Ground Rules

- Grading. You will be graded on the correctness as well as clarity of your solutions. You are required to prove any claims that you make. In particular, when you are asked to design an algorithm, you must argue its correctness and running time.

- Collaboration. You are allowed to discuss questions with other people in the class. However, you must solve and write your answers yourself without any help. You must also give explicit citations to any sources besides the textbook and class notes, including discussions with classmates. Solutions taken from external sources such as the WWW, even if cited, will receive no credit unless there is significant “value added”. In cases of doubt, you may be asked to explain your answer to the instructor and this will determine your grade.

- Lateness. Please see the class webpage for details on the lateness policy.

- This homework is due in one week. Start working early. Plan your work in such a way that you have the opportunity to put some problems on the back burner for a while and revisit them later. Good luck!

Problems

1. (Bottlenecks and Critical Edges.) In both the following parts, your algorithm should run in time polynomial in n, m and log F, where F is the max flow.
   
   (a) An edge in a flow network is called a bottleneck if increasing its capacity increases the max flow in the network. Give an efficient algorithm for finding all the bottlenecks in the network.

   (b) An edge in a flow network is called critical if decreasing its capacity decreases the max flow in the network. Give an efficient algorithm for finding all the critical edges in the network.

2. Problem 7.29 in the textbook (p. 433).

3. Problem 7.27 in the textbook (p. 431).
   
   Hint: Solve part (b) first. Then, for part (a), consider the same graph that you constructed to solve part (b). What is the mincut in this graph? Does this tell you something about the value of the max-flow and whether there exists a fair schedule?