

**Problem 1**

a.

P: I take History, Q: I take Economics

$$P \rightarrow \neg Q$$

b.

P: I take Japanese, Q: I take Italian

$$(P \wedge \neg Q) \vee (\neg P \wedge Q)$$

An alternate (and logically equivalent) form:  $(P \vee Q) \wedge \neg(P \wedge Q)$ 

c.

P: I take cs520, Q: I take cs552, R: I take cs564

$$(P \wedge Q) \vee (P \wedge R) \vee (Q \wedge R)$$

**Problem 2**

a. Truth Table:

P	Q	$\neg Q$	$(P \rightarrow Q)$	$(P \rightarrow \neg Q)$	$(P \rightarrow Q) \wedge (P \rightarrow \neg Q)$
F	F	T	T	T	T
F	T	F	T	T	T
T	F	T	F	T	F
T	T	F	T	F	F

 $(P \rightarrow Q) \wedge (P \rightarrow \neg Q)$  is true under some interpretations, so it's satisfiable.

b. Truth Table:

P	Q	$\neg Q$	R	$\neg R$	$(P \rightarrow Q)$	$(P \rightarrow R)$	$(\neg Q \wedge \neg R)$	Result
F	F	T	F	T	T	T	T	F
F	F	T	T	F	T	T	F	F
F	T	F	F	T	T	T	F	F
F	T	F	T	F	T	T	F	F
T	F	T	F	T	F	F	T	F
T	F	T	T	F	F	T	F	F
T	T	F	F	T	T	F	F	F
T	T	F	T	F	T	T	F	F

 $(P \rightarrow Q) \wedge (P \rightarrow R) \wedge (\neg Q \wedge \neg R) \wedge P$  is unsatisfiable, since all the interpretations make it false.

c. Truth Table:

P	Q	$(P \rightarrow Q)$	$(Q \rightarrow P)$	$(P \rightarrow Q) \vee (Q \rightarrow P)$
F	F	T	T	T
F	T	T	F	T
T	F	F	T	T
T	T	T	T	T

 $(P \rightarrow Q) \vee (Q \rightarrow P)$  is valid, because all the interpretations make it to be true.

d. Truth Table:

P	Q	R	$(P \rightarrow Q)$	$(Q \rightarrow R)$	$(P \rightarrow R)$	$(P \rightarrow Q) \rightarrow (Q \rightarrow R)$	Result
F	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	T	T	T	T	T	T	T
T	F	F	F	T	F	T	F
T	F	T	F	T	T	T	T
T	T	F	T	F	F	F	T
T	T	T	T	T	T	T	T

So  $((P \rightarrow Q) \rightarrow (Q \rightarrow R)) \leftrightarrow (P \rightarrow R)$  satisfiable.**Problem 3**

Proof:

#	Wff	Justification
1.	Q	given
2.	Y	given
3.	$Q \wedge Y \rightarrow W$	given
4.	$\neg\neg X$	given
5.	$\neg X \vee \neg Y \vee R$	given
6.	$Q \wedge Y$	And Introduction on 1 and 2
7.	W	Modus Ponens on 3 and 6
8.	X	Double Negation Elimination on 4
9.	$\neg Y \vee R$	Unit Resolution on 5 and 8
10.	R	Unit Resolution on 2 and 9
11.	$R \wedge W$	And introduction on 7, 10

**Problem 4**a. Replace  $(P \rightarrow Q)$  by  $(\neg P \vee Q)$ , and  $(X \rightarrow Y)$  by  $(\neg X \vee Y)$ , we get  
 $(\neg P \vee Q) \wedge (\neg X \vee Y)$ 

b.

$$\begin{aligned}
 & (P \wedge Q \rightarrow Z) \vee (X \wedge Y) \\
 &= (\neg(P \wedge Q) \vee Z) \vee (X \wedge Y) \\
 &= (\neg P \vee \neg Q \vee Z) \vee (X \wedge Y) \\
 &= (\neg P \vee \neg Q \vee Z \vee X) \wedge (\neg P \vee \neg Q \vee Z \vee Y)
 \end{aligned}$$

c.

$$\begin{aligned} & ((P \rightarrow Q) \rightarrow (X \rightarrow Y)) \leftrightarrow (A \wedge B) \\ & = (((P \rightarrow Q) \rightarrow (X \rightarrow Y)) \rightarrow (A \wedge B)) \wedge ((A \wedge B) \rightarrow ((P \rightarrow Q) \rightarrow (X \rightarrow Y))) \end{aligned}$$

left part:

$$\begin{aligned} & ((P \rightarrow Q) \rightarrow (X \rightarrow Y)) \rightarrow (A \wedge B) \\ & = ((\neg P \vee Q) \rightarrow (\neg X \vee Y)) \rightarrow (A \wedge B) \\ & = (\neg(\neg P \vee Q) \vee (\neg X \vee Y)) \rightarrow (A \wedge B) \\ & = ((P \wedge \neg Q) \vee (\neg X \vee Y)) \rightarrow (A \wedge B) \\ & = \neg((P \wedge \neg Q) \vee (\neg X \vee Y)) \vee (A \wedge B) \\ & = (\neg(P \wedge \neg Q) \wedge \neg(\neg X \vee Y)) \vee (A \wedge B) \\ & = ((\neg P \vee Q) \wedge (X \wedge \neg Y)) \vee (A \wedge B) \\ & = ((\neg P \vee Q) \wedge (X \wedge \neg Y)) \vee (A \wedge B) \\ & = (((\neg P \vee Q) \wedge X \wedge \neg Y) \vee A) \wedge (((\neg P \vee Q) \wedge X \wedge \neg Y) \vee B) \\ & = (\neg P \vee Q \vee A) \wedge (X \vee A) \wedge (\neg Y \vee A) \wedge (\neg P \vee Q \vee B) \wedge (X \vee B) \wedge (\neg Y \vee B) \end{aligned}$$

right left:

$$\begin{aligned} & ((A \wedge B) \rightarrow ((P \rightarrow Q) \rightarrow (X \rightarrow Y))) \\ & = (\neg A \vee \neg B \vee \neg X \vee Y \vee P) \wedge (\neg A \vee \neg B \vee \neg X \vee Y \vee \neg Q) \end{aligned}$$

$$\begin{aligned} \text{So Result} &= (\neg P \vee Q \vee A) \wedge (X \vee A) \wedge (\neg Y \vee A) \wedge (\neg P \vee Q \vee B) \wedge (X \vee B) \wedge (\neg Y \vee B) \\ & \quad + (\neg A \vee \neg B \vee \neg X \vee Y \vee P) \wedge (\neg A \vee \neg B \vee \neg X \vee Y \vee \neg Q) \end{aligned}$$

**Problem 5**

In CNF, the wff's in Problem 3 become:

- (1)  $\neg P \vee \neg Q \vee R$
- (2)  $\neg X \vee \neg Y \vee R$
- (3)  $\neg Q \vee \neg Y \vee W$
- (4)  $Q$
- (5)  $X$
- (6)  $Y$

The negated query is: (7)  $\neg R \vee \neg W$

