

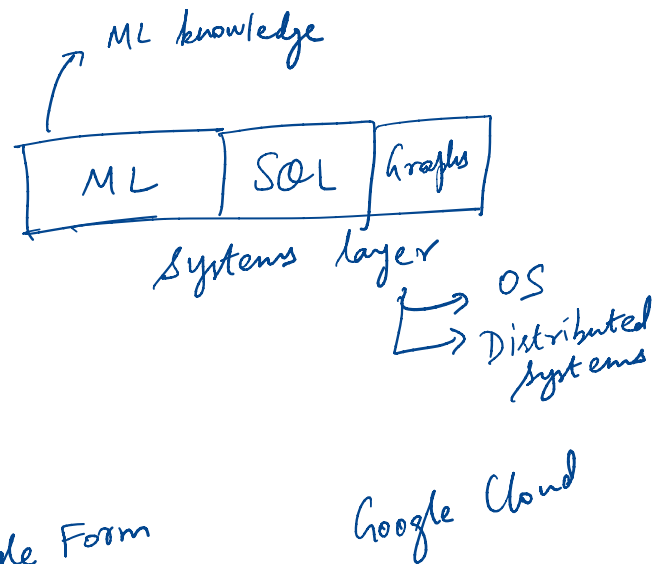
Good morning!

CS 744: DRF

Shivaram Venkataraman

Fall 2020

ADMINISTRIVIA



- Assignment 2 out! → Piazza
- Course Project →
 - Form groups? ~ 3 students
 - Project list by Monday (9/28) → Google Form
 - Submit project bids by Thursday (10/1)
 - Assigned project by Friday (10/2)

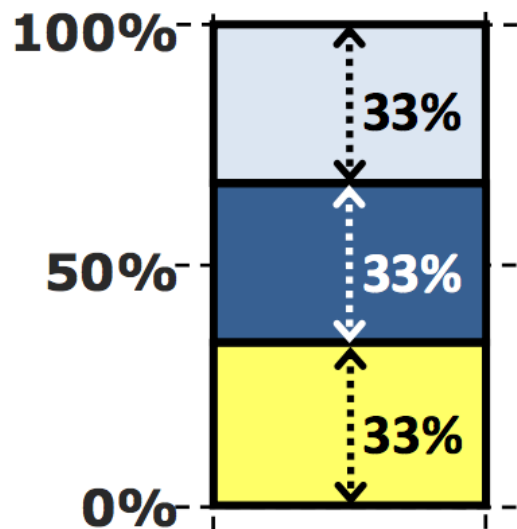
~ 2 months to work on
the project

SETTING: FAIR SHARING

3 users = $\frac{1}{3}$ rd of resources
each user

networking
OS \rightarrow lottery scheduling

Equal Share

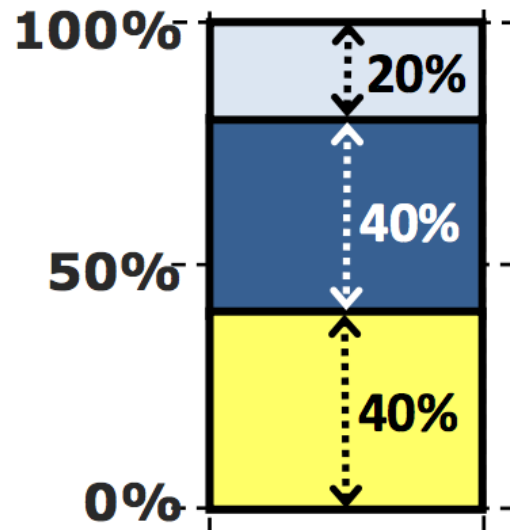


Max-Min Share

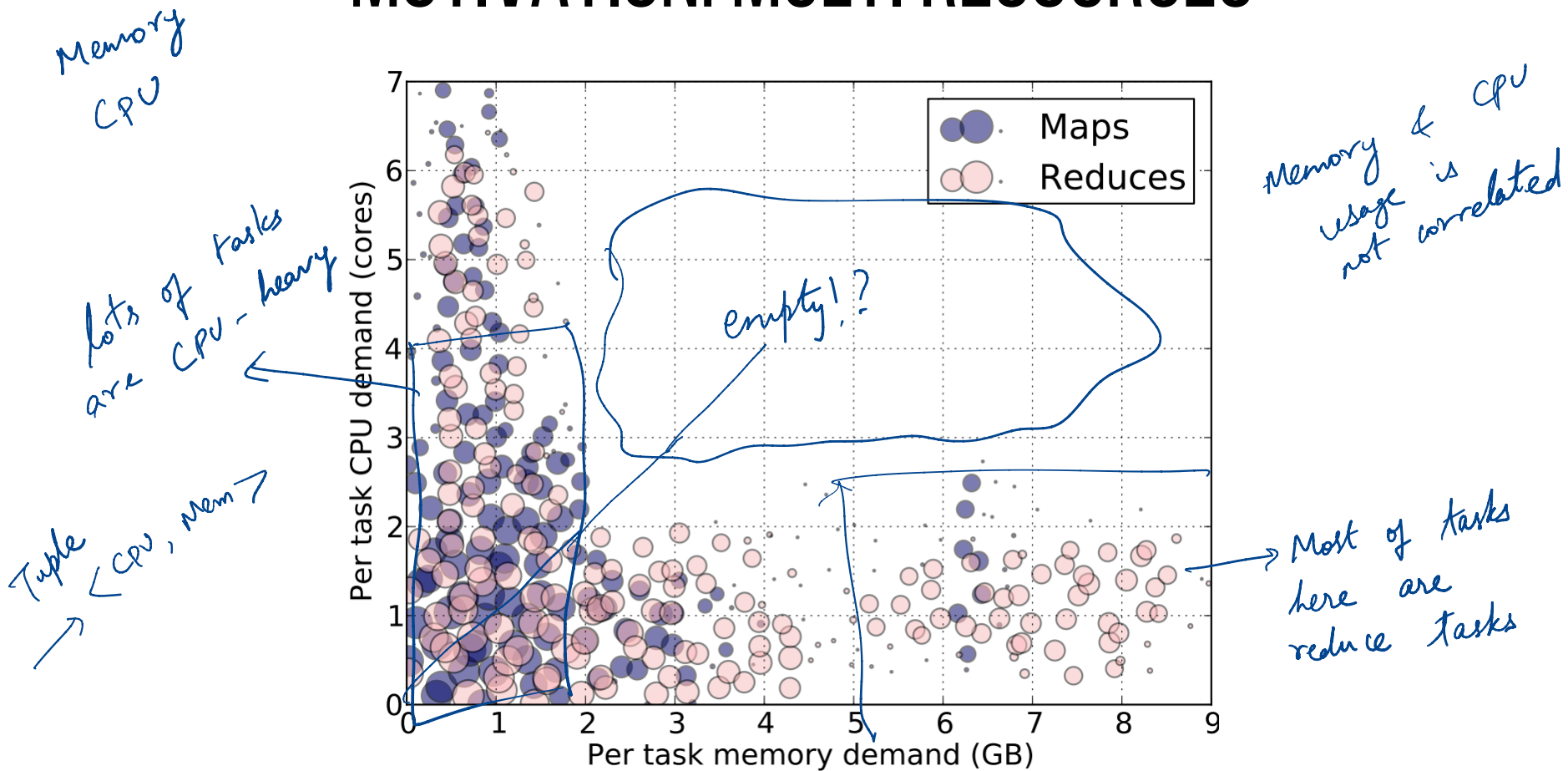
\rightarrow to handle different demands across users

Maximize the allocation
for most poorly treated
users

Maximize the minimum



MOTIVATION: MULTI RESOURCES



DRF: MODEL

Users have a **demand vector**

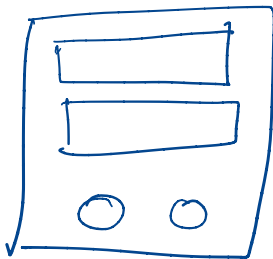
$\langle 2, 3, 1 \rangle$ means user's task needs 2 R1, 3 R2, 1 R3 \rightarrow one task

Resources given in multiples of demand vector

i.e., users might get $\langle 4, 6, 2 \rangle \leftarrow$ 2 tasks with their demand

slot based model

6 cores
10 GB



4 map slots \equiv 1 core
1.5 GB of mem
2 reduce slots

No containers
cgroups
linux

PROPERTIES

Sharing Incentive

User should get
at least $1/n$ of resources
No worse off than their
own cluster with $1/n$ resources

Strategy Proof

You can't lie about
what you need to get
more
Incentivizing truth
telling

Pareto Efficiency

If you allocate more
for one user, you need
to take away from others

Utilization

Envy free

Users should not envy
allocation of another
user

PROPERTIES

Sharing Incentive

User is no worse off than a cluster with
 $1/n$ resources

Strategy Proof

User should not benefit by
lying about demands

Pareto Efficiency

Not possible to increase
one user without
decreasing another

Envy free

User should not desire the
allocation of another user

DRF: APPROACH

Dominant Resource

Resource user has the **biggest** share of

Total: <10 CPU, 4 GB>

User 1: <1 CPU, 1 GB>

Dominant resource is **memory**

CPU
share $\frac{1}{10}$

mem share

$$\frac{1}{4}$$

Dominant Share

Fraction of the dominant resource user is allocated

E.g., for User 1 this is **25% or 1/4**

DRF: APPROACH

Equalize the dominant share of users

Total: <9 CPU, 18 GB>

User1: <1 CPU, 4 GB>
dom res: mem

User2: <3 CPU, 1 GB>
dom res: CPU

Total used: 9 CPU, 14 GB

User	Allocation	Dominant Share
User1	<0 CPU, 0 GB> <1 CPU, 4 GB> <2 CPU, 8 GB> [< <u>3</u> CPU, 12 GB>]	0 2/9 4/9 6/9
User2	<0 CPU, 0 GB> <3 CPU, 1 GB> [< <u>6</u> CPU, 2 GB>]	0 3/9 6/9

DRF: APPROACH

Total: <9 CPU, 18 GB>

User1: <1 CPU, 4 GB> per task
<3 CPU, 12 GB> for 3 tasks

dom res: mem

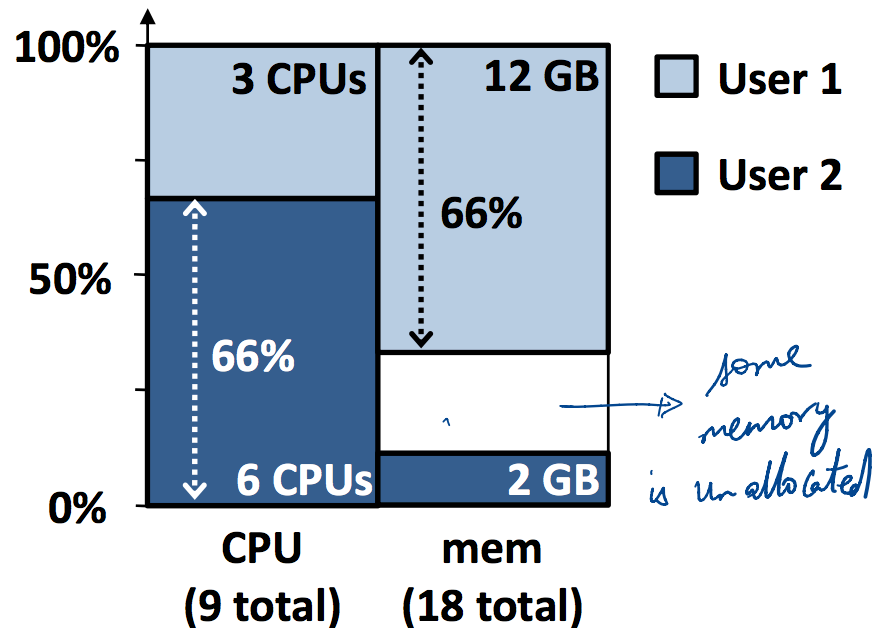
dom share: $12/18 = 2/3$

User2: <3 CPU, 1 GB>

<6 CPU, 2 GB> for 2 tasks

dom res: CPU

dom share: $6/9 = 2/3$



DRF ALGORITHM

Whenever there are available resources:

Schedule a task to the user with **smallest dominant share**

DRF ALGORITHM

Algorithm 1 DRF pseudo-code

$R = \langle r_1, \dots, r_m \rangle$ \triangleright total resource capacities

$C = \langle c_1, \dots, c_m \rangle$ \triangleright consumed resources, initially 0

s_i ($i = 1..n$) \triangleright user i 's dominant shares, initially 0

$U_i = \langle u_{i,1}, \dots, u_{i,m} \rangle$ ($i = 1..n$) \triangleright resources given to user i , initially 0

pick user i with lowest dominant share s_i

$D_i \leftarrow$ demand of user i 's next task $\leftarrow \langle 1 \text{ CPU}, 4 \text{ GB} \rangle$

if $C + D_i \leq R$ **then**

$C = C + D_i$ \triangleright update consumed vector

$U_i = U_i + D_i$ \triangleright update i 's allocation vector

$s_i = \max_{j=1}^m \{u_{i,j}/r_j\}$

else

return

\triangleright the cluster is full

end if

\longrightarrow cluster config

\longrightarrow running sum

\longrightarrow initialization

\longrightarrow track resources given to each user

$\left. \begin{array}{l} \text{if one resource}(r_j) \text{ is} \\ \text{full} \end{array} \right\} \longrightarrow \text{you can still offer} \\ \text{resources to tasks that} \\ \text{don't need } R_j$

COMPARISON: ASSET FAIRNESS

Asset Fairness: Equalize each user's sum of resource shares

Violates Sharing Incentive

Consider total of 70 CPUs, 70 GB RAM

U1 needs <2 CPU, 2 GB RAM> per task ≈ 4 units of resource

U2 needs <1 CPU, 2 GB RAM> per task ≈ 3 units of resource

Asset Fair Allocation:

U1: 4, 8, 12, ..., 60 ≈ 15 tasks for U1

U2: 3, 6, 9, 12, ..., 60 ≈ 20 tasks for U2

COMPARISON: ASSET FAIRNESS

Asset Fairness: Equalize each user's sum of resource shares

Violates Sharing Incentive

*U1 will be better off
with a 50% cluster that is dedicated!*

Consider total of 70 CPUs, 70 GB RAM

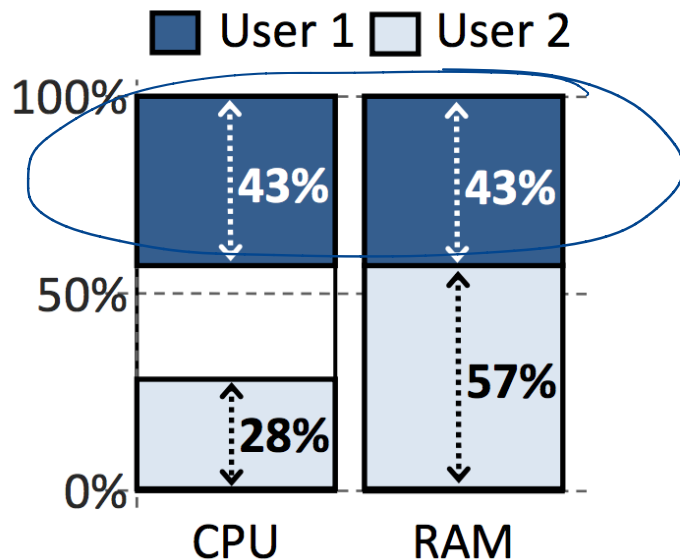
U1 needs <2 CPU, 2 GB RAM> per task

U2 needs <1 CPU, 2 GB RAM> per task

Asset Fair Allocation:

U1: 15 tasks: 30 CPU, 30 GB (Sum = 60)

U2: 20 tasks: 20 CPU, 40 GB (Sum = 60)



COMPARISON: CEEI

CEEI: Competitive Equilibrium from Equal Incomes

- Each user receives initially $1/n$ of every resource,
- Subsequently, each user can trade resources with other users in a perfectly competitive market
- Computed by maximizing **product of utilities** across users

COMPARISON: CEEI

Total: <9 CPU, 18 GB>

User1: <1 CPU, 4 GB>

User2: <3 CPU, 1 GB>

dominant resource

CEEI

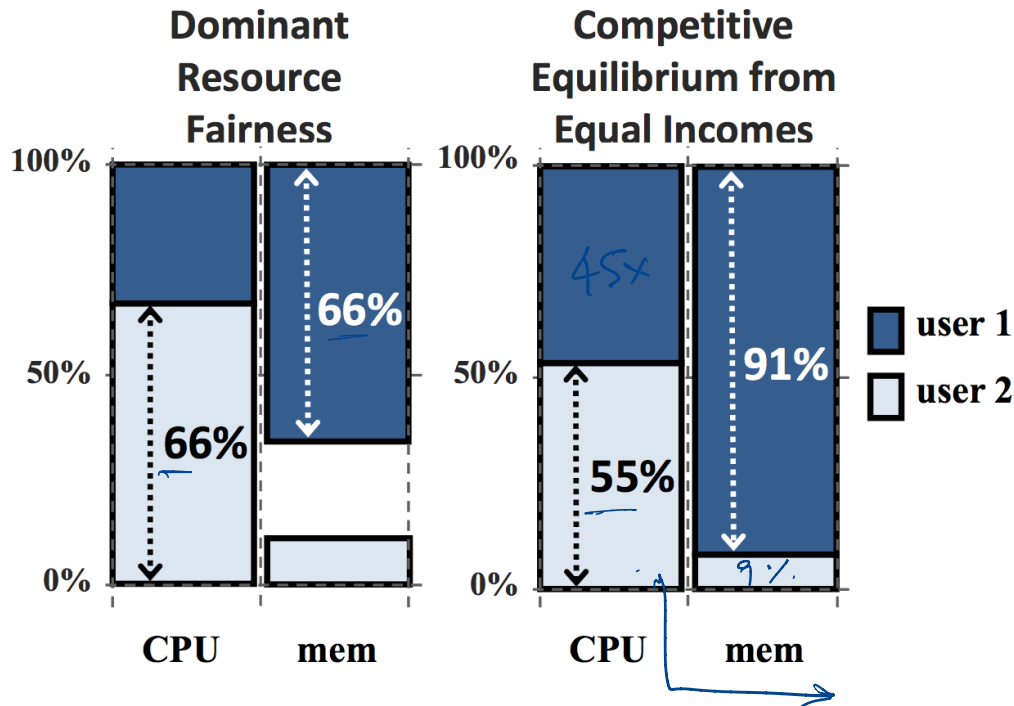
$$\max (x \cdot y)$$

subject to

$$x + 3y \leq 9 \quad \text{CPU}$$

$$4x + y \leq 18 \quad \text{Mem}$$

$$x = 4.05, \quad y = 1.62$$



CEEI: STRATEGY PROOFNESS

Total: <9 CPU, 18 GB>

x, y used to be higher

$$y = 1.62$$

$$x = 4.05$$

User2 Before:

CEEI: 55% CPU, 9% mem

3.6 tasks ??

↳ discrete number of tasks !!

Total: <9 CPU, 18 GB>

User1: <1 CPU, 4 GB>

User2: <3 CPU, 2 GB>

$$x + 3y \leq 9$$

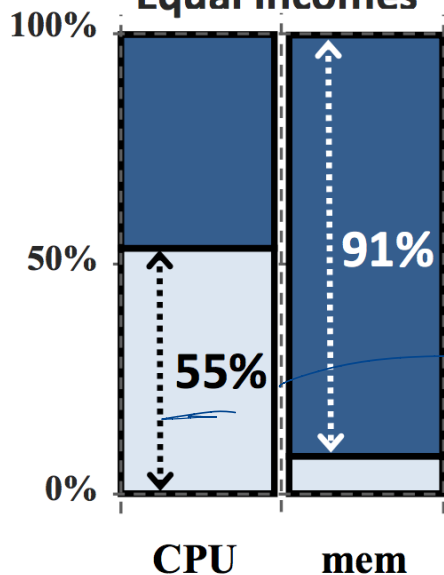
$$4x + 2y \leq 18$$

max x, y

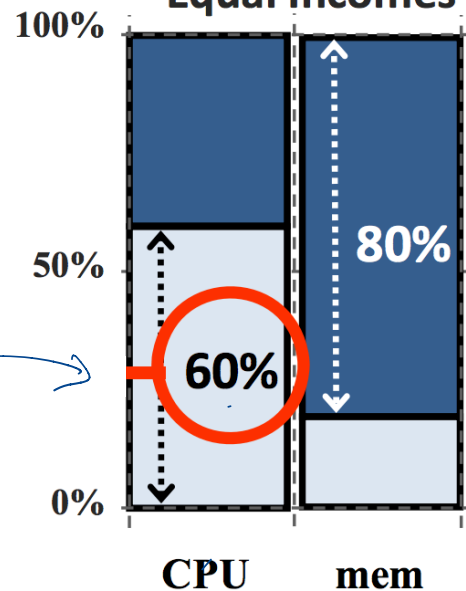
$$x = 3.6$$

$$y = 1.8$$

Competitive
Equilibrium from
Equal Incomes



Competitive
Equilibrium from
Equal Incomes



■ user 1
■ user 2

COMPARISON

Property	Allocation Policy		
	Asset	CEEI	DRF
Sharing Incentive		✓	✓
Strategy-proofness	✓		✓
Envy-freeness	✓	✓	✓
Pareto efficiency	✓	✓	✓
Single Resource Fairness	✓	✓	✓
Bottleneck Fairness		✓	✓
Population Monotonicity	✓		✓
Resource Monotonicity			

Table 2: Properties of Asset Fairness, CEEI and DRF.

SUMMARY

DRF: Dominant Resource Fairness

Allocation policy for scheduling

Provides multi-resource fairness → *generalizes max-min fairness*

Ensures sharing incentive, strategy proofness

DISCUSSION

<https://forms.gle/i7m7xXxKhtfvL9UD9>

Consider a system with 100 units of CPU, 50 units of memory and 200 units of disk. Consider three users with the following requirements

Alice (4 CPU, 1 memory, 1 disk)

Bob (1 CPU, 4 memory and 4 disk)

Carol (1 CPU, 2 memory and 16 disk)

List the dominant resource as defined in DRF for Alice, Bob and Carol

Alice : CPU 2/50

Bob : Memory 4/50

Carol : Disk 4/50

What would be the final task allocation in the given cluster for Alice, Bob and Carol ?

x, y, z num tasks of Alice, Bob, Carol

$$\frac{4x}{100} = \frac{4y}{50} = \frac{16z}{200}$$

$$4x + y + z \leq 100$$

$$x + 4y + 2z \leq 50$$

$$x + 4y + 16z \leq 200$$

replace x in terms
of y, z
& solve

$$x = 12.5$$

$$y = 6.25 = z$$

Every time Alice is
allocated ~~is~~ $8/200$

$$\text{Bob} = \text{Carol} = 16/200$$

Alice get two turns
for every 1 turn of
Bob, Carol

$$\vdots$$
$$\text{Alice} = 12$$

$$\text{Bob} = 6, \text{ Carol} = 6$$

either $(14, 6, 6)$ or $(12, 6, 7)$ ✓

What could be one workload / cluster scenario where DRF implemented on Mesos will NOT be optimal?

- If there aren't enough resources
if at least one task can run
- Heterogeneous tasks
[over time?] → Instantaneously fair?
- Locality pref?

Mesos:
 resource offer (9 CPU, 18 GB)
 → task <1 CPU, 3 GB> at Ex1

NEXT STEPS

$$R: \langle 21, 21 \rangle$$

$$D_1: \langle 3, 2 \rangle: D_2: \langle 4, 1 \rangle$$

3 tasks

3 tasks

$$\approx \frac{9}{21} = \frac{3}{7}$$

$$\frac{12}{21} = \frac{4}{7}$$

Next Week: Machine Learning Assignment 2 out!

Strategy proof: assuming rational actors

DRF starvation → one long running tasks
 & very highly contended cluster

DRF allocation

D1:	3/21	$\left \begin{array}{l} \frac{3x}{21} = \frac{4y}{21} \\ 3x + 4y \\ \leq 21 \\ ?? \end{array} \right.$
D2:	4/21	
D1:	6/21	
D2:	8/21	
D1:	9/21	
D2:	12/21	

optimization problem

$\equiv \max \min \langle \text{dom share} \rangle$
 subject to
 resource constraints

equate

$$\frac{3x}{21} = \frac{4y}{21}$$