CS 577: Introduction to Algorithms — Homework 10

Out: 12/06/10 — **Due: 12/13/10**

Ground Rules

- **Grading.** You will be graded on the correctness as well as clarity of your solutions. You are required to prove any claims that you make. In particular, when you are asked to design an algorithm, you must argue its correctness as well as running time. You may use without proof any theorems proven in class or in the textbook, as long as you state and cite them properly.
- **Collaboration.** You may work on and submit solutions in pairs.
- **Lateness.** Homework is due promptly at the start of class. Late homework will receive zero credit.
- **Extra credit questions.** Extra credit questions will not directly contribute to your score. However, they will be taken into account in the final grading and may improve your overall grade if your total score is close to the boundary between two grades. Furthermore they are fun to solve and improve your understanding of the course.
- Start working on your homework early. Plan your work in such a way that you have the opportunity to put some problems on the back burner for a while and revisit them later. Good luck!

Problems

1. **(3 points.)** Problem 8.1 in the textbook (p. 505).

2. **(6 points.)** Consider the following variant of 3-SAT: Given a 3-CNF formula, decide whether there exists an assignment to the variables of the formula such that every clause contains at least one literal that is true and at least one literal that is false. Show that this problem, *Not-All-Equal-SAT (NAE-SAT)*, is NP-complete.

   (Hint: Use Circuit-SAT in your reduction and follow the proof we did in class to reduce Circuit-SAT to 3-SAT.)

3. **(6 points.)** In the Max-Cut problem, our goal is to find a partition of the vertex set of an unweighted graph with the maximum number of edges going across. Thus, Max-Cut is the opposite of (global) Min-Cut. Surprisingly (as you will show in this question), the complexity of finding a Max-Cut is very different from that of finding a Min-Cut!

   Formally, the decision version of the problem is stated as follows:

   Given a graph G with edge set E and vertex set V, and a number k, does there exist a partition of the vertices into two sets V_1 and V_2 with $V_1, V_2 \neq \emptyset$ and $V_1 \cup V_2 = V$, such that $|(V_1 \times V_2) \cap E| \geq k$.

   Prove that the Max-Cut problem is NP-complete.

   (Hint: Use NAE-SAT from problem 2 in your reduction. You may assume that NAE-SAT is NP-complete.)

4. **(Extra credit.)** In this question we consider the special case of the Max-Cut problem with $k = |E|/2$. Surprisingly a $1/2$-Cut, that is, a cut with at least $|E|/2$ edges, always exists. Give a polynomial time algorithm for finding such a $1/2$-Cut.