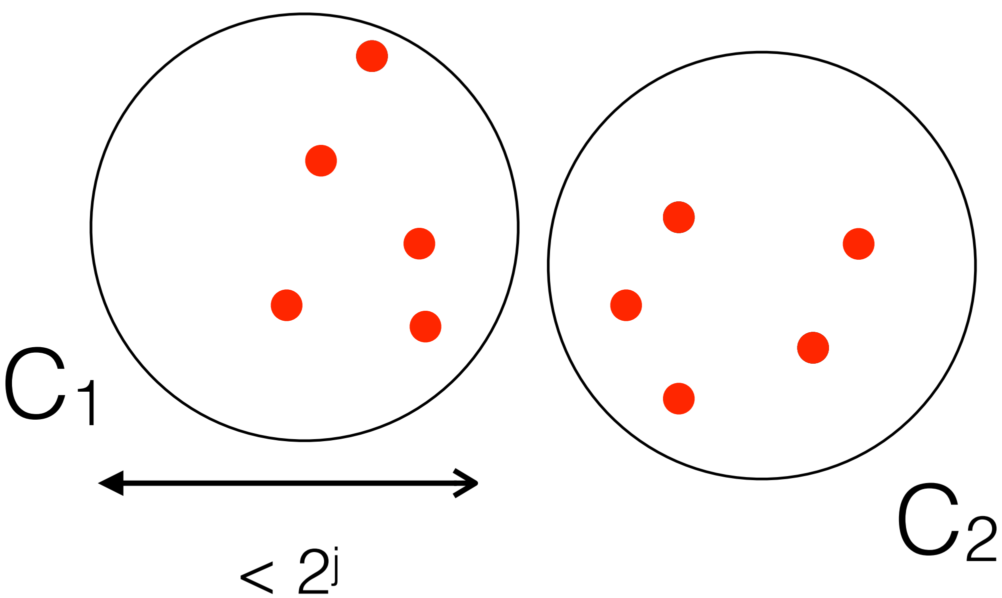
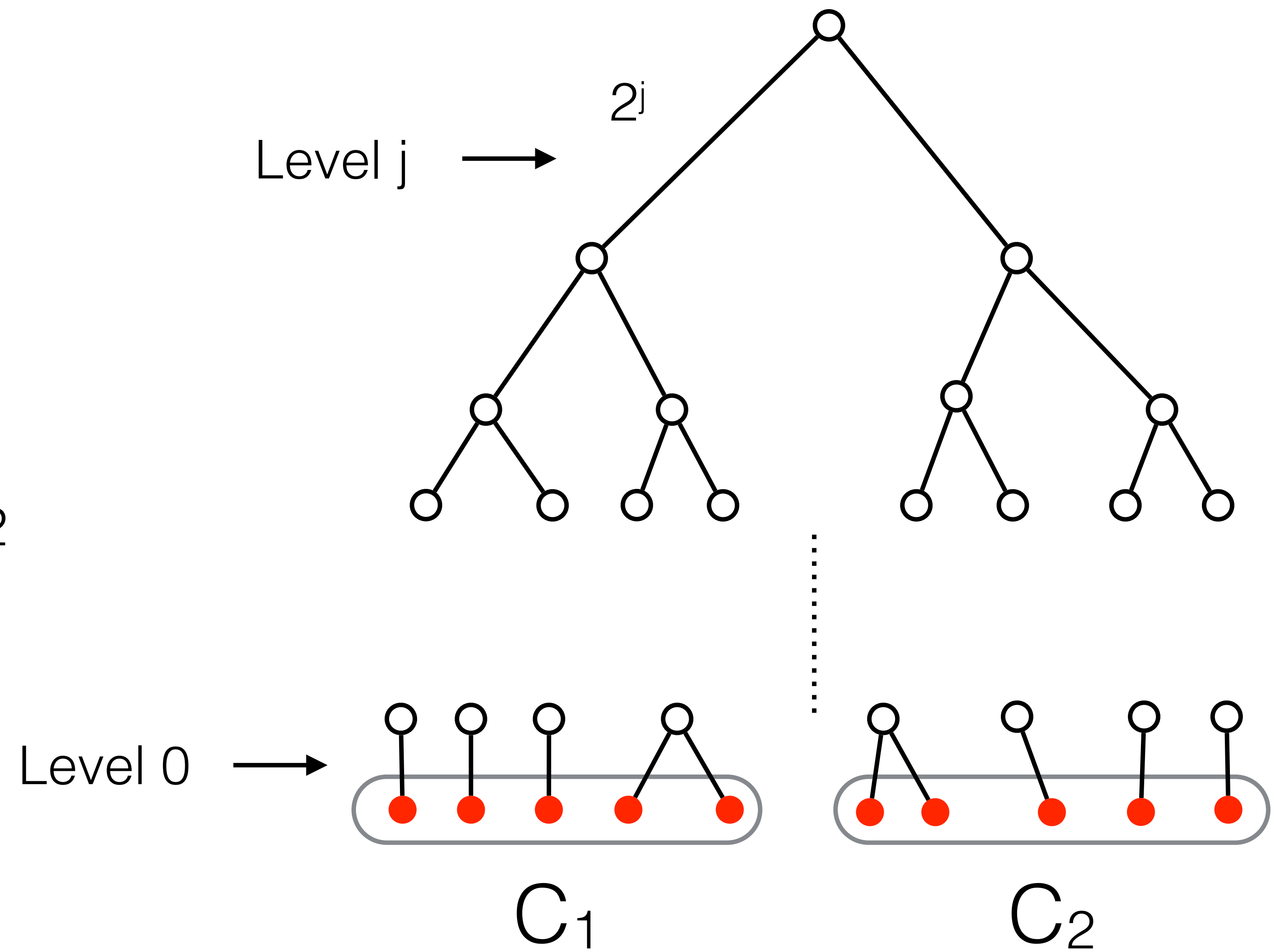


1. Leaves of level-j edge → 2^j-diam subset

HST Embeddings



2^j-diam partition



1. Leaves of level-j edge → 2^j-diam subset
2. Level-j edges → 2^j-diam decomposition

Goal: ALG $\leq$ O(1) OPT(T) for any HST embedding T

Proof Strategy:

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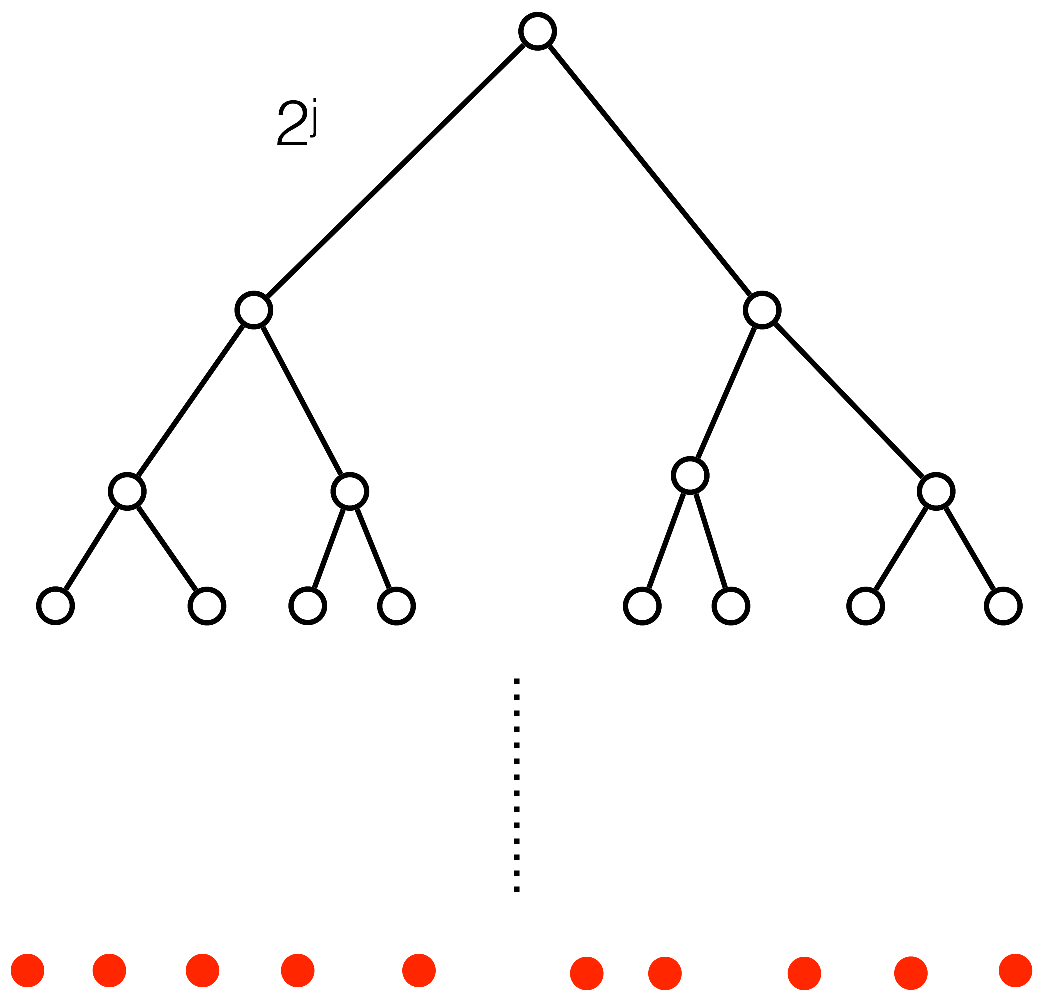

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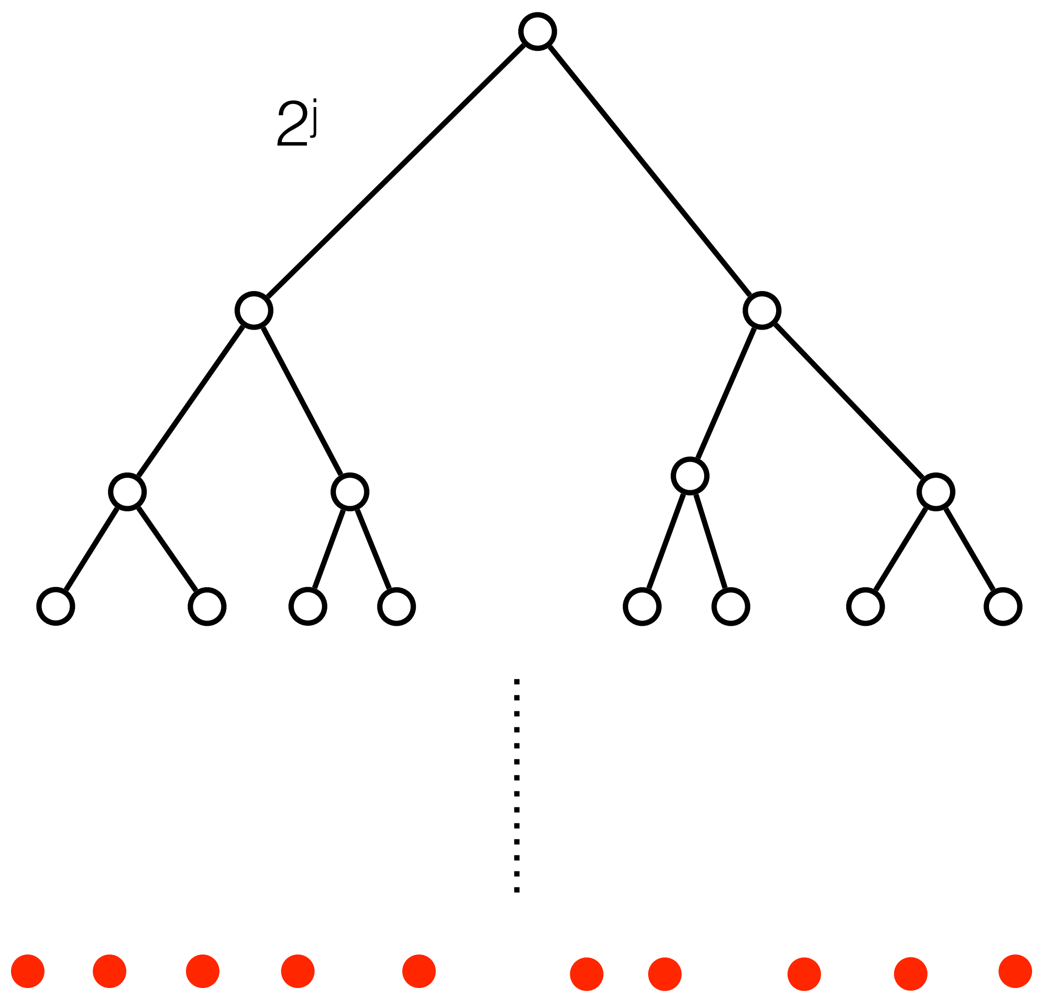

1. Decompose OPT(T) into contributions from tree edges

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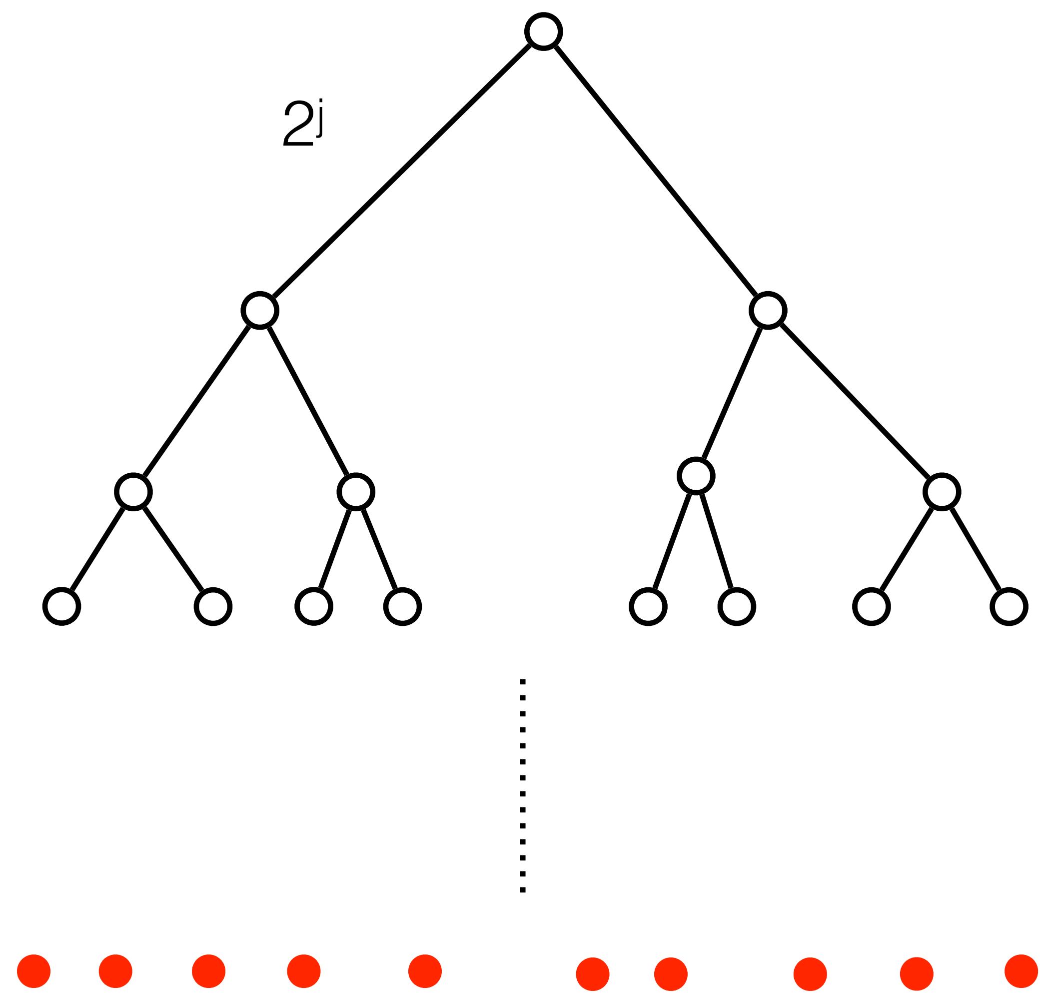

1. Decompose OPT(T) into contributions from tree edges
2. Charge ALG to tree edges

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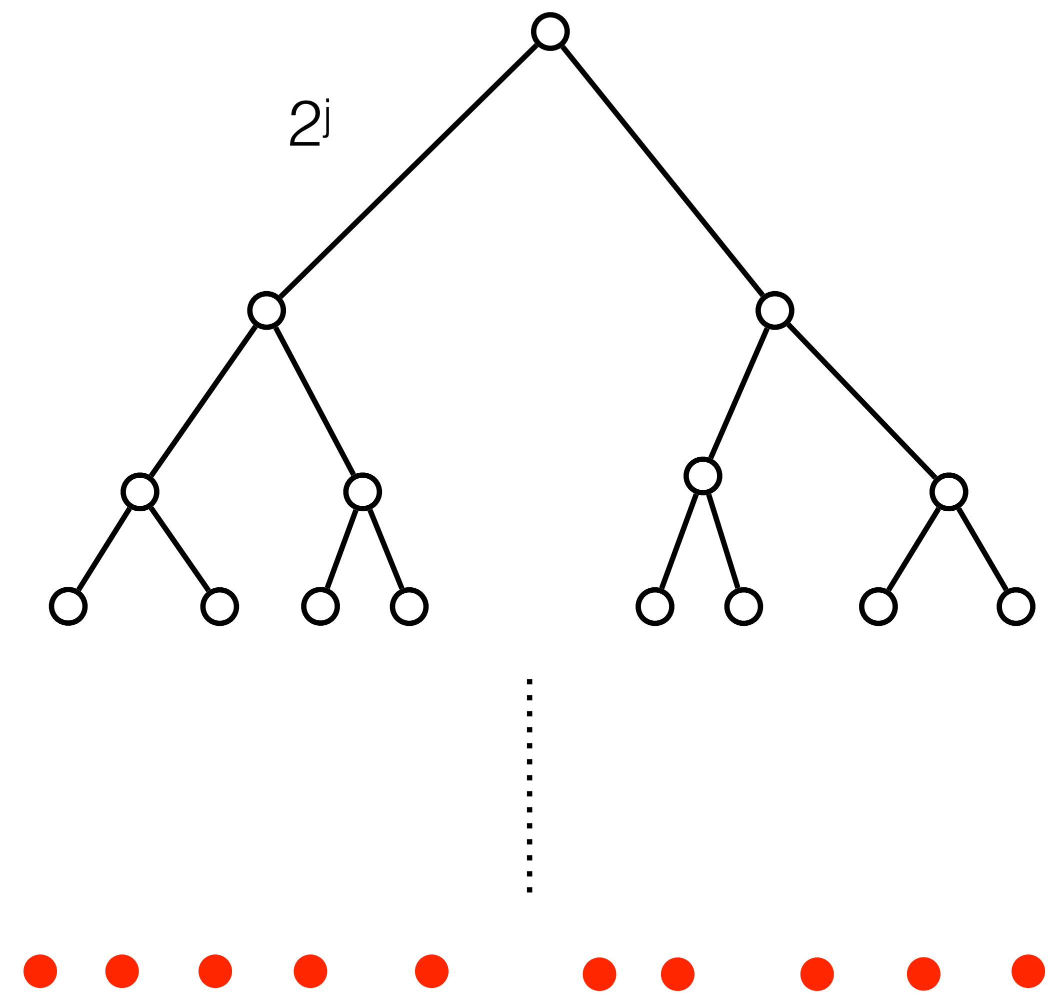
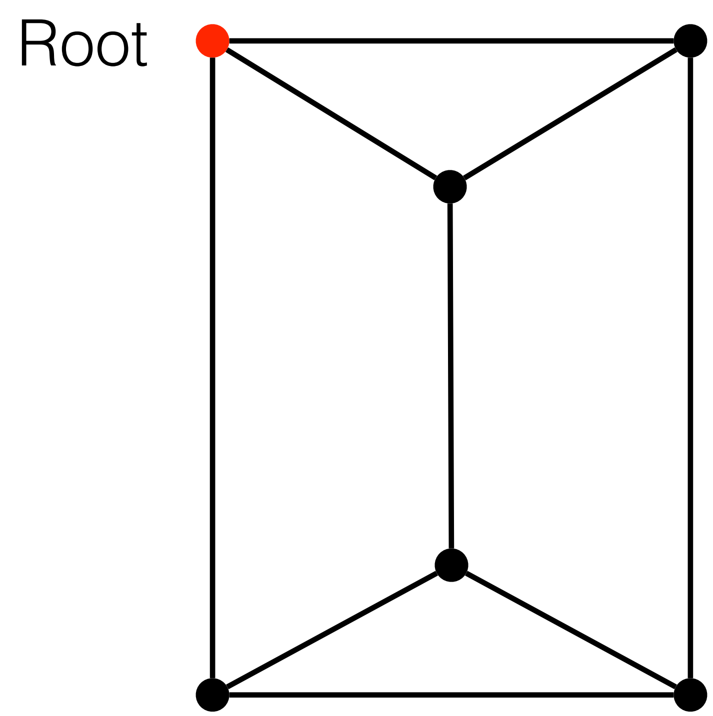


1. Decompose OPT(T) into contributions from tree edges
2. Charge ALG to tree edges
3. Use bounded-diameter property to argue no edge is overcharged

Outline

1. Overview of Analysis Framework ✔
2. Warm-Up: Steiner Tree
3. Rent-or-Buy
4. Steiner Network

Online Steiner Tree



- Terminals arrive over time
- Maintain min-cost subgraph connecting terminals to root

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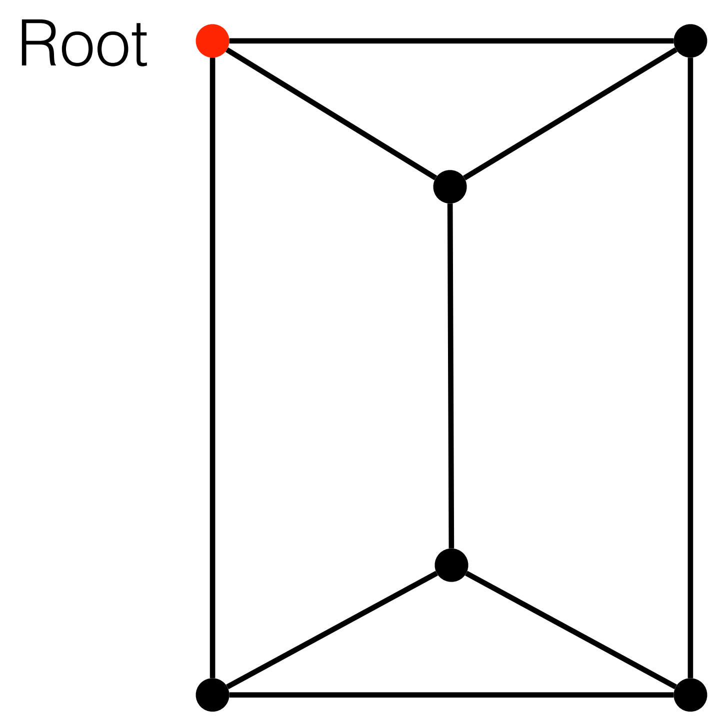

- Terminals arrive over time
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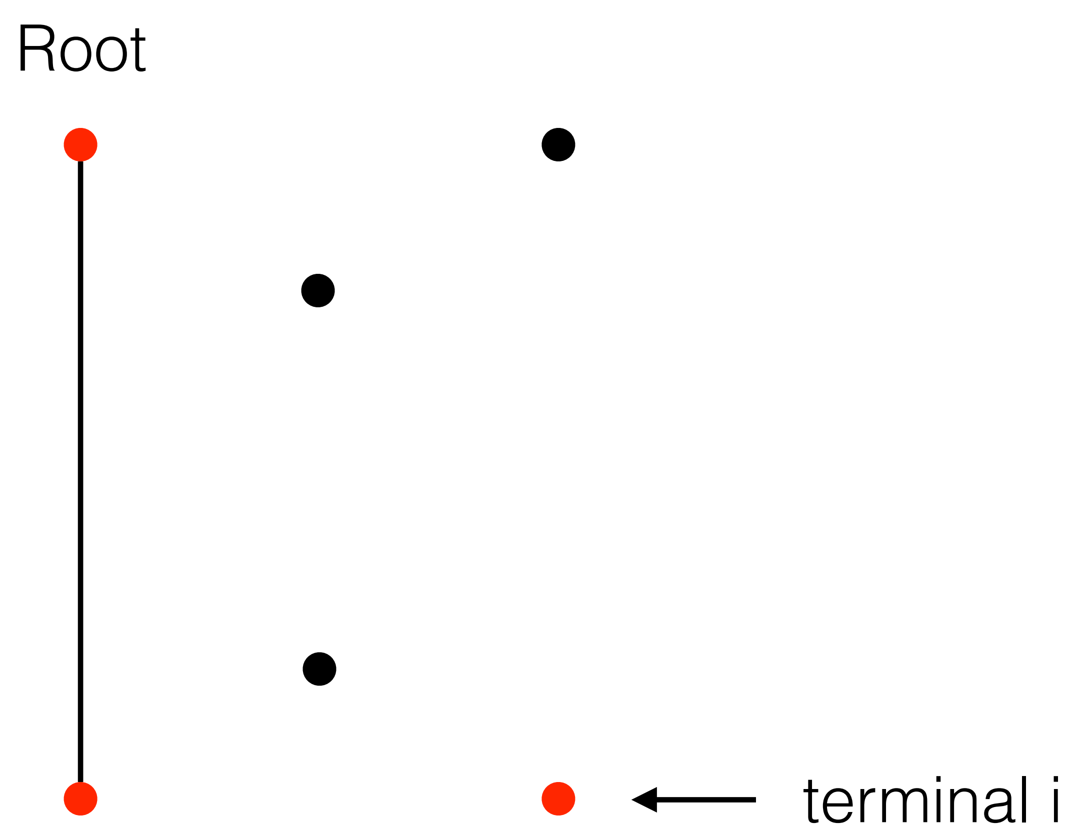
We will show: greedy algorithm is O(log k)-competitive [Imase-Waxman 91]

Greedy Algorithm

Connect current terminal to nearest previous terminal



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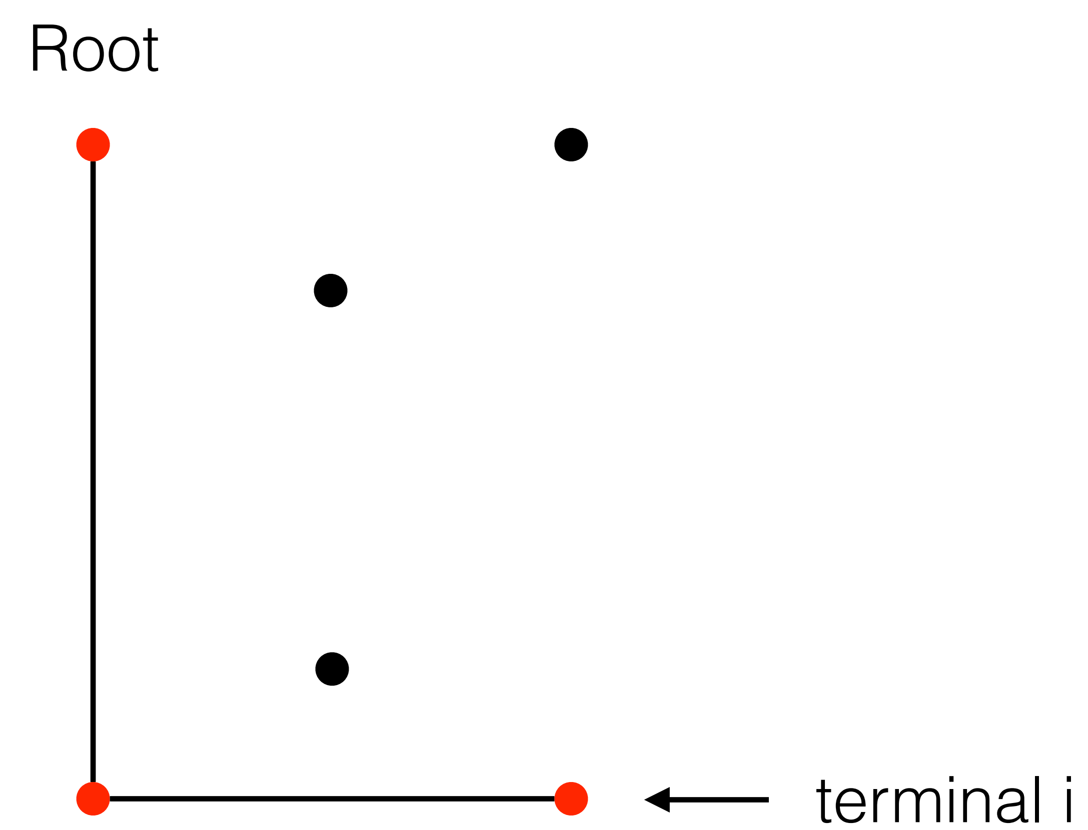

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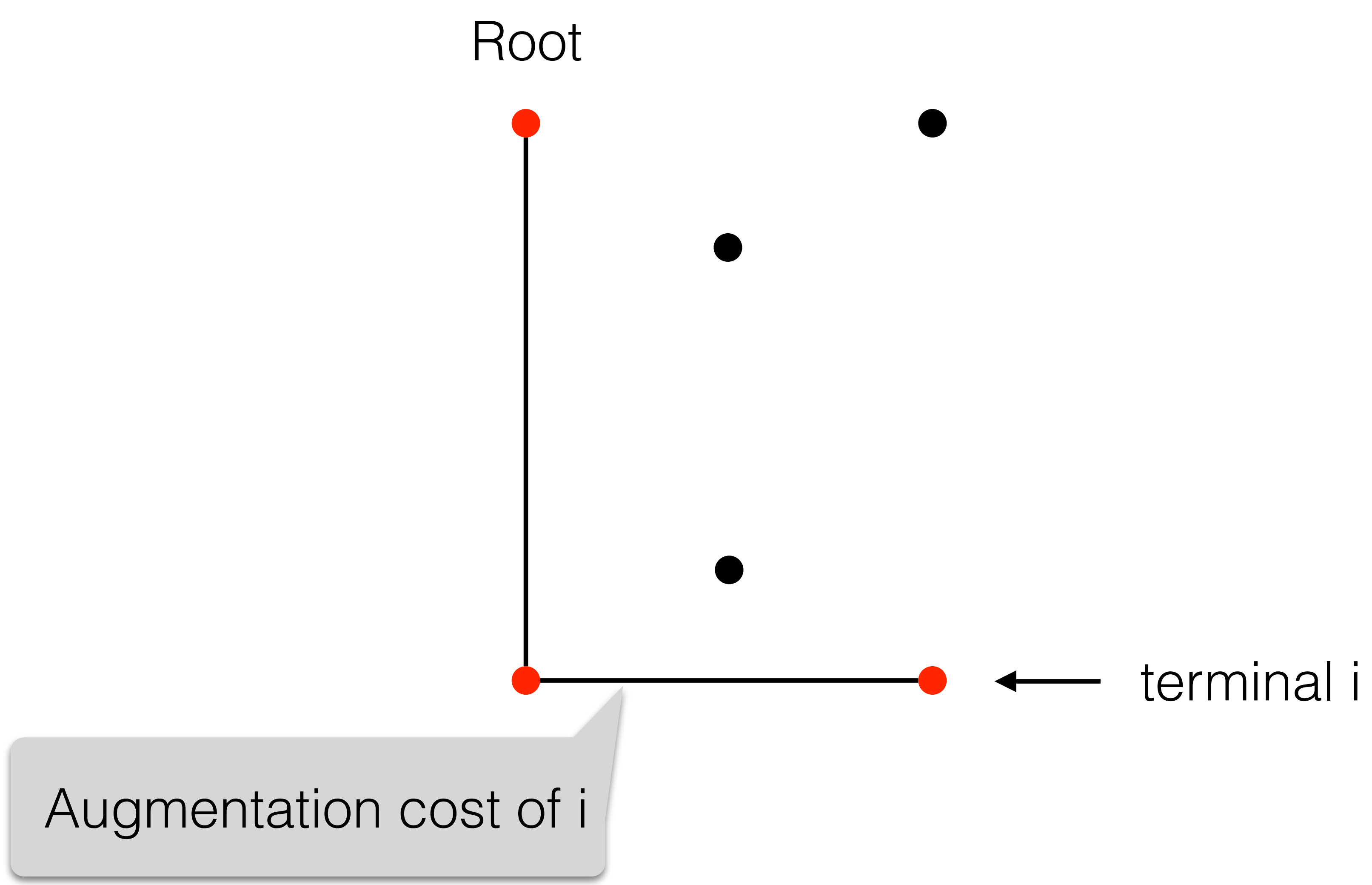

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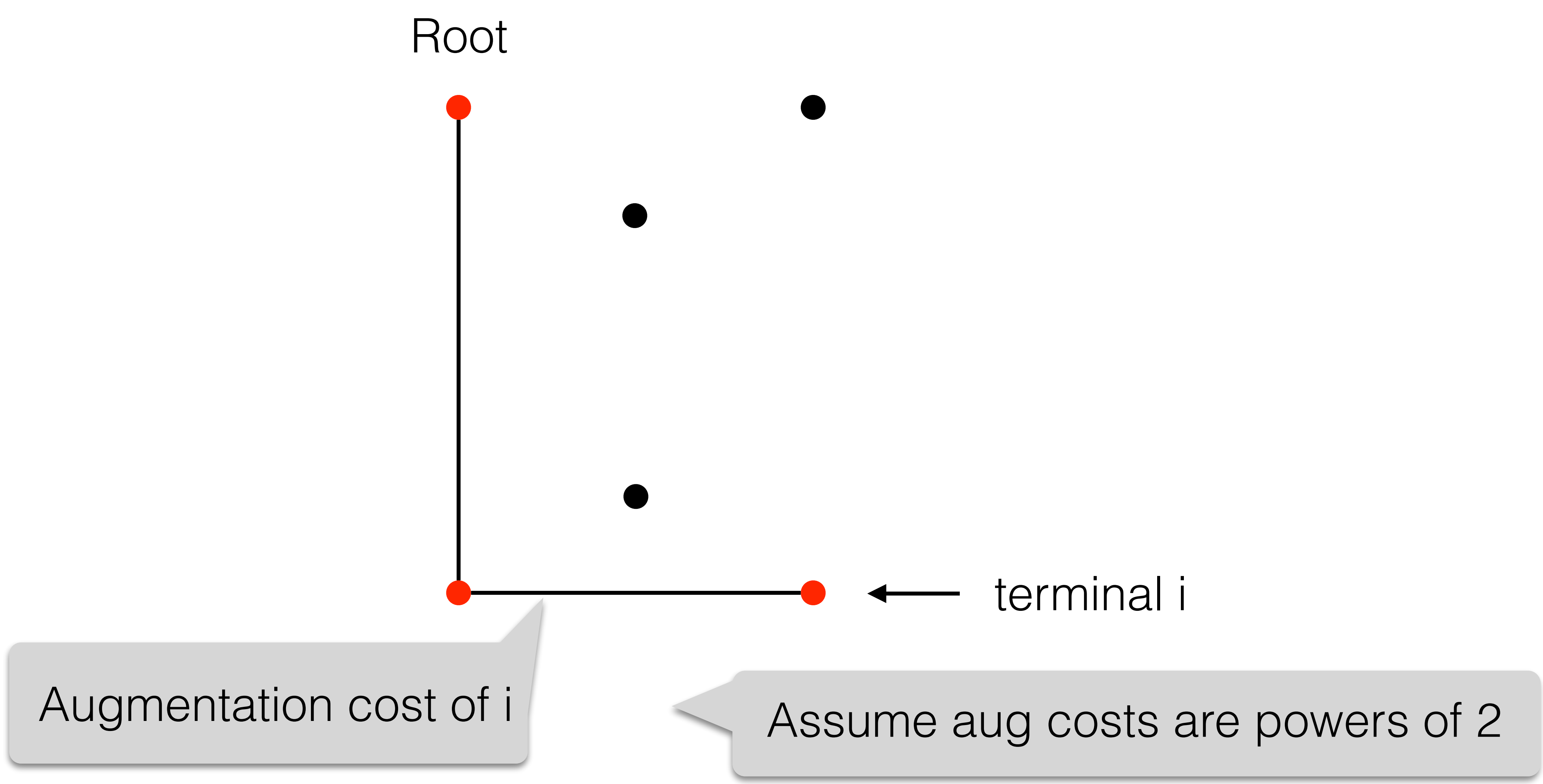

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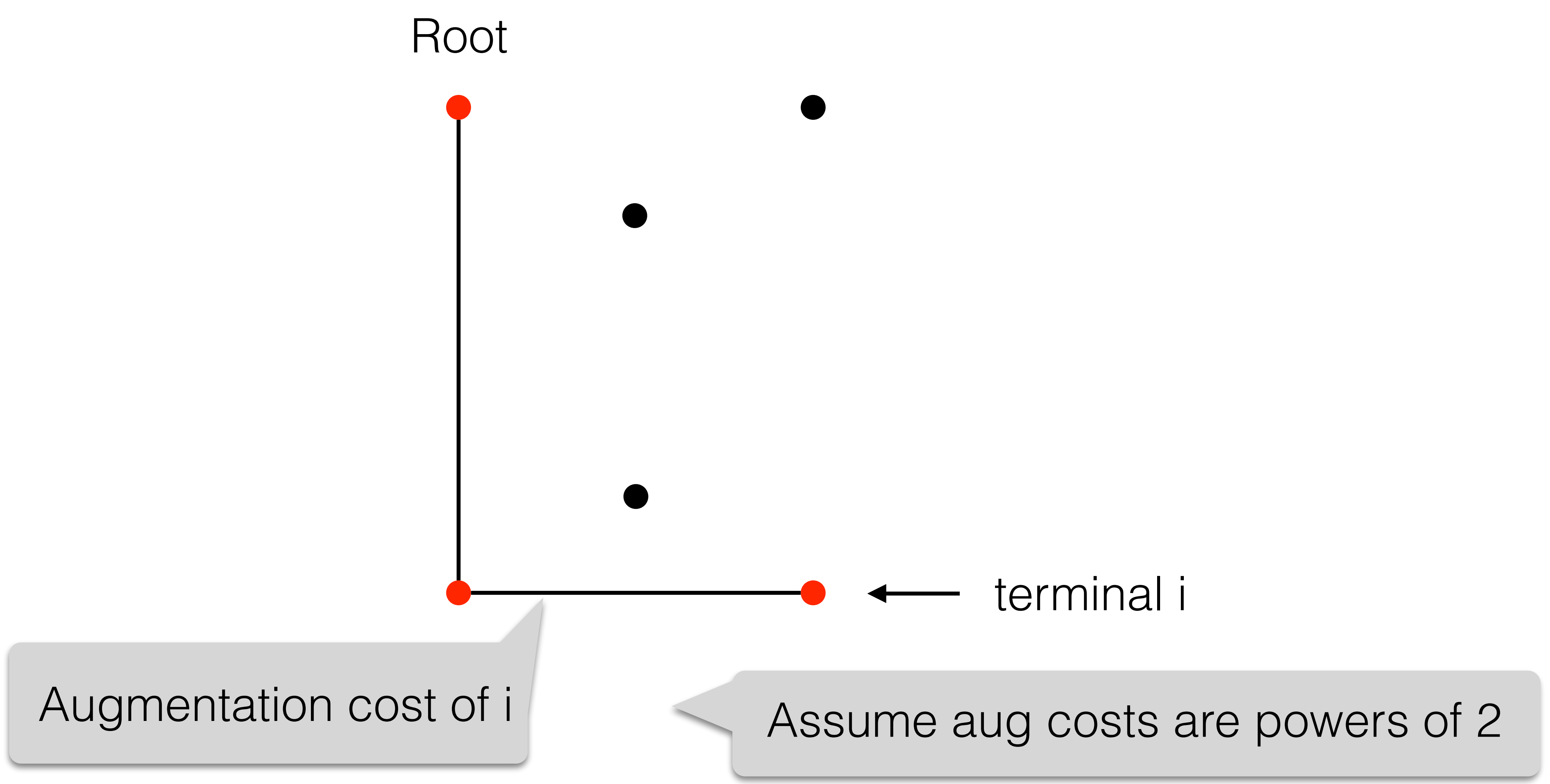
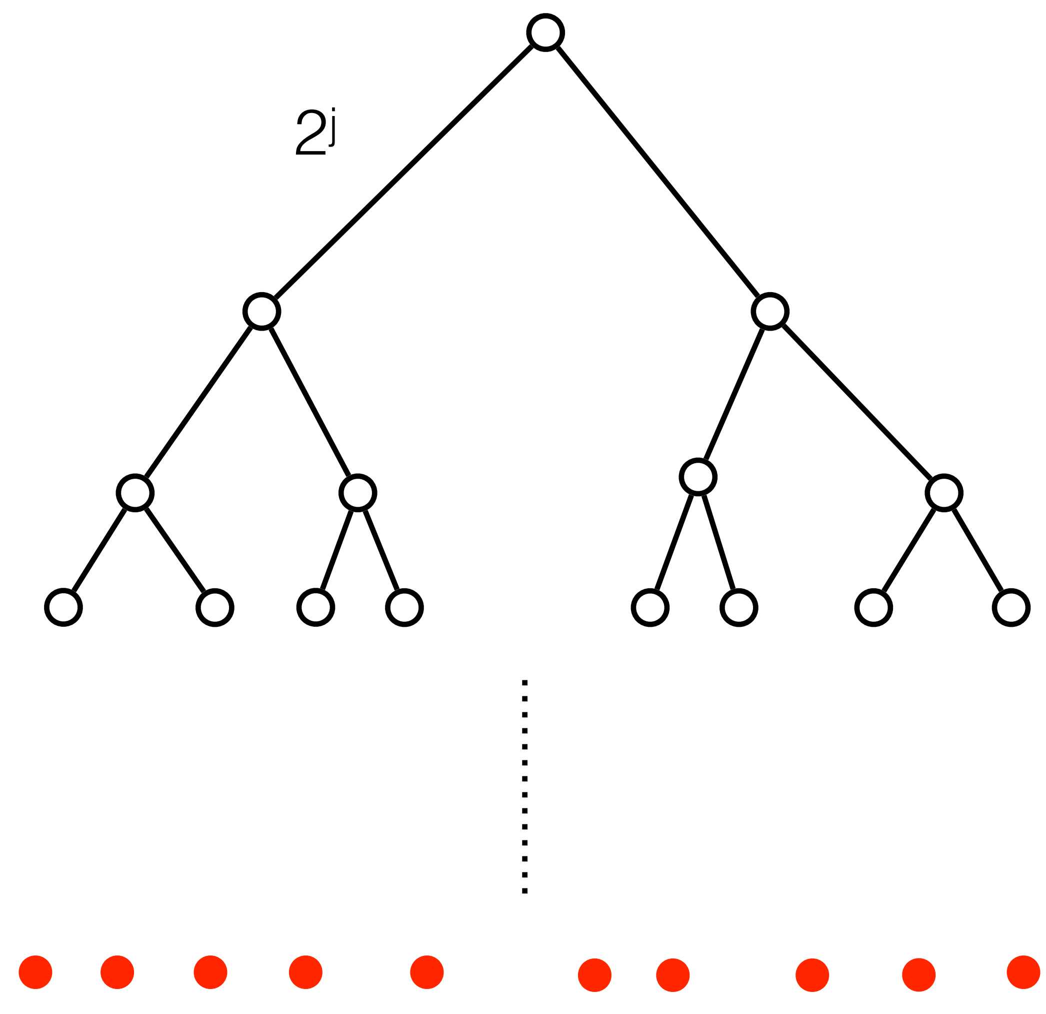


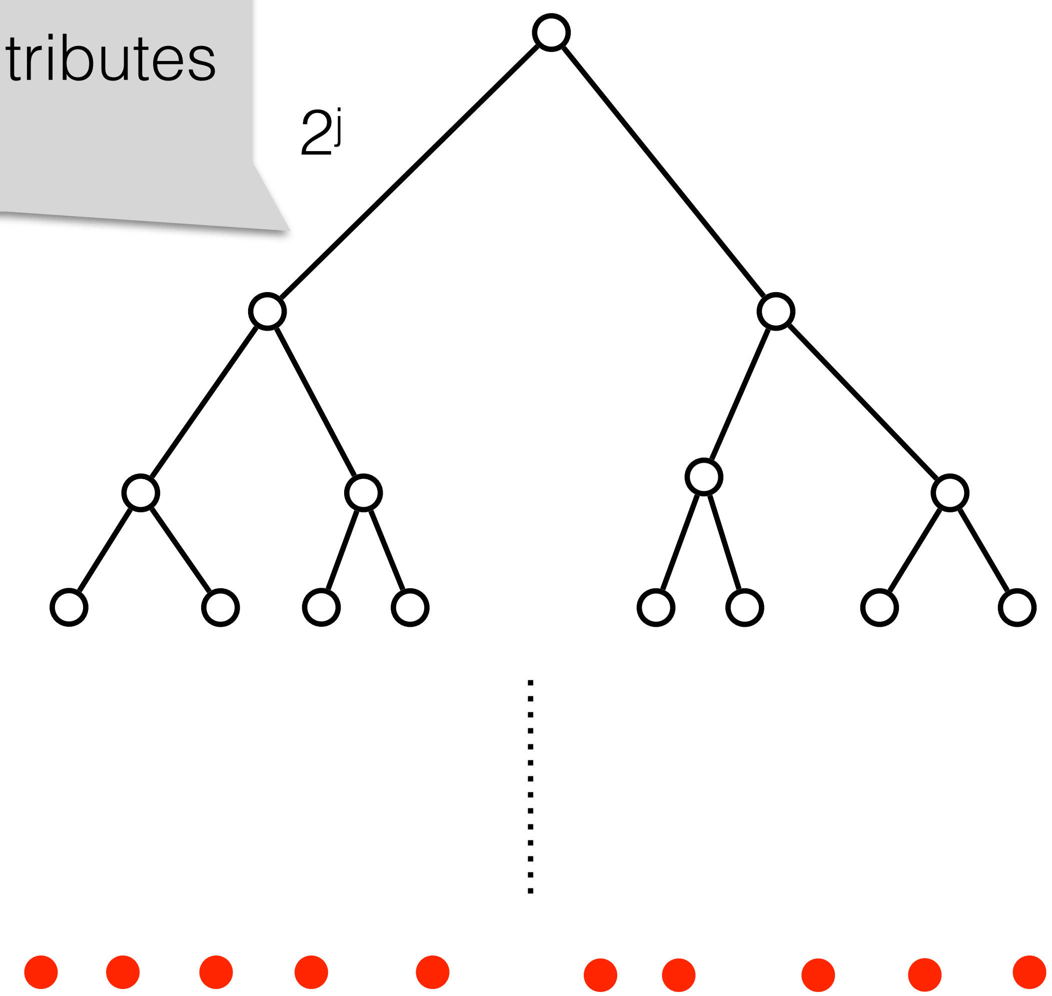
Claim: Total augmentation cost $\leq$ OPT(T)

Analysis

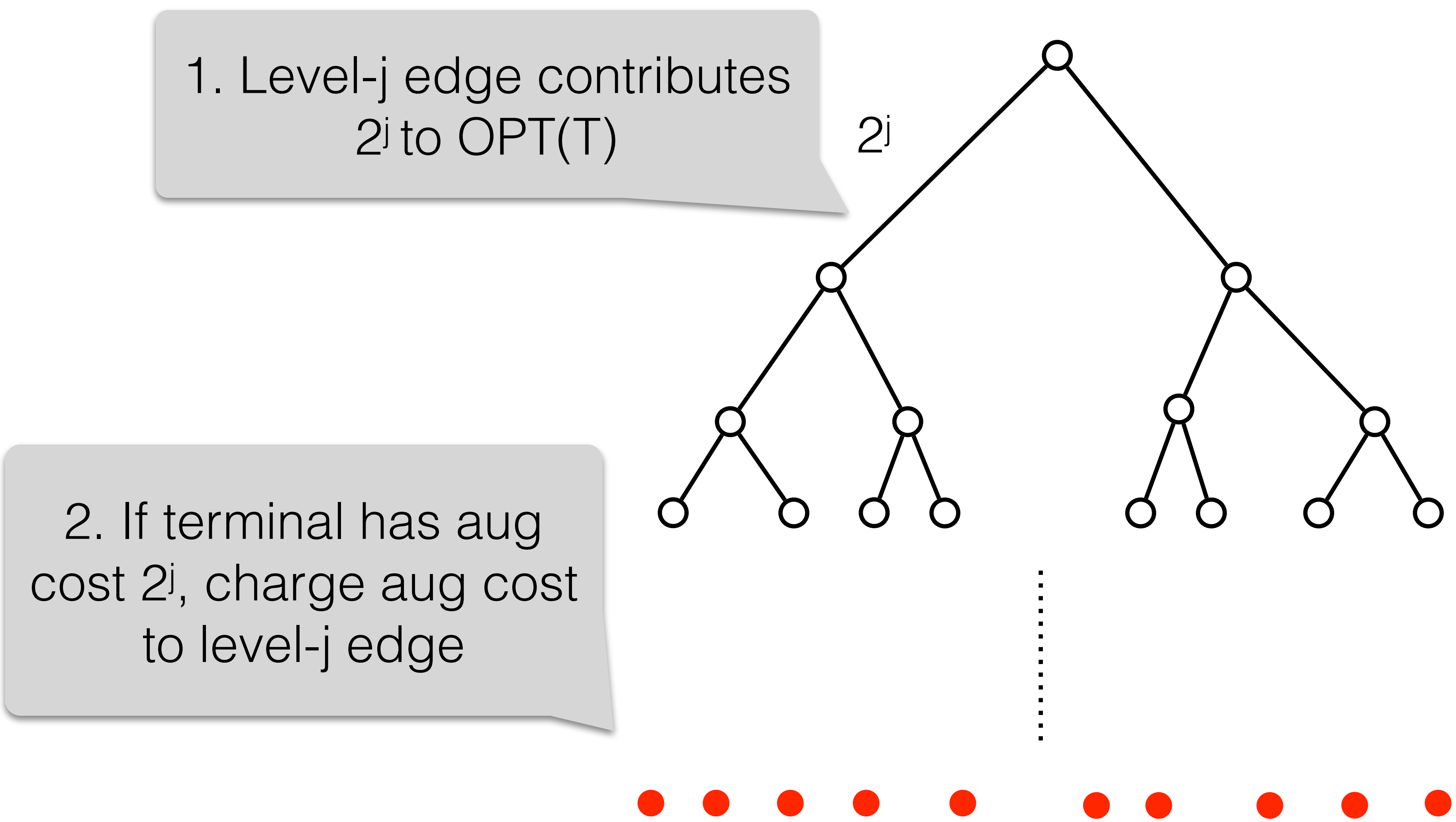


Analysis

1. Level-j edge contributes 2^j to OPT(T)



Analysis


1. Level-j edge contributes 2^j to OPT(T)
2^j
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Analysis

3a. Charged 2^j for each terminal in C with aug cost 2^j

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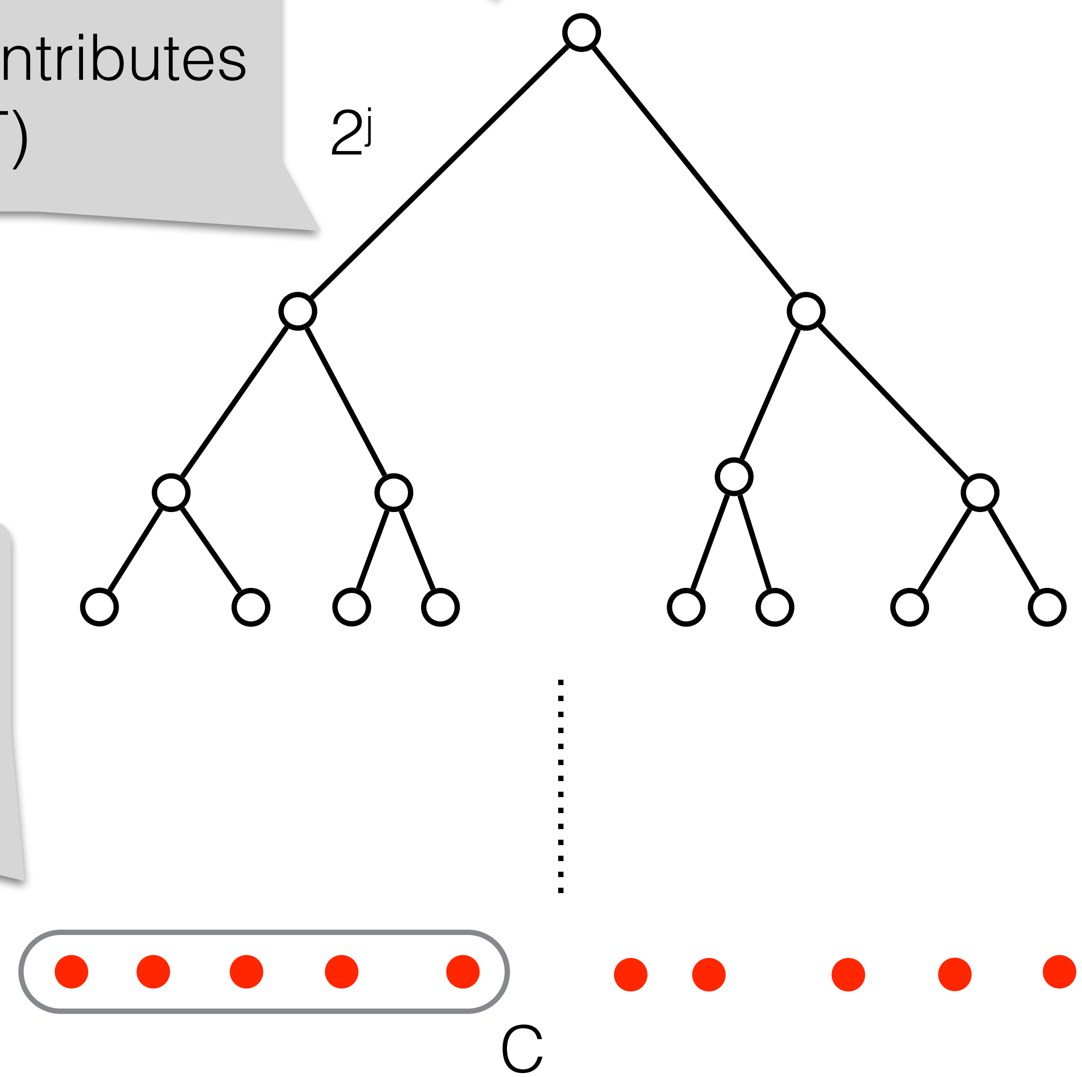

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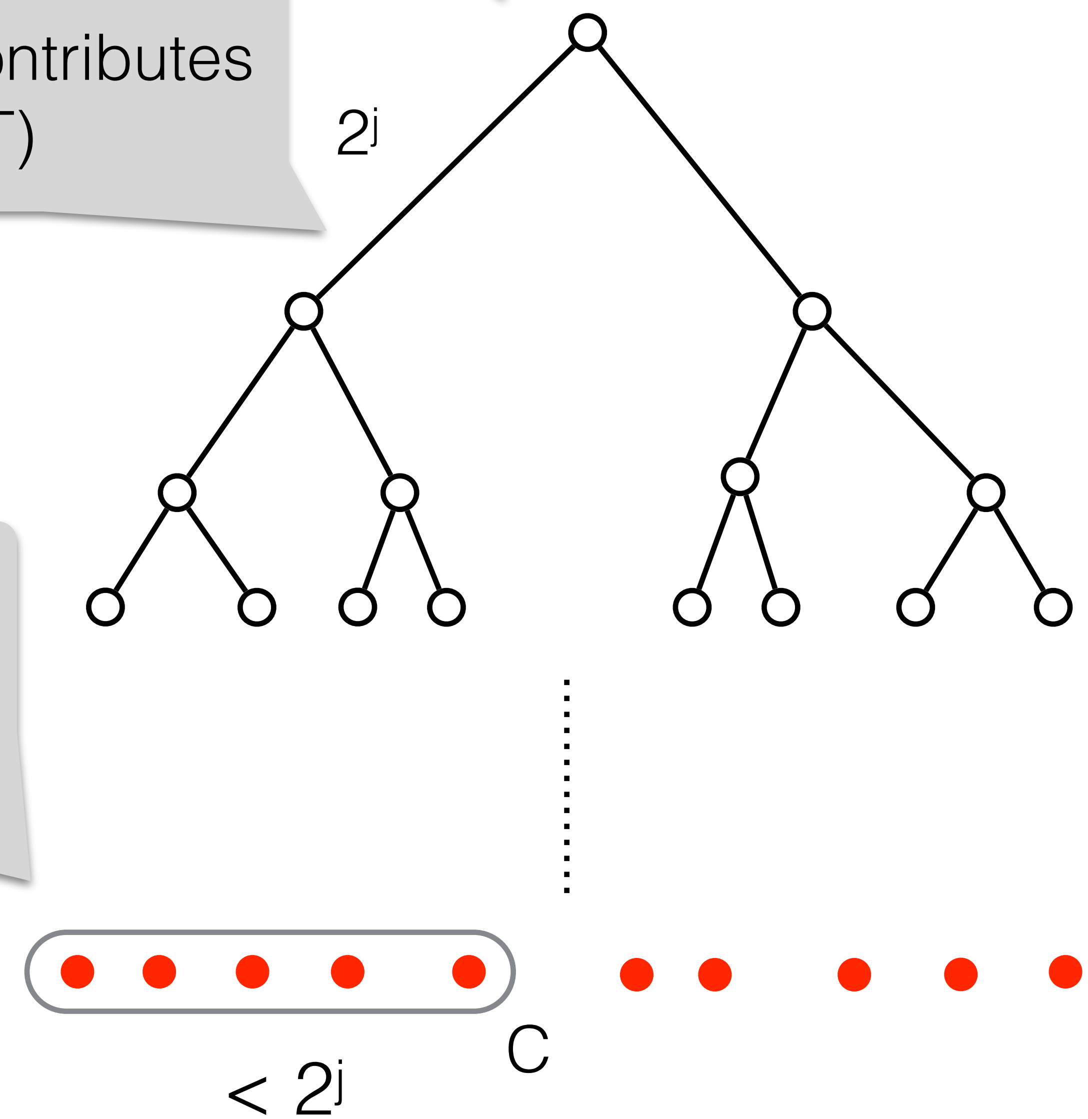

Analysis

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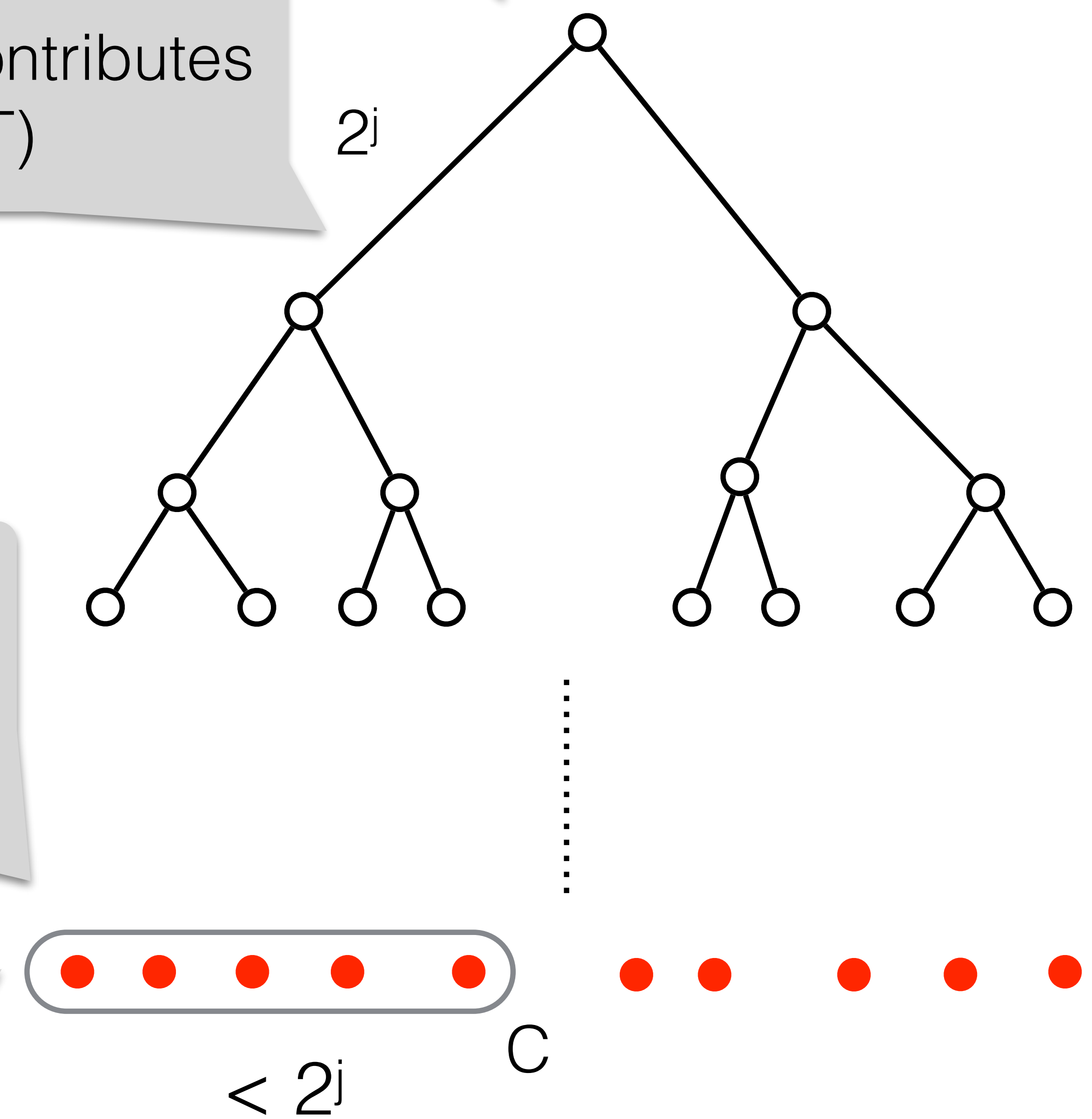
2. If terminal has aug cost 2^j, charge aug cost to level-j edge

3b. At most one terminal in C with aug cost 2^j



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Greedy is O(log k)-competitive for online Steiner tree

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Theorem [Imase-Waxman 91]:
Greedy is O(log k)-competitive for online Steiner tree

We also get a simpler analysis for the online Steiner forest algorithm of Berman-Coulston.

Outline

1. Overview of Analysis Framework ✔
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Rent-or-Buy

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- Initially, given root and parameter $M \geq 1$

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Cost:

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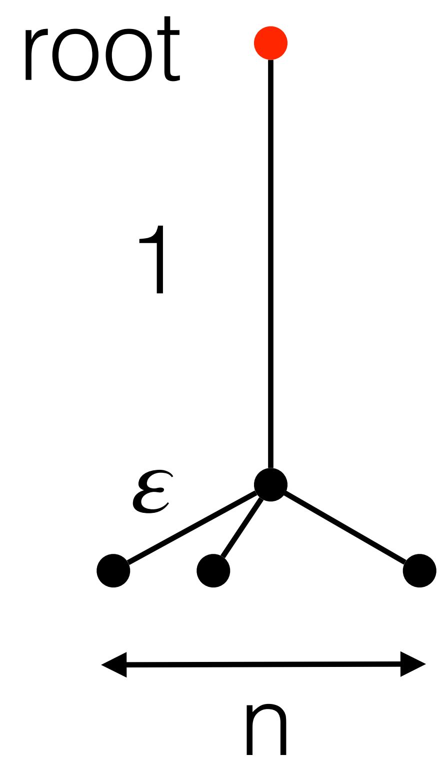
Cost: $\underbrace{Mc(H)}_{\text{Buy cost}} + \sum_i \underbrace{c(Q_i)}_{\text{Rent cost of i}}$

Example

Tree instance



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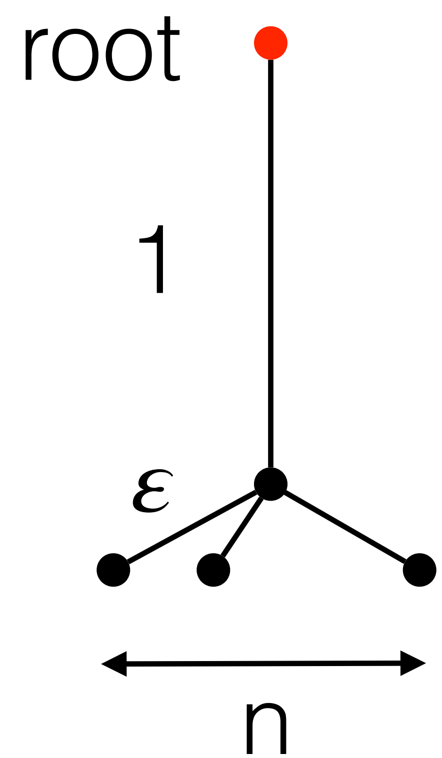
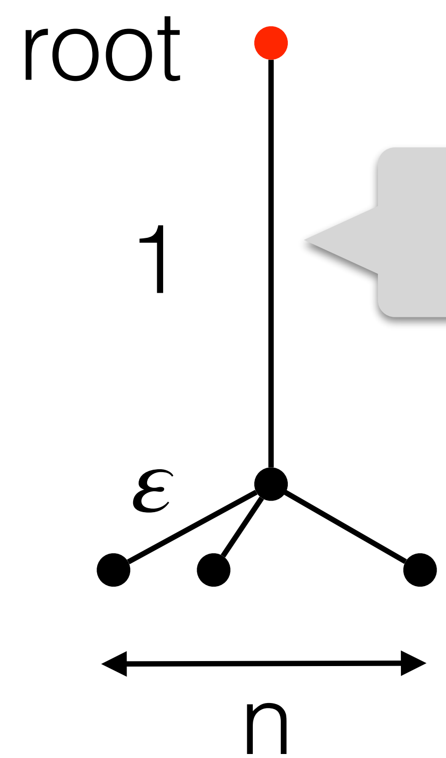
Tree instance



Terminals are leaves

Example

Tree instance

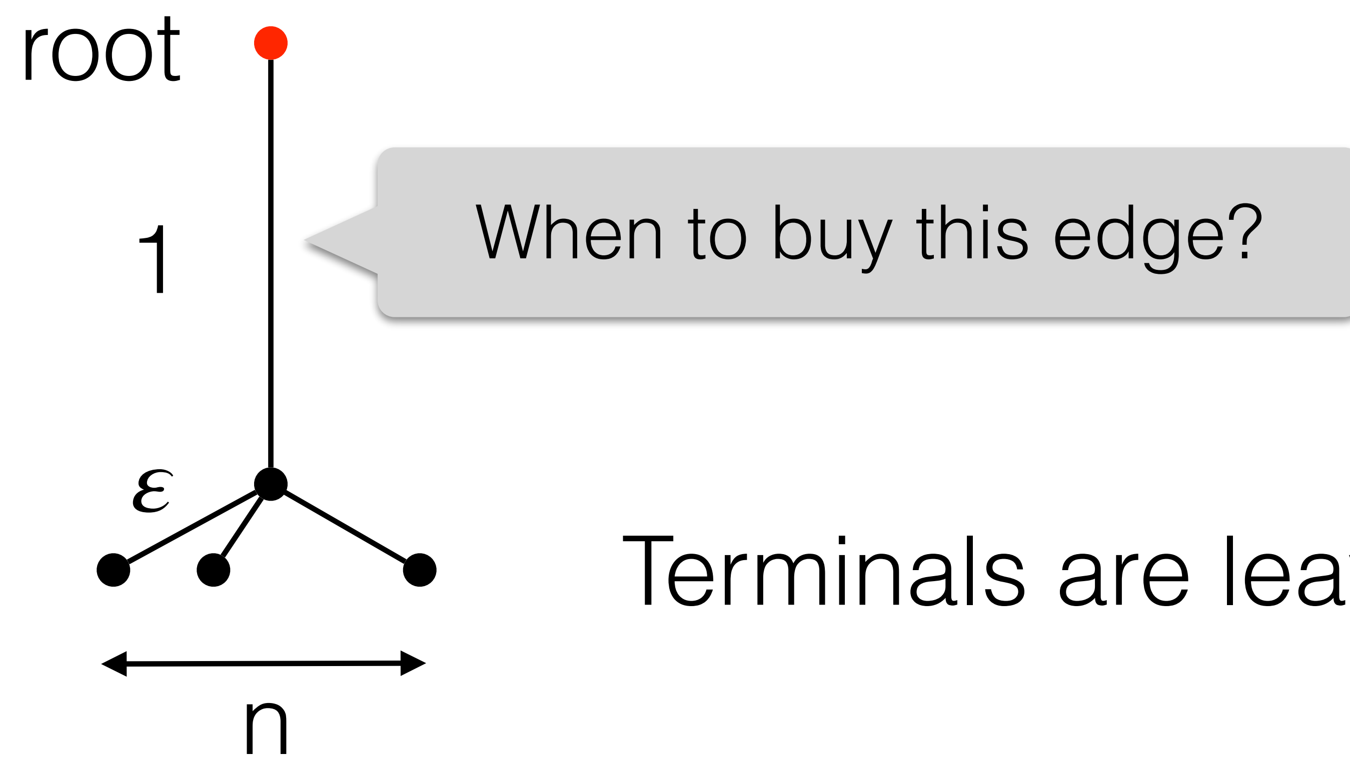


When to buy this edge?

Terminals are leaves

Example

Tree instance

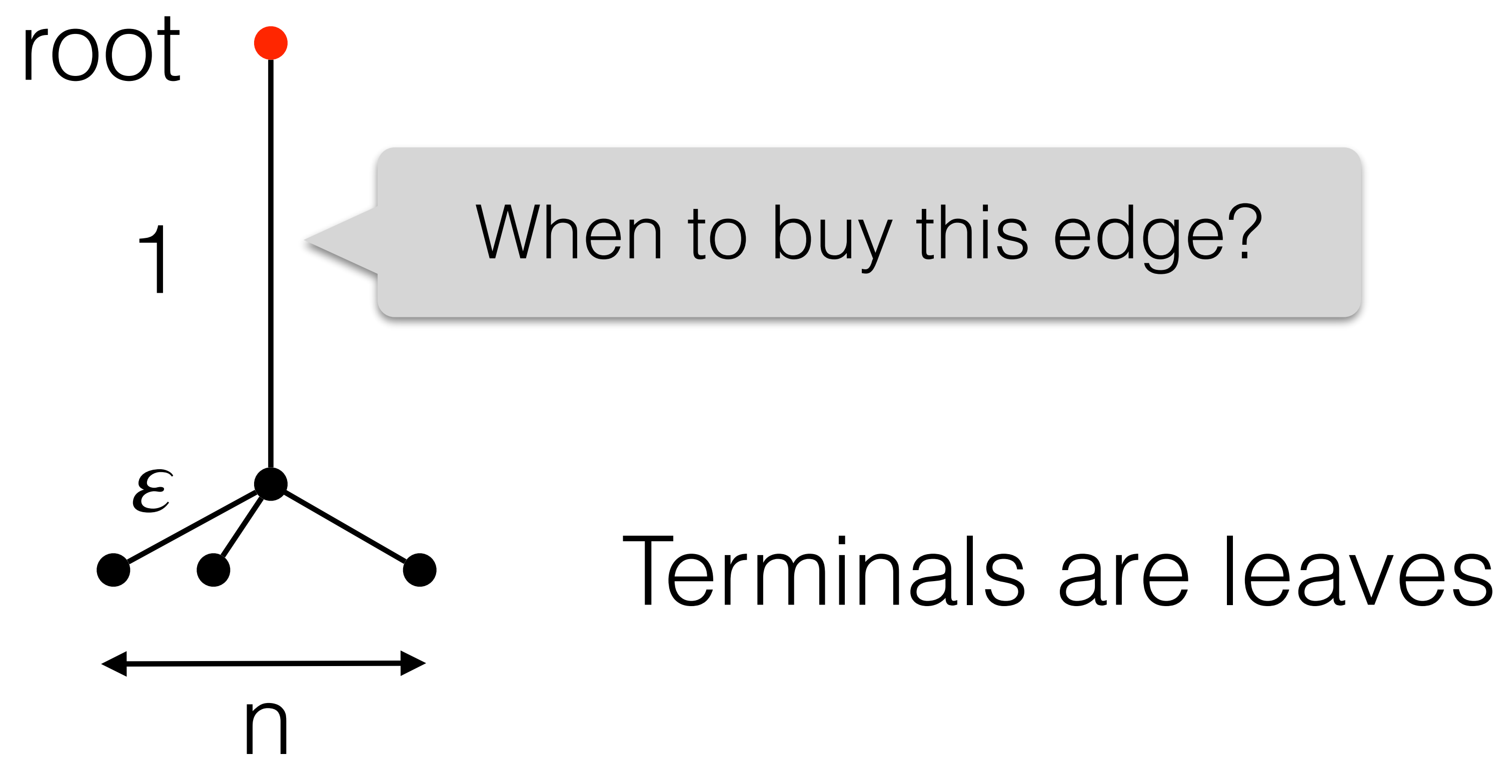


Terminals are leaves

OPT = min {#terminals, M}

Example

Tree instance



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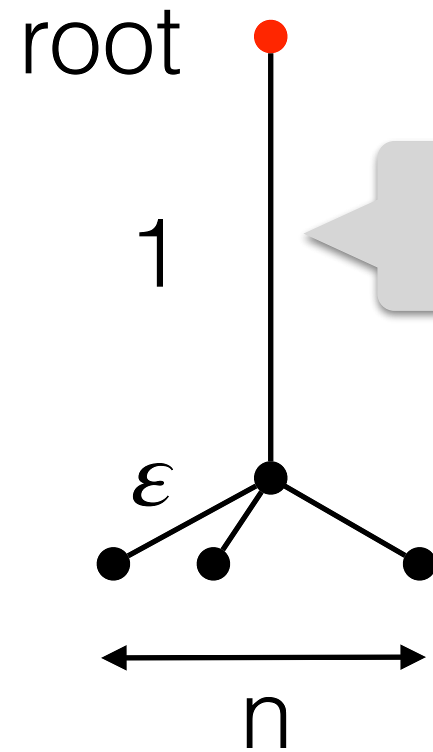
<u>Online Algorithm</u>

First M terminals: Rent
(M+1)th terminal: Buy

Example

Tree instance



When to buy this edge?

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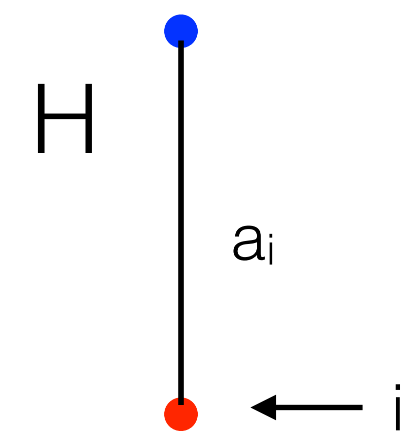
<u>Online Algorithm</u>

First M terminals: Rent
(M+1)th terminal: Buy

Buy when sufficient rent cost to pay for buy cost (break-even rule)

<u>Algorithm:</u>

- When a terminal i arrives:
    - Let e be shortest edge to H and a_i its length
    - If $\geq$ M terminals with rent cost $\geq a_i$ within distance a_i, buy e at cost Ma_i
    - Otherwise, rent e at cost a_i



Algorithm:

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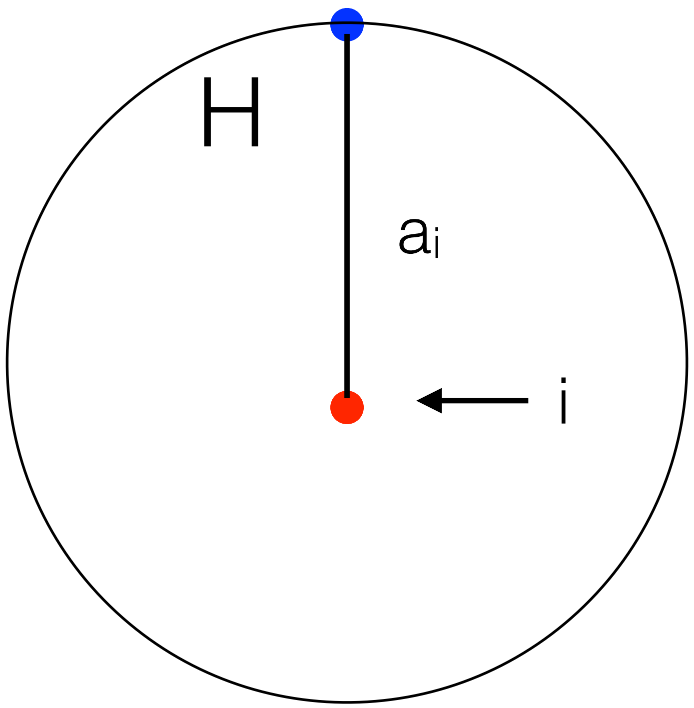

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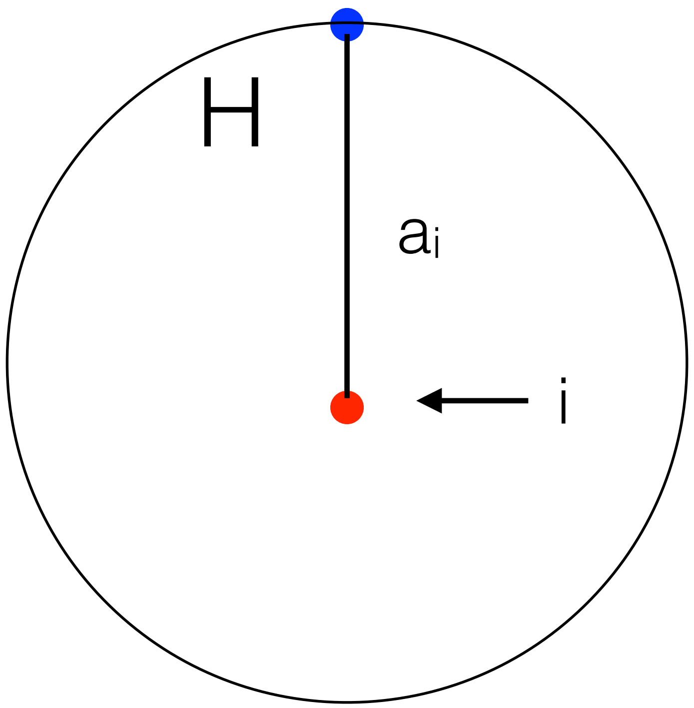

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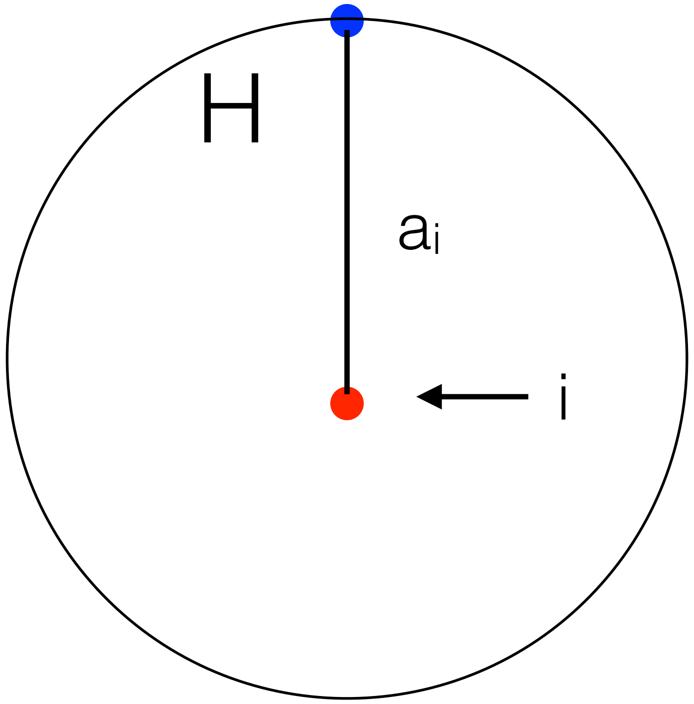

Analysis:

Lemma: Buy cost $\leq$ O(1) Rent cost

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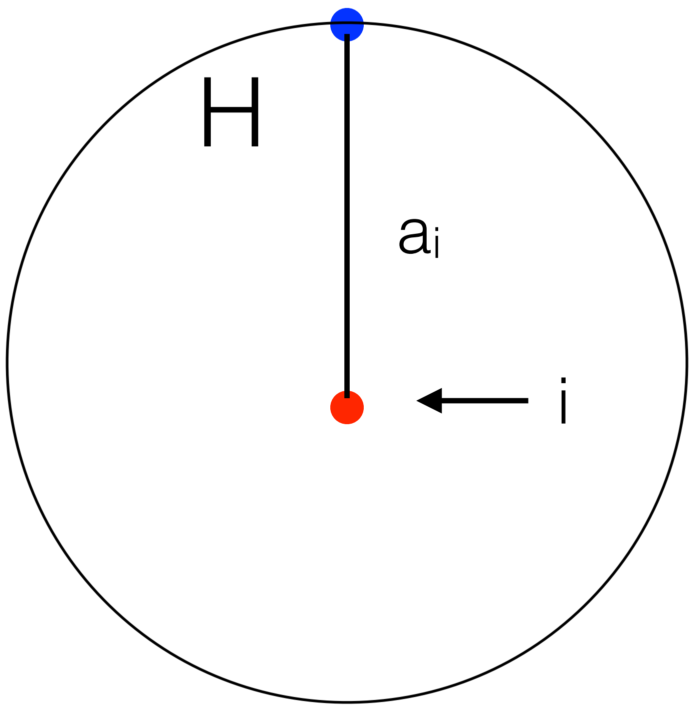

Analysis:

Lemma: Buy cost $\leq$ O(1) Rent cost

Lemma: Rent cost $\leq$ O(1)OPT(T) for any HST embedding T

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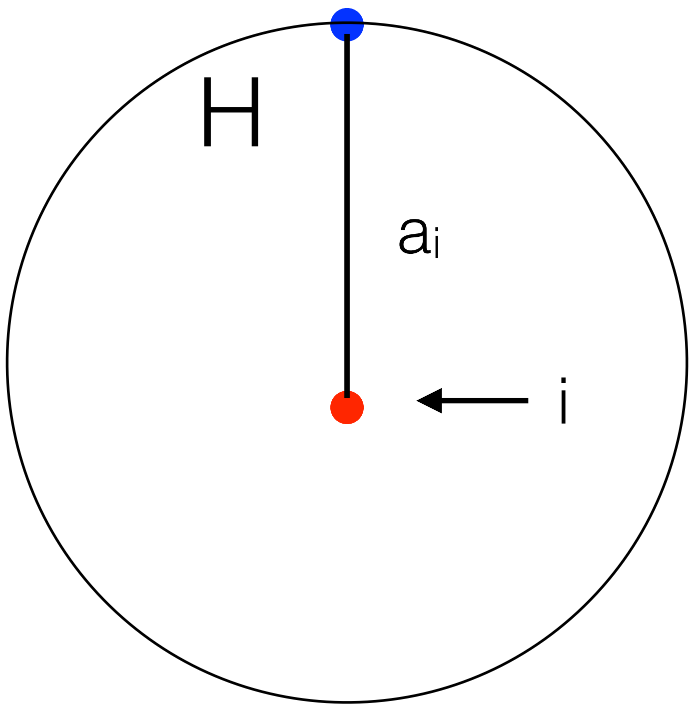

Analysis:

Lemma: Buy cost $\leq$ O(1) Rent cost

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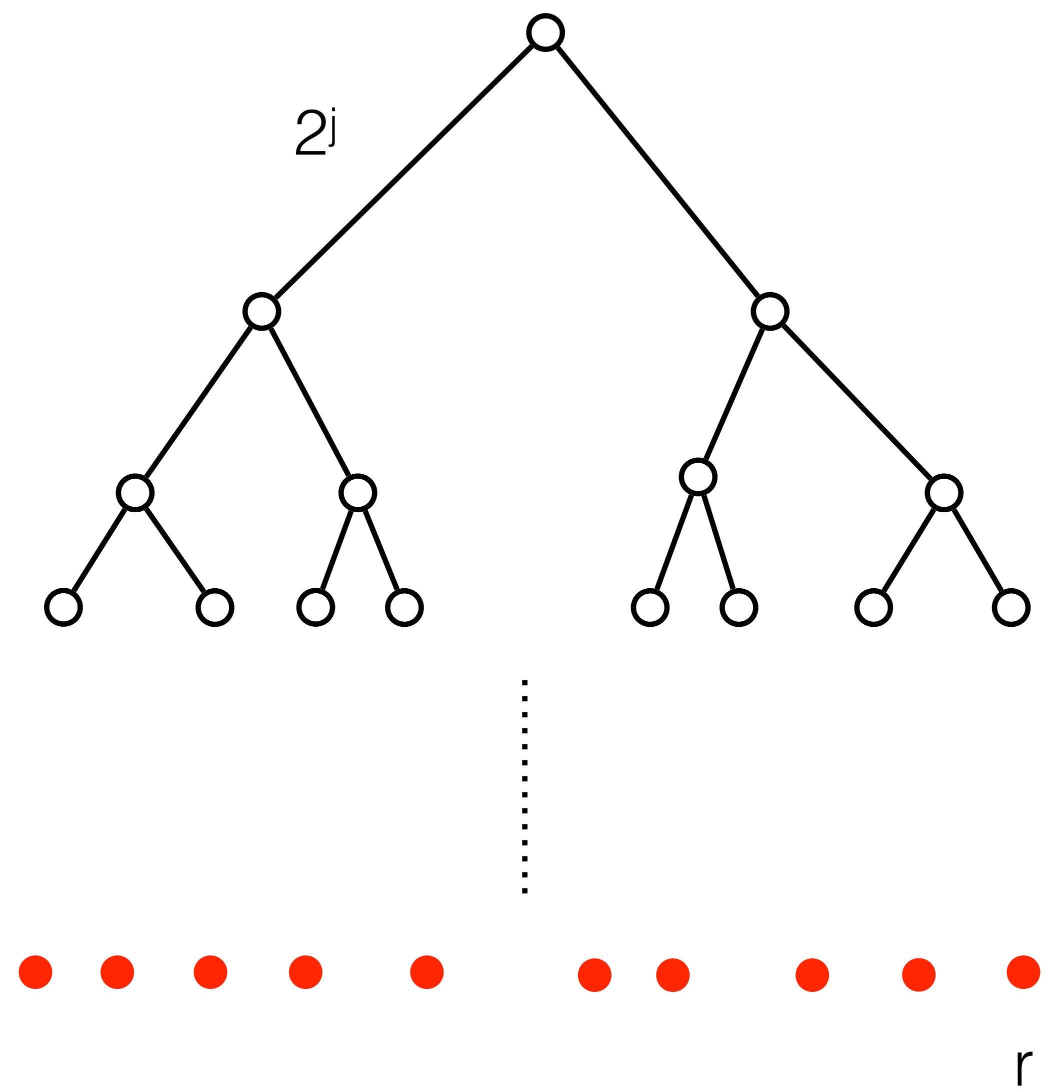
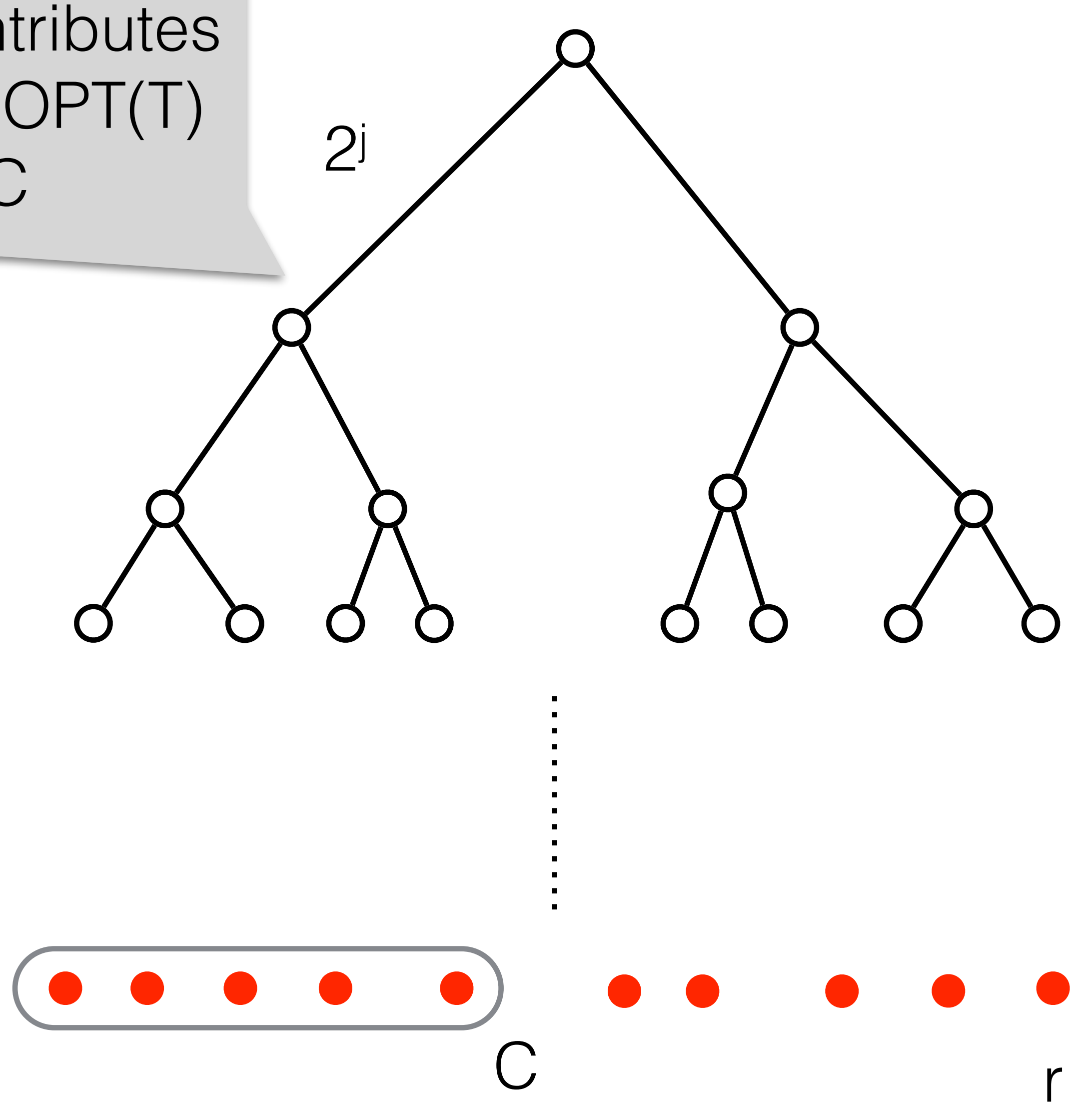
Theorem: ALG is O(log k)-competitive for ROB

Analysis



Analysis

1. Level-j edge contributes $2^j \min\{M, |C|\}$ to OPT(T) if r not in C



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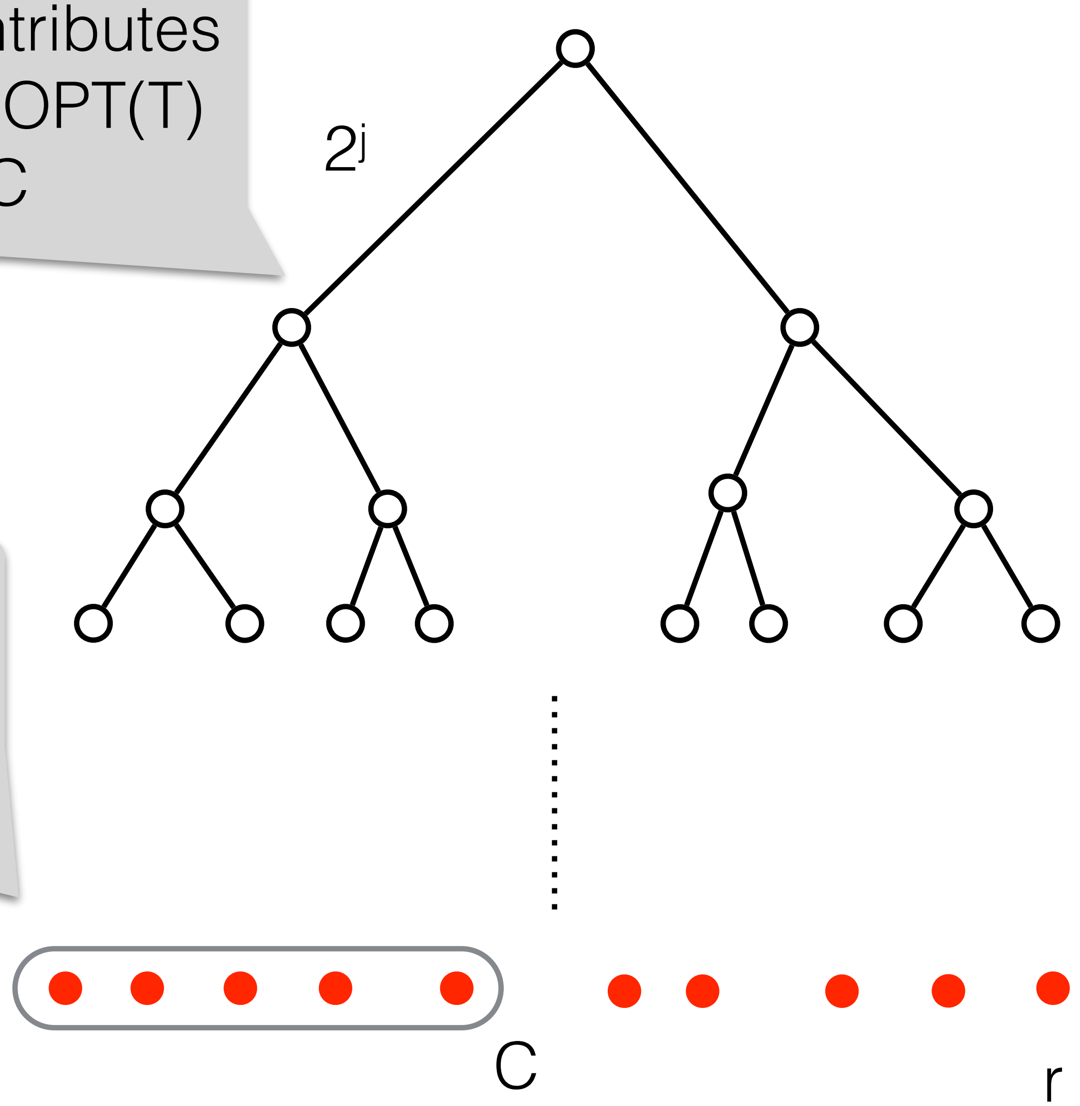

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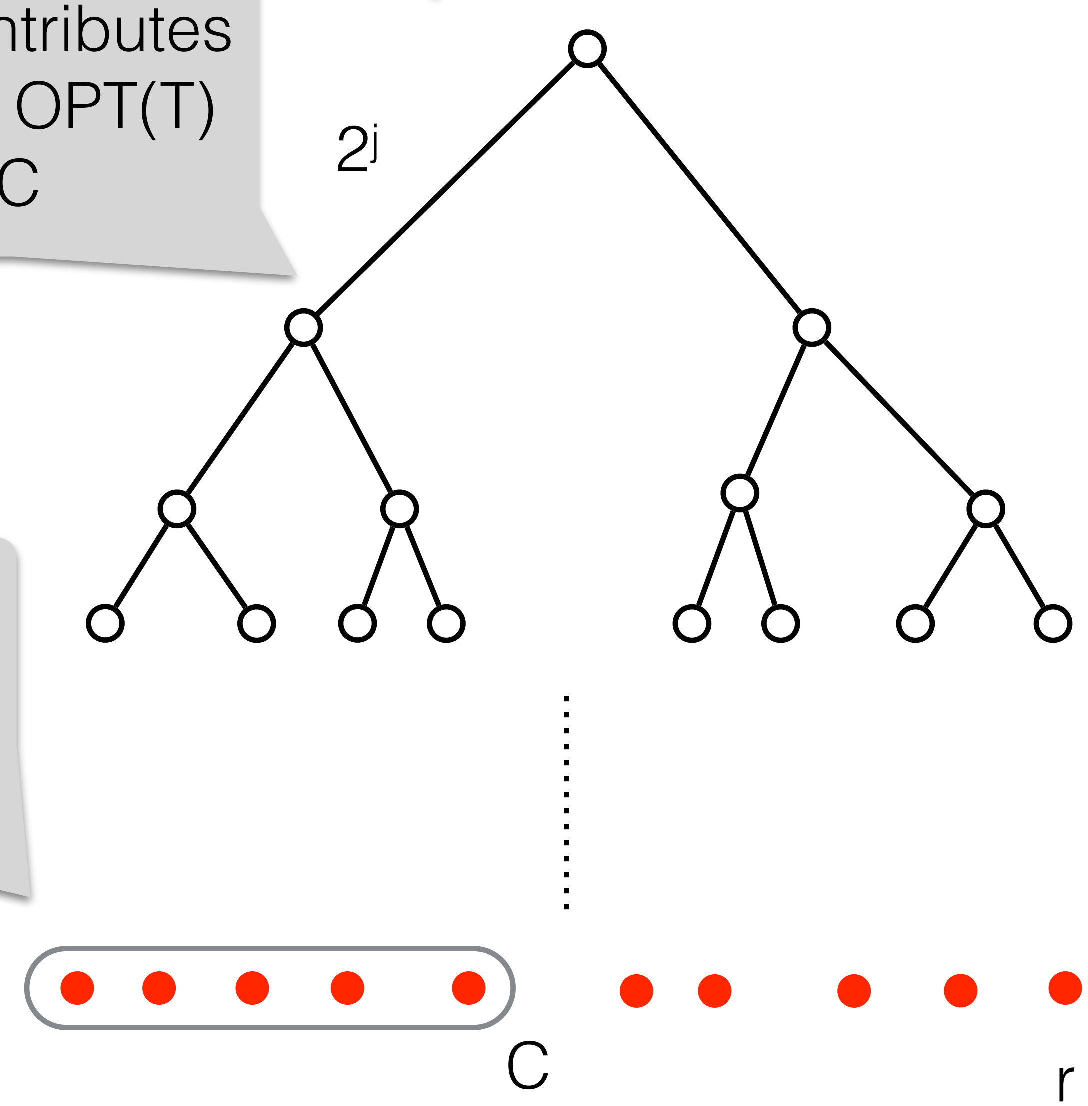

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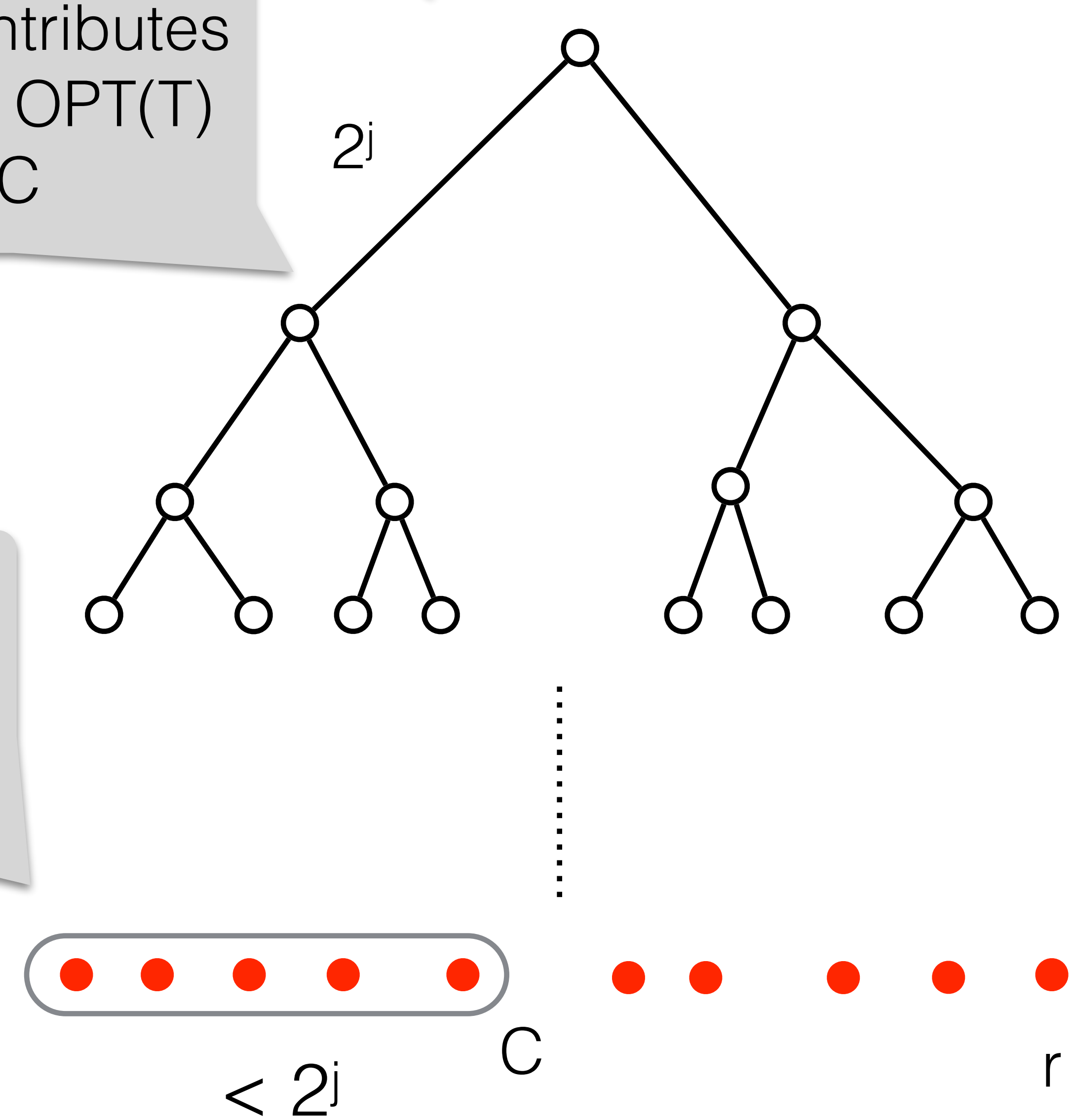

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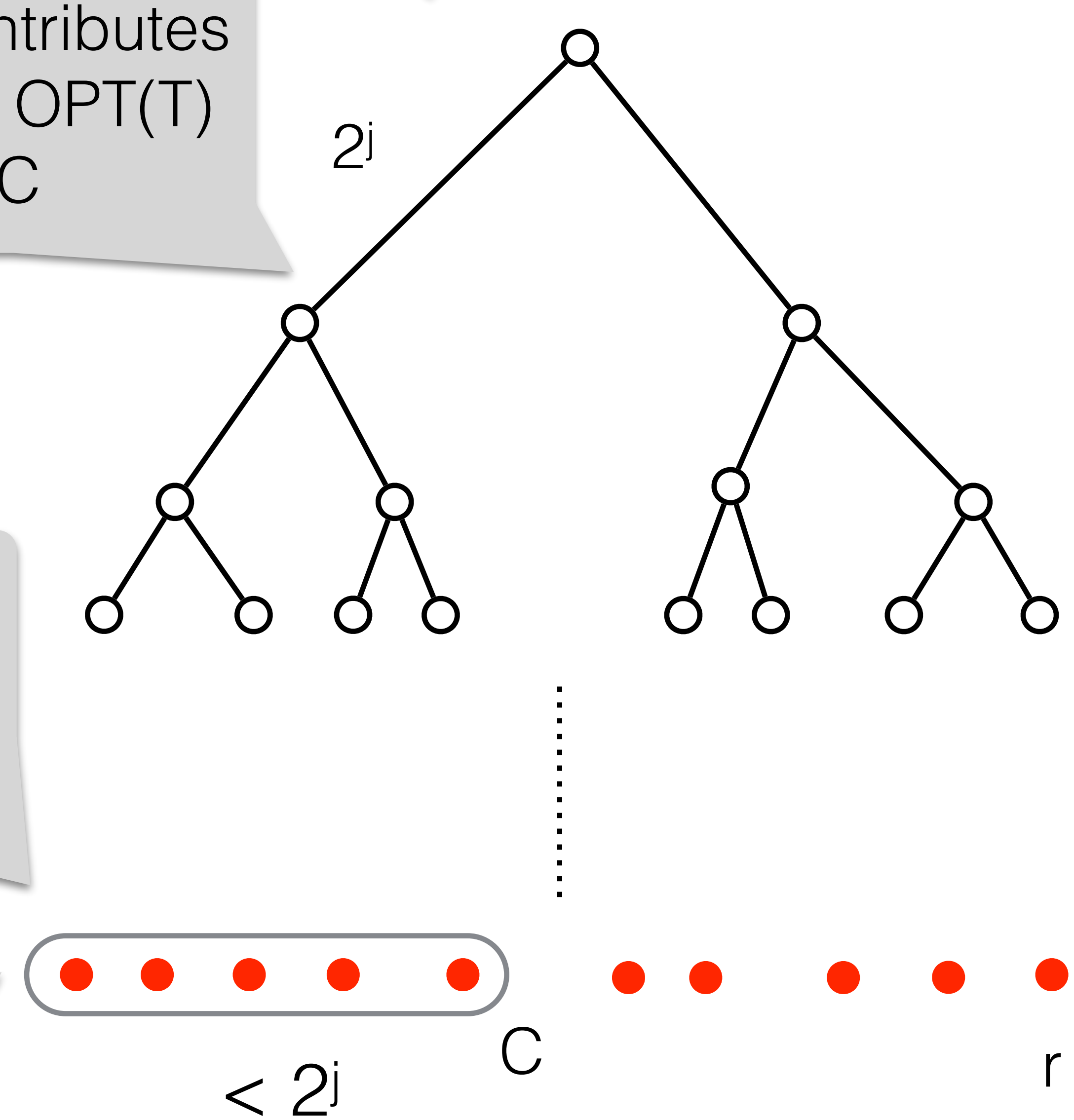

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2. If terminal has rent
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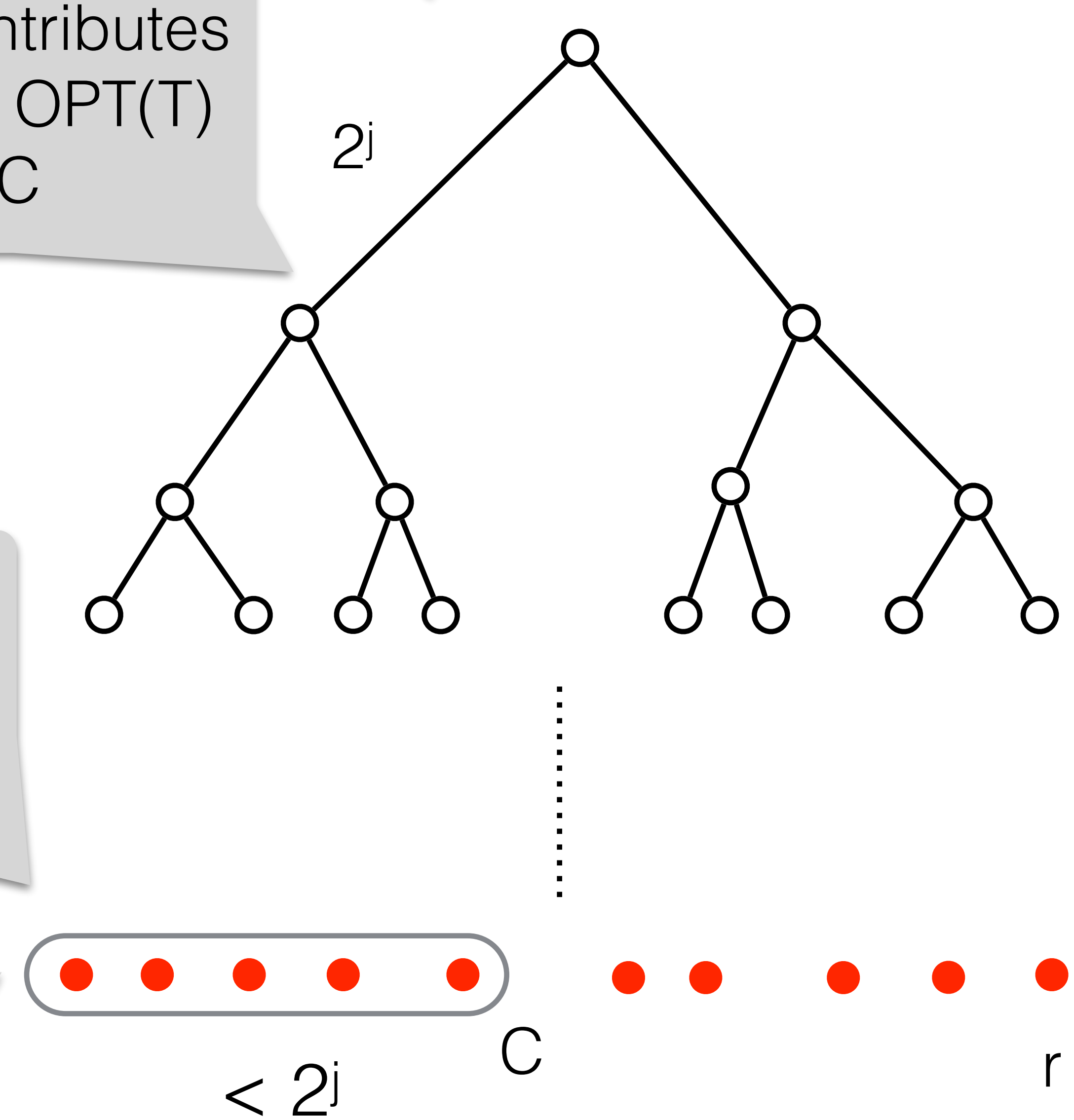
edge

Algorithm was designed with this in mind!

3b. At most M terminals in C with rent cost 2^j



Other Problems

- Works for many other “rent-or-buy”-type problems
- Connected facility location
- Prize-collecting Steiner forest (simpler analysis than [Qian-Williamson 11])

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Single-Source Steiner Network

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- Given metric space and root
- Terminals with requirements R_i arrive one-by-one
- Maintain multigraph (edge-duplication allowed) s.t. terminal i has R_i edge-disjoint paths to root

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Assume

Single-Source Steiner Network

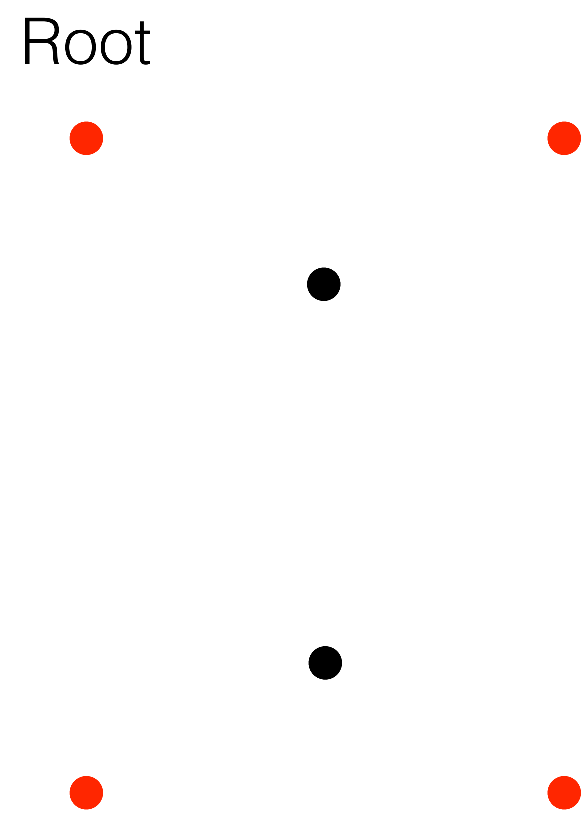
- Given metric space and root
- Terminals with requirements R_i arrive one-by-one
- Maintain multigraph (edge-duplication allowed) s.t. terminal i has R_i edge-disjoint paths to root

Assume

- Given maximum requirement (R_{max})
- Each requirement R_i is a power of 2

Intuition

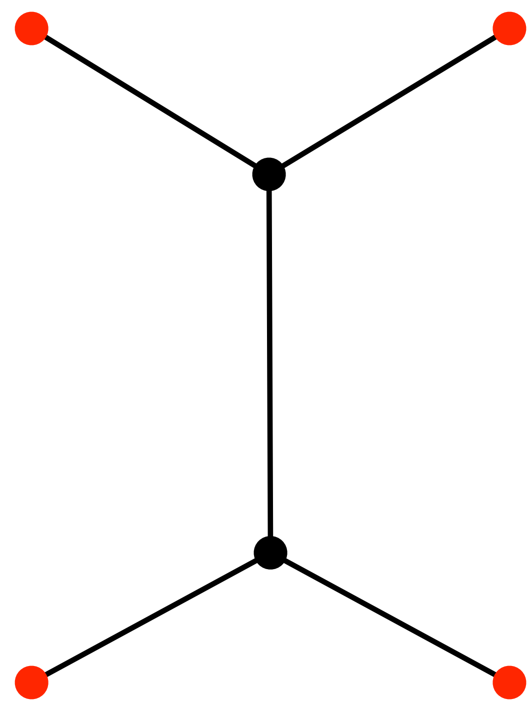
Intuition



$R_i = 2$ for all i

Intuition

Root



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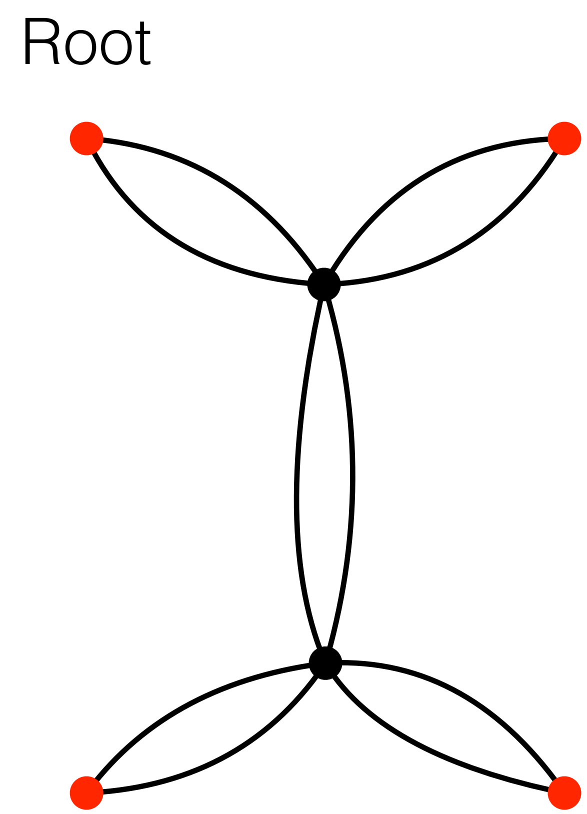

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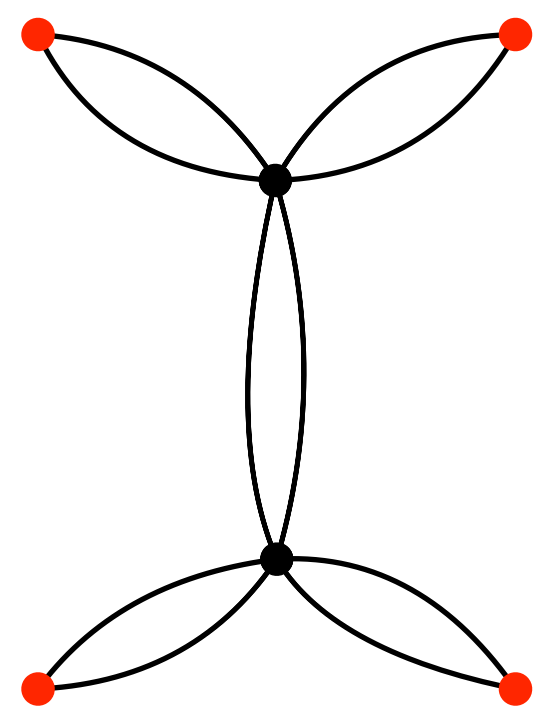

R_i = 2 for all i

If all requirements = 2^m, then OPT = 2^m SteinerTree

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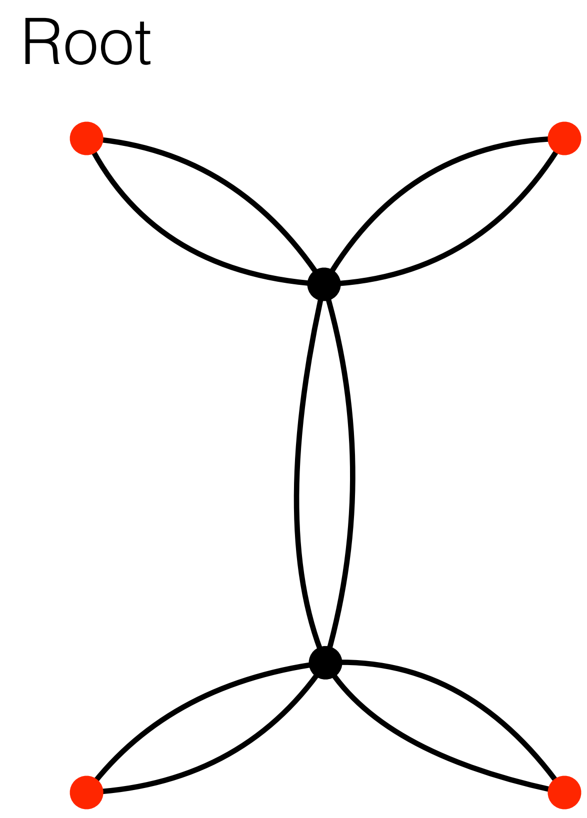

R_i = 2 for all i

If all requirements = 2^m, then OPT = 2^m SteinerTree

Key Idea: Decompose into several Steiner tree instances

Offline

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[Goemans-Bertsimas 93]:

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- For each scale $0 \leq m \leq \log R_{max}$

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Offline

[Goemans-Bertsimas 93]:

- For each scale $0 \leq m \leq \log R_{max}$
  - X_m = terminals with requirement 2^m and root
  - buy 2^m copies of $MST(X_m)$
- $OPT \geq 2^m SteinerTree(X_m) \implies 2(\log R_{max})$-approx

Offline

[Goemans-Bertsimas 93]:

- For each scale $0 \le m \le \log R_{max}$
  - X_m = terminals with requirement 2^m and root
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Tight

- $\mathrm{OPT} \ge 2^m \mathrm{SteinerTree}(X_m) \implies 2(\log R_{max})$-approx

Online

Our algorithm:

- For each scale $0 \leq m \leq \log R_{max}$
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- $\text{OPT} \geq 2^m \text{SteinerTree}(X_m) \implies O(\log k \log R_{max})$-approx

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Surprisingly, our analysis gives O(log k)!

Analysis

Fix a HST embedding T

Let $OPT_m(T)$ = optimal Steiner tree for X_m in T

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<u>Theorem</u>:
ALG is O(log k)-competitive for SS Steiner network

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Thank you!