

**Question 1**

**Note: there may be multiple FOPC answers for each sentence. Only one is given as your reference.**

- a.  $\forall x \text{ Basketball}(x) \rightarrow \text{MadeOfRubber}(x)$
- b.  $\forall x \text{ Flower}(x) \wedge \text{At}(x, \text{YardOf}(Sue)) \rightarrow \text{Red}(x)$
- c.  $\exists x \text{ Question}(x) \wedge \text{On}(x, \text{HW4}) \wedge \text{Hard}(x)$
- d.  $\forall x \text{ Bird}(x) \wedge \neg \text{Penguin}(x) \wedge \neg \text{Ostrich}(x) \wedge \neg \text{BrokenWing}(x) \rightarrow \text{Fly}(x)$
- e.  $\text{Mary} = \text{bossOf}(\text{John})$
- f.  $\forall x \forall y \forall b \forall w \forall s$   
 $\text{owner}(x, b, s) \wedge \text{writer}(w, b, s) \wedge \text{book}(b) \wedge \text{state}(s) \wedge \text{person}(x) \wedge \text{person}(y) \wedge \text{person}(w)$   
 $\rightarrow \text{owner}(y, b, \text{result}(\text{sell}(x, b, y), s)) \wedge \text{writer}(w, b, \text{result}(\text{sell}(x, b, y), s)) \wedge \text{state}(\text{result}(\text{sell}(x, b, y), s))$

**Question 2**

Suppose Bob has two hats, H1 and H2. Now Bob lost hat H1, but still has hat H2, which will make the sentence “A hat of Bob’s is missing” true.

Our interpretation contains the constants: {Bob, H1, H2}. We next need to 'define' all the predicates, for all possible arguments.

$\neg \text{hat}(\text{Bob})$	$\text{owner}(\text{Bob}, \text{Bob})$	$\text{owner}(\text{H2}, \text{Bob})$
$\text{hat}(\text{H1})$	$\neg \text{owner}(\text{Bob}, \text{H1})$	$\neg \text{owner}(\text{H2}, \text{H1})$
$\text{hat}(\text{H2})$	$\neg \text{owner}(\text{Bob}, \text{H2})$	$\neg \text{owner}(\text{H2}, \text{H2})$
	$\text{owner}(\text{H1}, \text{Bob})$	
$\neg \text{missing}(\text{Bob})$	$\neg \text{owner}(\text{H1}, \text{H1})$	
$\text{missing}(\text{H1})$	$\neg \text{owner}(\text{H1}, \text{H2})$	
$\neg \text{missing}(\text{H2})$		

From the given FOPC  $\forall x [\text{hat}(x) \wedge \text{owner}(x, \text{Bob})] \rightarrow \text{missing}(x)$ , by replacing  $x$  with H1 H2 and Bob separately, we get the following table:

$x$	$\text{hat}(x)$	$\text{owner}(x, \text{Bob})$	$\text{hat}(x) \wedge \text{owner}(x, \text{Bob})$	$\text{missing}(x)$	Result
Bob	False	True	False	False	True
H1	True	True	True	True	True
H2	True	True	True	False	<b>False</b>

Since the body of the ForAll is not true for each  $x$ , the ForAll's truth value is FALSE. Since the English is true in this interpretation, whereas the FOPC is false, the FOPC doesn't represent the English.

**Question 3**

- a. None (since two sentences contain different numbers of variables/terms)
- b.  $\{?x/1, ?y/f(2), ?z/f(g(a,b))\}$
- c. None, since  $f(3) \neq 1$
- d. None, since different predicates in two sentences
- e.  $\{?x/f(?a,?b), ?y/?a, ?a/?b\}$

**Question 4** (Natural Deduction)

#	WFF	Justification
1.	$\text{box}(\text{Box1}) \wedge \text{box}(\text{Box2}) \wedge (\text{Box1} \neq \text{Box2}) \wedge \text{paint}(\text{Red}) \wedge \text{paint}(\text{Blue})$ $\wedge (\text{Red} \neq \text{Blue}) \wedge \text{location}(\text{Loc1}) \wedge \text{location}(\text{Loc2}) \wedge (\text{Loc1} \wedge \text{Loc2})$	Given
2.	$\forall b,p,s \text{ box}(b) \wedge \text{paint}(p) \wedge \text{at}(b, \text{Loc2}, s) \rightarrow \text{color}(b, p, \text{result}(\text{Paint}(b, p), s))$	Given
3.	$\forall b,x,s \text{ box}(b) \wedge \text{location}(x) \rightarrow \text{at}(b, x, \text{result}(\text{Move}(b, x), s))$	Given
4.	$\text{box}(\text{Box1}) \wedge \text{location}(\text{Loc2})$	And Elimination on 1
5.	$\text{at}(\text{Box1}, \text{Loc2}, \text{result}(\text{Move}(\text{Box1}, \text{Loc2}), S_0))$	Generalized Modus Ponens on 3, 4, using binding list $\Theta = \{b/\text{Box1}, x/\text{Loc2}, s/S_0\}$
6.	$\text{box}(\text{Box1}) \wedge \text{paint}(\text{Red})$	And Elimination on 1
7.	$\text{box}(\text{Box1}) \wedge \text{paint}(\text{Red}) \wedge \text{at}(\text{Box1}, \text{Loc2},$ $\text{result}(\text{Move}(\text{Box1}, \text{Loc2}), S_0))$	And Introduction on 5, 6
8.	$\text{color}(\text{Box1}, \text{Red}, \text{result}(\text{Paint}(\text{Box1}, \text{Red}),$ $\text{result}(\text{Move}(\text{Box1}, \text{Loc2}), S_0))$	Generalized Modus Ponens on 2, 7 Binding list, $\Theta = \{b/\text{Box1}, p/\text{Red}, s/\text{result}(\text{Move}(\text{Box1}, \text{Loc2}), S_0)\}$
9.	$\exists s \text{ color}(\text{Box1}, \text{Red}, s)$	Existential Introduction on 8

So, after moving Box to Loc2 and painting Box1 at Loc2, we reach a state S where Box1 is

red.  $S = \text{result}(\text{Paint}(\text{Box1}, \text{Red}), \text{result}(\text{Move}(\text{Box1}, \text{Loc2}), S_0))$

Notice we can read out a PLAN for achieving our goal by "reading" the final state from right-to-left.