Problem 1

i) The full joint table should sum to 1, so to find the missing probability \( P(A, 7B, C, D) \) we can do:

1 - sum of given probabilities

\[
1 - (0.002 + 0.018 + 0.013 + \ldots) = 0.14
\]

ii) \( P(7B, 7C, D) = 0.018 + 0.019 = 0.037 \)

iii) \( P(A \lor 7B) = P(A) + P(7B) - P(A, 7B) \)

\[
= 0.031 + 0.019 + 0.150 + 0.140 + 0.080 + 0.070 + 0.060 + 0.090 \\
+ 0.002 + 0.018 + 0.013 + 0.127 + 0.031 + 0.019 + 0.160 + 0.140 \\
- (0.031 + 0.019 + 0.150 + 0.140)
\]

\[
= 0.800
\]

iv) \( P(A \mid 7B, 7C, D) = \frac{P(A, 7B, 7C, D)}{P(7B, 7C, D)} = \frac{0.019}{0.037} = 0.514 \)

v) \( P(7B, 7C, D \mid A) = \frac{P(A, 7B, 7C, D)}{P(A)} = \frac{0.019}{0.640} = 0.0296 \)

\[
P(A) = 0.031 + 0.019 + 0.150 + 0.140 + 0.080 + 0.070 + 0.060 + 0.090 \\
= 0.640
\]
Problem 2

i) \[ P(A, B, C, D) = P(A) \cdot P(B \mid A) \cdot P(C \mid A, B) \cdot P(D \mid B, C) \]
\[ = 0.7 \cdot 0.8 \cdot 0.6 \cdot 0.2 = 0.0672 \]

ii) \[ P(7B, 7C, 7D) = P(A, 7B, 7C, 7D) + P(7A, 7B, 7C, 7D) \]
\[ P(A, 7B, 7C, 7D) = P(A) \cdot P(7B \mid A) \cdot P(7C \mid A, 7B) \cdot P(7D \mid 7B, 7C) \]
\[ = 0.7 \cdot 0.2 \cdot 0.5 \cdot 0.1 = 0.007 \]
\[ P(7A, 7B, 7C, 7D) = P(7A) \cdot P(7B \mid 7A) \cdot P(7C \mid 7A, 7B) \cdot P(7D \mid 7B, 7C) \]
\[ = 0.3 \cdot 0.9 \cdot 0.7 \cdot 0.1 = 0.0189 \]
\[ P(7B, 7C, 7D) = 0.007 + 0.0189 = 0.0259 \]

iii) \[ P(A \mid B, C, 7D) = \frac{P(A, B, C, 7D)}{P(B, C, 7D)} \]
\[ P(A, B, C, 7D) = P(A) \cdot P(B \mid A) \cdot P(C \mid A, B) \cdot P(7D \mid B, C) \]
\[ = 0.7 \cdot 0.8 \cdot 0.6 \cdot 0.8 = 0.2688 \]
\[ P(B, C, 7D) = P(A, B, C, 7D) + P(7A, B, C, 7D) \]
\[ P(7A, B, C, 7D) = P(7A) \cdot P(B \mid 7A) \cdot P(C \mid 7A, B) \cdot P(7D \mid B, C) \]
\[ = 0.3 \cdot 0.1 \cdot 0.4 \cdot 0.8 = 0.0096 \]
\[ P(B, C, 7D) = 0.2688 + 0.0096 = 0.2784 \]
\[ P(A \mid B, C, 7D) = \frac{0.2688}{0.2784} = 0.9655 \]
You could have also observed that due to the Markov Blanket Property,
\[ P(A \mid B, C, 7D) = P(A \mid B, C) \], which would have expanded into
\[ P(A \mid B, C) = \frac{P(A, B, C)}{P(B, C)} \]
\[ P(A, B, C) = P(A) \cdot P(B \mid A) \cdot P(C \mid A, B) \]
\[ = 0.7 \cdot 0.8 - 0.6 \]
\[ = 0.336 \]
\[ P(B, C) = P(A, B, C) + P(7A, B, C) \]
\[ P(7A, B, C) = 0.3 \cdot 0.1 \cdot 0.4 = 0.012 \]
\[ P(B, C) = 0.336 + 0.012 = 0.348 \]
\[ P(A \mid B, C) = \frac{0.336}{0.348} = 0.9655 \]

iv) \[ P(7D \mid A, B, C) = \frac{P(A, B, C, 7D)}{P(A, B, C)} \] (without Markov Blanket)
\[ P(A, B, C, 7D) = 0.2688 \] computed on previous page
\[ P(A, B, C) = 0.336 \] computed above
\[ P(7D \mid A, B, C) = \frac{0.2688}{0.336} = 0.8 \]
Problem 2 Continued

iv continued \( P(7D|B,C) = \frac{P(B,C,7D)}{P(B,C)} \)

\[ P(B,C,7D) = 0.2784 \text{ (computed on page 2)} \]

\[ P(B,C) = 0.348 \text{ (computed on page 3)} \]

\[ P(7D|B,C) = \frac{0.2784}{0.348} = 0.8 = P(7D|A,B,C) \]

v) \( P((A,D) \lor (B,C)) = P(A,D) + P(B,C) - P(A,B,C,D) \)


\[ P(A,B,C,D) = 0.0672 \text{ page 2} \]

\[ P(A,7B,C,D) = P(A) \cdot P(7B|A) \cdot P(C|A,7B) \cdot P(D|7B,C) = 0.7 \cdot 0.2 \cdot 0.6 \cdot 0.3 = 0.021 \]

\[ P(A,7C,D) = 0.7 \cdot 0.8 \cdot 0.4 \cdot 0.4 = 0.0896 \]

\[ P(A,7B,7C,D) = 0.7 \cdot 0.2 \cdot 0.5 \cdot 0.9 = 0.063 \]

\[ P(A,D) = 0.0672 + 0.021 + 0.0896 + 0.063 = 0.2408 \]

\[ P(B,C) = 0.348 \text{ page 3} \]

\[ P((A,D) \lor (B,C)) = 0.2408 + 0.348 - 0.0672 = 0.5216 \]
Problem 3

\[ P(A, B, 7C, 7D) = P(A)P(B|A)P(7C|A, B)P(7D|B, 7C) \]
\[ = 0.7 \cdot 0.8 \cdot 0.4 \cdot 0.6 \]
\[ = 0.1344 \]

\[ P(A, B, 7C, D) = 0.0896 \quad \text{page 4} \]

\[ P(A, B, C, 7D) = 0.2688 \quad \text{page 2} \]

\[ P(A, B, C, D) = 0.0672 \quad \text{page 3} \]

The main point of this question is to highlight that a Bayes Net can fill all the cells in a full joint table.
Problem 4

i) \[ P(\text{flu} \mid \text{7shot}) = \frac{P(\text{7shot} \mid \text{flu}) P(\text{flu})}{P(\text{7shot})} \] using Bayes' Rule

We know \( P(\text{flu}) \) and \( P(\text{7shot}) \), so let's focus on \( P(\text{7shot} \mid \text{flu}) \).

\[ P(\text{7shot} \mid \text{flu}) = 1 - P(\text{shot} \mid \text{flu}) \]

\[ P(\text{shot} \mid \text{flu}) = \frac{P(\text{flu} \mid \text{shot}) P(\text{shot})}{P(\text{flu})} = \frac{(0.1)(0.7)}{(0.2)} = 0.35 \]

\[ P(\text{7shot} \mid \text{flu}) = 1 - 0.35 = 0.65 \]

Now solving the original question,

\[ P(\text{flu} \mid \text{7shot}) = \frac{P(\text{7shot} \mid \text{flu}) P(\text{flu})}{P(\text{7shot})} = \frac{(0.65)(0.2)}{1 - 0.7} = 0.433 \]

ii) \[ P(\text{shot} \mid \text{7flu}) = \frac{P(\text{7flu} \mid \text{shot}) P(\text{shot})}{P(\text{7flu})} \] using Bayes’ Rule

\[ P(\text{7flu} \mid \text{shot}) = 1 - P(\text{flu} \mid \text{shot}) = 1 - 0.1 = 0.9 \]

\[ P(\text{shot} \mid \text{7flu}) = \frac{(0.9)(0.7)}{0.8} = 0.788 \]

iii) \[ P(\text{DE}) = \frac{57}{1000000} \] \[ P(\text{T4}) = P(\text{T4}, \text{DE}) + P(\text{T4}, \text{7DE}) \]

\[ P(\text{T4} \mid \text{DE}) = 0.95 \] \[ P(\text{T4} \mid 7\text{DE}) = 0.05 \]

\[ P(\text{DE} \mid \text{T4}) = \frac{P(\text{T4} \mid \text{DE}) P(\text{DE})}{P(\text{T4})} \]

\[ = \frac{(0.95)(0.57)}{0.05} = 0.00108 \]
Problem 4

1) Alternative Solution

\[ P(\text{flu} \mid \text{7shot}) = ? \]

Using conditioning, we observe the following.

\[
P(\text{flu}) = P(\text{flu} \mid \text{shot}) \cdot P(\text{shot}) + P(\text{flu} \mid \text{7shot}) \cdot P(\text{7shot})
\]

\[
0.2 = 0.1 \cdot 0.7 + P(\text{flu} \mid \text{7shot}) \cdot (1 - 0.7)
\]

\[
0.2 - 0.07 = 0.3 \cdot P(\text{flu} \mid \text{7shot})
\]

\[
\frac{0.2 - 0.07}{0.3} = P(\text{flu} \mid \text{7shot})
\]

\[
P(\text{flu} \mid \text{7shot}) = 0.4133
\]